The Common Core State Standards in Mathematics

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Macon, Georgia, 31 July 2012
The Common Core State Standards in Mathematics

Figure: Georgia school districts
Grade level where different states introduced addition and subtraction of fractions (CSMC)

<table>
<thead>
<tr>
<th>Grade</th>
<th>Addition &amp; Subtraction of Fractions</th>
</tr>
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<tbody>
<tr>
<td>$1^{\text{st}}$ grade</td>
<td>2 states</td>
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<td>$2^{\text{nd}}$ grade</td>
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<td>$3^{\text{rd}}$ grade</td>
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<td>$8^{\text{th}}$ grade</td>
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</table>
Mathematics topics intended at each grade by at least two thirds of 21 U.S. States.

On average a state would have 6–8 more topics per grade level in its complete curriculum.

Composite of high achieving countries

Note that topics are introduced and sustained in a coherent fashion, producing a clear upper-triangular structure.

<table>
<thead>
<tr>
<th>TOPIC</th>
<th>GRADE:</th>
<th>1</th>
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<th>3</th>
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<td>Number of topics covered by at least 67% of the A+ countries</td>
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<td>20</td>
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<td>Number of additional topics intended by A+ countries to complete a typical curriculum at each grade level</td>
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<td>5</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>3</td>
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</tbody>
</table>

- A+ countries from 1995 TIMSS
- Topics intended at each grade by at least two thirds of A+ countries.
- On average an A+ country would have 1–6 more topics per grade level in its complete curriculum.
COMMON CORE STATE STANDARDS FOR Mathematics
Led by two organizations of states, the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO)—not directed by the federal government.

- 48 states participated in the development of the standards
- 3-person writing team, 59-person work team, 19-person feedback group
- Based on progressions provided by the work team
- Rapid release and review by work team, feedback group, states, national organizations
- Finished Summer 2010, validated by a validation committee, states started making decisions whether or not to adopt the standards
Sources considered

- Standards of other countries and states
- Mathematics education research
- The structure of the subject
- National reports
- Commissioned research reviews and progressions
- Opinions of many groups and people
### Secondary One

#### Topic/Sub-topics

- **Algebraic representation and formulae**
  - Include:
    - using letters to represent numbers
    - interpreting notations:
      - \( ab \) as \( a \times b \)
      - \( \frac{a}{b} \) as \( a \div b \)
      - \( a^2 \) as \( a \times a \), \( a^3 \) as \( a \times a \times a \), \( a^2b \) as \( a \times a \times b \), ...
      - \( 3y \) as \( y - y - y \) or \( 3 \times y \)
      - \( \frac{3 \pm y}{5} \) as \( (3 \pm y) \div 5 \) or \( \frac{1}{5}(3 \pm y) \)
    - evaluation of algebraic expressions and formulae
    - translation of simple real-world situations into algebraic expressions
    - recognising and representing number patterns (including finding an algebraic expression for the \( n \)th term)

- **Algebraic manipulation**
  - Include:
Number and Operations

Expectations

In grades 6–8 all students should—

• work flexibly with fractions, decimals, and percents to solve problems;
• compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line;
• develop meaning for percents greater than 100 and less than 1;
• understand and use ratios and proportions to represent quantitative relationships;
• develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation;
• develop meaning for integers and represent and compare quantities with them.

Understand numbers, ways of representing numbers, relationships among numbers, and number systems
MATHEMATICS STANDARD ARTICULATED BY GRADE LEVEL

GRADE 6

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

### Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

| PO 1. | Express fractions as ratios, comparing two whole numbers (e.g., $\frac{3}{4}$ is equivalent to 3:4 and 3 to 4). |
| PO 2. | Compare two proper fractions, improper fractions, or mixed numbers. |
| PO 3. | Order three or more proper fractions, improper fractions, or mixed numbers. |
| PO 4. | Determine the equivalency between and among fractions, decimals, and percents in contextual situations. |
What should standards look like?

- NCTM: 14 standards + 7 pages of narrative for Grades 6–8 Number and Operations
- State: 82 standards + 0 pages of narrative for Grade 6 Number and Operations
What should standards look like?

- Flat lists of performance objectives of even grain size, designed to be delivered into the hands of assessment writers without further human intervention?
- Guides to curriculum? A list of things that teachers have to “cover”?
- A statement of what we want our children to understand and be able to do?
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- Guides to curriculum? A list of things that teachers have to “cover”?
- A statement of what we want our children to understand and be able to do?
Design principles for the Common Core

Focus attending to fewer topics in greater depth at any given grade level, giving teachers and students time to complete that grade’s learning.

Coherence attending to the structure of mathematics and the natural pathways through that structure, where “natural” means taking into account both the imperatives of logic and the imperatives of cognitive development in designing the sequence of ideas.

