Georgia Standards of Excellence Monitoring Guidance Document

Mathematics

Foundations of Algebra

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Foundations of Algebra is a first year high school mathematics course option aimed at students who have reported low standardized test performance in prior grades and/or have demonstrated significant difficulties in previous mathematics classes. This course will provide many opportunities to revisit and expand the understanding of foundational algebra concepts, will employ diagnostic means to offer focused interventions, and will incorporate varied instructional strategies to prepare students for required high school mathematics courses.

In order to successfully implement Foundations of Algebra, and, hence, the successful completion of the course by students who need this extra mathematics help, this guidance document has been written with input from K – Postsecondary Georgia educators. The document details the expectations of the school, district, and state to ensure the following: fidelity of implementation, in-course student success, post-course student success, data-driven course modification process.

As is stated in the Georgia Mathematics Graduation Requirement document... "Successful preparation for both postsecondary education and employment requires learning the same rigorous mathematics content and skills. No longer do students planning to enter the workplace after high school need a different and less rigorous mathematics curriculum than those planning to go to college. (Achieve, Inc., 2004)." With careful planning and monitoring, Foundations of Algebra can help students complete the high school mathematics courses needed to be prepared for postsecondary education and employment.

This Monitoring Guidance Document is separated into four parts:

- Teacher's Student Success Plan,
- Administrator's Student Success Plan
- District's Student Success Plan
- State's Student Success Plan

Teacher's Student Success Plan

The **Teacher's Student Success Plan** suggests the following:

- An overarching monitoring conducted with Individual Knowledge Assessment of Number (IKAN), Global Strategy Stage Assessment (GloSS) data, and the course pre- and post-assessment available through the Georgia Online Formative Assessment Resource (GOFAR).
- The targeted monitoring consists of the module pre- and post- assessments and the formative assessment lessons embedded in the modules.
- Ongoing monitoring can be used every day in order to adapt teaching and learning to meet immediate learning needs minute by minute and day to day.
- A Teacher and Student Pre-Post Survey of Knowledge and Skills can be used at the beginning and end of the course in an effort to see a positive change in students' attitudes towards mathematics. (Appendix A)

In addition to the Student Success Plan, the teacher of Foundations of Algebra should utilize deliberate, research-based instructional practices. Instruction and assessment should include the appropriate use of manipulatives and technology. Mathematics concepts should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used, where appropriate, in the context of realistic experiences.

The Standards for Mathematical Practice will provide the foundation for instruction and assessment. The following are thoughts to keep in mind when teaching the class:

- Grading practices should emphasize mastery of standards through the frequent use of aligned quizzes and tests, both formative and summative.
- Continual progress monitoring should be used to assess and diagnose each student's strengths and weaknesses based on the standards of the course and to provide appropriate interventions.
- Opportunities should be provided for students to review content with a focus on standards not previously mastered.
- Attention needs to be given to the vocabulary of the current course. The academic language of mathematics should be explicitly taught as concepts are introduced and reinforced.
- Proven strategies for success in mathematics should be utilized on a daily basis. Students should be engaged in doing mathematics, explaining their thinking, and justifying their work. Multiple representations of concepts (tables, charts, graphs, verbal descriptions) should be used as often as possible.
- There should be strong emphasis on building a positive disposition toward learning mathematics.

Teachers will be able to begin the course with a course pre-assessment and will be able to analyze student growth with a post-assessment at the end of the course. Each module will include a pre- and post- assessment as well as spot diagnostics and appropriate intervention mini-lessons. The assessments include test item commentaries and a standards alignment document that reflects the depth of knowledge level of each test item.

Foundations of Algebra teachers will be able to take advantage of an instruction manual to assist them in the continuing administration of the IKAN. This screener is part of the criteria for placement in this course. IKAN is easy to use and will help pinpoint a student's mathematics misconceptions. In a February 25th webinar, the state math team provided the information and simple step-by-step instructions. This recording is available for reference using the link below.

http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/Mathematics.aspx

The goal is to support instruction in order to assist students in the successful completion of Foundations of Algebra. Assignments, quizzes and tasks should be aligned to the standards in the course. Individuals should be given multiple opportunities to show mastery of the content, including opportunities to demonstrate mastery of material; additional practice or re-testing can be beneficial. The value of formative assessment and feedback cannot be overstated. Continuous progress monitoring with both feedback and commentary is essential in this course. Students should not feel pressure to "make grades" in this class as much as they should be motivated and encouraged to master standards. Documented continuous communication with students on an individual basis is the most appropriate way to maintain records of progress.

Administrator's Student Success Plan

As the instructional leaders of the school, administrator support is the key to any curriculum initiative. A short informational video is available for administrators and counselors at http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/Mathematics.aspx

Some possible suggestions for following the progress of students in Foundation of Algebra would be

- Student Interview Questions (Appendix B)
- Surveys of Instruction Practice from the Teacher Keys Effectiveness System
- Professional development opportunities based on the strategies for formative assessment http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/GeorgiaFIP.aspx
- Data talks with walkthroughs using the Standards for Mathematical Practice (Appendix C)
- Analysis of IKAN administrations throughout the year

Whenever possible, school support personnel or support agencies can conduct periodic check-ins with districts/schools to discuss successes and concerns. As resources permit, focus groups can meet periodically to discuss the implementation of Foundations of Algebra and the success of the students enrolled in the course.

District's Student Success Plan

Districts must adhere to the eligibility requirements to receive state funding for the Foundations of Algebra course. Local school districts will have the flexibility to utilize Foundations of Algebra resources to accommodate specific student needs both prior to enrollment in ninth grade or in high school mathematics support courses.

Suggestions for district analysis of the Foundations of Algebra Course uses the following timeline:

During the Year

- Establishment of Professional Learning Communities with Teacher Focus Groups
- Analysis of Common Module Assessment Results
- Examination of District Benchmark Assessment Data, if available
- Implementation of Student Standards Progress
- Monitoring of IKAN and GloSS Growth Data (assessment administered throughout the school year)

At the End of the Course

- Analysis of Grade Distributions
- Examination of Student Growth

At the End of Coordinate Algebra

- Evaluation of each specific cohort's End of Course (EOC) data for Coordinate Algebra or Algebra I to determine success

At the End of Analytic Geometry

- Evaluation of each specific cohort's EOC data for Analytic Geometry or Geometry to determine success
- Comparisons of the PSAT performance of students who completed Foundations of Algebra to those who started high school with Coordinate Algebra or Algebra I

At the End of High School Matriculation

- Analysis of graduation rate for each specific cohort beginning with the 2019 cohort

State's Student Success Plan

The State Board of Education (SBOE) on March 12, 2015, approved requirements for a four-step eligibility process for student placement in the Foundations of Algebra course.

Step I: Middle school administrators/teachers will identify students who **Did Not Meet** expectations on at least two of the following three assessments: 5th, 6th, and 7th grade CRCT. These are students who may have significant gaps in their mathematics achievement.

Step II: Middle schools will administer Individual Knowledge Assessment of Number (IKAN) diagnostic to students identified in Step I. Only students who score at IKAN Stage 5 (equivalent to 4th grade mathematics) or below will be eligible for Foundations of Algebra (No costs will be incurred by the state or districts.).

Step III: During the scheduling process, high schools will enroll students identified as eligible in Step II in Foundations of Algebra.

Step IV: By the October 2015 FTE count date, districts implementing the Foundations of Algebra course will report the GTID, student name, the IKAN score of every student assessed in Step III, and an indicator of whether the student was enrolled in the course to the GaDOE.

The Georgia Department of Education will compare the Algebra I or Coordinate Algebra performance of students who completed the Foundations of Algebra course with the performance of students who did not meet the Individual Knowledge Assessment of Number (IKAN) eligibility requirement and thus were enrolled directly into Algebra I or Coordinate Algebra. The analysis of data collected in Step IV, as indicated in the attachment, will allow the Georgia Department of Education to determine student success and needed adjustments to eligibility requirements and instructional resources.