Rigor balancing conceptual understanding, procedural fluency, and meaningful applications of mathematics.
Standards for Mathematical Content, K–8

<table>
<thead>
<tr>
<th>Grade</th>
<th>Math Topics</th>
</tr>
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<tbody>
<tr>
<td>K</td>
<td>Number and Operations in Base Ten, Operations and Algebraic Thinking, Counting and Cardinality</td>
</tr>
<tr>
<td>1</td>
<td>Number and Operations—Fractions</td>
</tr>
<tr>
<td>2</td>
<td>Geometry, Measurement and Data</td>
</tr>
<tr>
<td>3</td>
<td>Geometry, Measurement and Data</td>
</tr>
<tr>
<td>4</td>
<td>Statistics and Probability, The Number System, Expressions and Equations</td>
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<tr>
<td>5</td>
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<td>6</td>
<td>Statistics and Probability, The Number System, Expressions and Equations</td>
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<td>7</td>
<td>Statistics and Probability, The Number System, Expressions and Equations</td>
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<tr>
<td>8</td>
<td>Functions</td>
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</tbody>
</table>

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**Content standards** define what students should understand and be able to do

**Clusters** are groups of related standards

**Domains** are larger groups that progress across grades

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**Number and Operations in Base Ten**

3.NBT

**Use place value understanding and properties of operations to perform multi-digit arithmetic.**

1. Use place value understanding to round whole numbers to the nearest 10 or 100.

2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., $9 \times 80$, $5 \times 60$) using strategies based on place value and properties of operations.
Standards for Mathematical Content, High School

- Number and Quantity
- Algebra
  - Seeing Structure in Expressions
  - Arithmetic with Polynomials and Rational Expressions
  - Creating Equations
  - Reasoning with Equations and Inequalities
- Functions
  - Interpreting Functions
  - Building Functions
  - Linear, Quadratic, and Exponential Models
  - Trigonometric Functions
- Modeling
- Geometry
- Statistics and Probability
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning
Standards not in the number operations domains often support that work:

2.MD.10

Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems.
Taking focus seriously: the case of patterns

- Trickle of pattern standards in elementary school
  - **3.OA.9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
  - **4.OA.5.** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
  - **5.OA.3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

- In middle school, further preparation for functions is provided in the domains Ratios and Proportional Relationships and Expressions and Equations.
- The function concept finally makes its appearance in its own domain in Grade 8, and becomes a major conceptual category in high school.

[math.arizona.edu/~wmc The Common Core State Standards in Mathematics]
Focus in Grade 3

Operations and Algebraic Thinking
- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations — Fractions
- Develop understanding of fractions as numbers.

Measurement and Data
- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry
- Reason with shapes and their attributes.
### Ratios and Proportional Reasoning
- Analyze proportional relationships and use them to solve real-world and mathematical problems.

### The Number System
- Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.

### Expressions and Equations
- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

### Geometry
- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area and volume.

### Statistics and Probability
- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.
Say goodbye to . . .

- The love affair with patterns
- Mindless medians
- Least common denominators
- Factor trees
- Reducing fractions
- “Simplifying”

\[ p^{-\frac{1}{4}} \]

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The Common Core State Standards in Mathematics
There is a world of difference between a student who can summon a mnemonic device to expand a product such as \((a + b)(x + y)\) and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics . . .

— CCSSM, p. 5
Focus in high school

- **Algebra**
  - Seeing Structure in Expressions
  - Arithmetic with Polynomials and Rational Expressions
  - Creating Equations
  - Reasoning with Equations and Inequalities

- **Functions**
  - Interpreting Functions
  - Building Functions
  - Linear, Quadratic, and Exponential Models
  - Trigonometric Functions

- **Modeling**
  - ...

- **Geometry**
  - ...

- **Statistics and Probability**
  - ...
Taking focus seriously means delaying favored topics until their time, which will be a difficult shift for the educational system in the U.S.

In high school, the subject matter focus broadens: algebra, functions, geometry, statistics, probability, modeling.

Focus in high school means not so much a small number of topics as a concentration of skills and practice into a small number of underlying principles.
A Grecian Urn
Coherence

A Grecian Urn

Standards for a Grecian Urn

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The Common Core State Standards in Mathematics
## Understand place value (Grade 2)

**2.NBT.1.** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

- **a.** 100 can be thought of as a bundle of ten tens—called a “hundred.”

- **b.** The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

**2.NBT.2.** Count within 1000; skip-count by 5s, 10s, and 100s.

**2.NBT.3 . . .**

**2.NBT.4 . . .**
Coherence flows from focus

- Operations and Algebraic Thinking
  - Expressions and Equations
    - The Number System
      - Algebra

F1

- Number and Operations—Base Ten
  - Number and Operations—Fractions

F2

K 1 2 3 4 5 6 7 8 High School
Understanding 1.OA.7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

Fluency 2.OA.2. Fluently add and subtract within 20 using mental strategies.¹ By end of Grade 2, know from memory all sums of two one-digit numbers.

Applications 2.MD.5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

¹See standard 1.OA.6 for a list of mental strategies.
“Understand”

- Understand and apply properties of operations and the relationship between addition and subtraction (Grade 2)
- Understand concepts of area and relate area to multiplication and to addition (Grade 3)
- Understand ratio concepts and use ratio reasoning to solve problems (Grade 6)
- Understand congruence and similarity using physical models, transparencies, or geometry software (Grade 8)
- Understand solving equations as a process of reasoning and explain the reasoning (High School)
- Understand and evaluate random processes underlying statistical experiments (High School)
Fluency requirements

- Single digit addition facts in Grade 2
- Single digit multiplication facts in Grade 3
- Addition and subtraction within 1000 in Grade 3
- Addition and subtraction using standard algorithm in Grade 4
  (addition and subtraction starts in Grade 1)
- Multiplication using standard algorithm in Grade 5
  (multiplication and division starts in Grade 3)
- Division using standard algorithm in Grade 6
- All operations on decimals using standard algorithm in Grade 6
- Solving linear equations with rational coefficients in Grade 7
7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

SMP.4. Model with mathematics.
The paper clip below is just over 4 cm long.

How many paper clips like this may be made from a straight piece of wire 10 meters long?
Focus, coherence, and rigor work together

A Grecian Urn

It's pretty clear what the focus is here; nobody is going to say this is all about grape leaves. But the grape leaves are important; details large and small make this a coherent work of art. Craftsmanship shows in the balance between art, technique, and story. Rigor is the balance that produces students who can use what they know; who show craft in using mathematics.

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The Common Core State Standards in Mathematics
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Craftsmanship shows in the balance between between art, technique, and story.

Rigor is the balance that produces students who can use what they know; who show craft in using mathematics.
The danger of checklists

Standards cannot be read individually, in isolation.

Standards for a Grecian Urn
The danger of checklists

Standards for a Grecian Urn

- Standards cannot be read individually, in isolation.
- The cluster headings are part of the standards.
The danger of checklists

Standards cannot be read individually, in isolation.
The cluster headings are part of the standards.
The domain headings are part of the standards.

Standards for a Grecian Urn
Standards cannot be read individually, in isolation.

The cluster headings are part of the standards.

The domain headings are part of the standards.

The connections across and within grade levels are part of the standards.

Standards for a Grecian Urn
The danger of checklists

Standards cannot be read individually, in isolation.

The cluster headings are part of the standards.

The domain headings are part of the standards.

The connections across and within grade levels are part of the standards.

The structure is the standards.

Standards for a Grecian Urn
Tools

- Progressions
  (http://ime.math.arizona.edu/progressions/)
- Illustrative Mathematics Project
  (http://illustrativemathematics.org).
- My blog, Tools for the Common Core,
  http://commoncoretools.wordpress.com
- Institute for Mathematics & Education
  (http://ime.math.arizona.edu/commoncore/)
### Comparison of CCSS with A+ composite

#### The number of extra topics per grade level in CCSS is comparable with A+ countries.
Look for and express regularity in repeated reasoning

For every 5 cups grape juice, mix in 2 cups peach juice.

<table>
<thead>
<tr>
<th>x cups grape</th>
<th>y cups peach</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{2}{5}$</td>
</tr>
<tr>
<td>2</td>
<td>$2 \cdot \frac{2}{5}$</td>
</tr>
<tr>
<td>3</td>
<td>$3 \cdot \frac{2}{5}$</td>
</tr>
<tr>
<td>4</td>
<td>$4 \cdot \frac{2}{5}$</td>
</tr>
</tbody>
</table>

$y = \frac{2}{5}x$
For each 1 unit you move to the right, move up $\frac{2}{5}$ of a unit.

When you go 2 units to the right, you go up $2 \cdot \frac{2}{5}$ units.
When you go 3 units to the right, you go up $3 \cdot \frac{2}{5}$ units.
When you go 4 units to the right, you go up $4 \cdot \frac{2}{5}$ units.
When you go $x$ units to the right, you go up $x \cdot \frac{2}{5}$ units.

Starting from $(0, 0)$, to get to a point $(x, y)$ on the graph, go $x$ units to the right, so go up $x \cdot \frac{2}{5}$ units.

Therefore $y = x \cdot \frac{2}{5}$
Do the equations have a solution? Give a reason for your answer that does not depend on solving the equation.

1. \[\frac{t + 2}{3 + t} = 1\]
2. \[\frac{3 + t}{3 - t} = 1\]
3. \[\frac{t - 2}{2 - t} = 1\]
Shifts in the Standards

A sequence of Grade 3 tasks about placing fractions on the number line, arranged in order of increasing sophistication.

- Locating Fractions Less Than One on the Number Line (http://illustrativemathematics.org/illustrations/168)
- Locating Fractions Greater than One on the Number Line (http://illustrativemathematics.org/illustrations/173)
- Closest to 1/2 (http://illustrativemathematics.org/illustrations/171)
- Find 1 (http://illustrativemathematics.org/illustrations/169)
- Find 2/3 (http://illustrativemathematics.org/illustrations/170)
- Which is Closer to 1? (http://illustrativemathematics.org/illustrations/172)