APPENDIX A: TEACHER AND STUDENT PRE-POST SURVEY OF KNOWLEDGE AND SKILLS

Teacher Pre-Post Survey of Knowledge and Skills

Standards for Mathematical Practice: Teacher Behaviors	Please Circle Your Current Instructional Practices
1. Make sense of problems and persevere in solving them. *Teachers are:* • Providing rich problems aligned to the standards (TKES 1) • Providing appropriate time for students to engage in the productive struggle of problem solving (TKES 8) *Examples of questions teachers ask:* • What information do you have? What do you need to find out? What do you think the answer might be? • Can you draw a picture? How could you make this problem easier to solve? • How is ___'s way of solving the problem like/different from yours? Does your plan make sense? Why or why not? • What tools/manipulatives might help you? What are you having trouble with? How can you check this?	**Standard 1 is part of my instructional framework—I allow students to plan a solution pathway and I give them time to engage in and adjust problem solving as needed.** **Always** **Almost Always** **Sometimes** **Rarely**
2. Reason abstractly and quantitatively. *Teachers are:* • Providing a variety of problems in different contexts that allow students to arrive at a solution in different ways (TKES 4) • Using think aloud strategies as they model problem solving (TKES 3) • Attentively listening for strategies students are using to solve problems (TKES 5) *Examples of questions teachers ask:* • What does the number ____ represent in the problem? How can you represent the problem with symbols and numbers? • Can you make a chart, table or graph?	**Standard 2 is part of my instructional framework—I make sure my students can generalize concepts based on what they observe.** **Always** **Almost Always** **Sometimes** **Rarely**
3. Construct viable arguments and critique the reasoning of others. *Teachers are:* • Posing tasks that require students to explain, argue, or critique (TKES 8) • Providing many opportunities for student discourse in pairs, groups, and during whole group instruction (TKES 4) *Examples of questions teachers ask:* • Why or why not? How do you know? Can you explain that? Do you agree? • How is your answer different than _____'s? What math language will help you prove your answer? • What examples could prove or disprove your argument? What questions do you have for ____?	**Standard 3 is part of my instructional framework—I provide my students the opportunity to make conjectures with support and reason through the use of objects, drawings, diagrams and actions.** **Always** **Almost Always** **Sometimes** **Rarely**

Standards for Mathematical Practice: Teacher Behaviors	Please Circle Your Current Instructional Practices
4. Model with mathematics. *Teachers are:* • Providing opportunities for students to solve problems in real life contexts (TKES 3) • Identifying problem solving contexts connected to student interests (TKES 4) *Examples of questions teachers ask:* • Can you write a number sentence to describe this situation? What do you already know about solving this problem? • What connections do you see? Why do the results make sense? Is this working or do you need to change your model?	**Standard 4 is part of my instructional framework—I require students to put mathematics in the context of real world situations and identify those relationships.** **Always** **Almost Always** **Sometimes** **Rarely**
5. Use appropriate tools strategically. *Teachers are:* • Making a variety of tools readily accessible to students and allowing them to select appropriate tools for themselves (TKES 3) • Helping students understand the benefits and limitations of a variety of math tools (TKES 8) *Examples of questions teachers ask:* • How could you use manipulatives or a drawing to show your thinking? • Which tool/manipulative would be best for this problem? What other resources could help you solve this problem?	**Standard 5 is part of my instructional framework—I make it possible and encourage my students to use familiar, grade appropriate tools and I make sure they understand the benefits and limitations of each tool.** **Always** **Almost Always** **Sometimes** **Rarely**
6. Attend to precision. *Teachers are:* • Explicitly teaching mathematics vocabulary (TKES 1) • Insisting on accurate use of academic language from students (TKES 8) • Modeling precise communication (TKES 10) • Requiring students to answer problems with complete sentences, including units (TKES 10) • Providing opportunities for students to check the accuracy of their work (TKES 5) *Examples of questions teachers ask:* • What does the word ____ mean? Explain what you did to solve the problem. • Compare your answer to _____'s answer What labels could you use? • How do you know your answer is accurate? Did you use the most efficient way to solve the problem?	**Standard 6 is part of my instructional framework—I require my students to communicate precisely, to calculate accurately and efficiently, and to show flexibility with strategies.** **Always** **Almost Always** **Sometimes** **Rarely**
7. Look for and make use of structure. *Teachers are:* • Providing sense making experiences for all students (TKES 2) • Allowing students to do the work of using structure to find the patterns for themselves rather than doing this work for students (TKES 8) *Examples of questions teachers ask:* • Why does this happen? How is ____ related to ____? Why is this important to the problem? • What do you know about ____ that you can apply to this situation? How can you use what you know to explain why this works?	**Standard 7 is part of my instructional framework—I require my students to look closely at patterns and structure and to identify and understand the make-up and inclusion of number properties.** **Always** **Almost Always** **Sometimes** **Rarely**

Standards for Mathematical Practice: Teacher Behaviors	Please Circle Your Current Instructional Practices
• What patterns do you see?	
8. Look for and express regularity in repeated reasoning. *Teachers are:* • Providing sense making experiences for all students (TKES 2) • Allowing students to do the work of finding and using their own shortcuts rather than doing this work for students (TKES 8) *Examples of questions teachers ask:* • What generalizations can you make? Can you find a shortcut to solve the problem? • How would your shortcut make the problem easier? How could this problem help you solve another problem?	**Standard 8 is part of my instructional framework—I require my students to continually evaluate the reasonableness of intermediate results.** **Always** **Almost Always** **Sometimes** **Rarely**

Student Pre-Post Survey of Knowledge and Skills

Standards for Mathematical Practice: Student Behaviors	Please Circle Your Current Learning Practices
1. . Make sense of problems and persevere in solving them. *Students are:* • Working and reading rich problems carefully (TKES 3) • Drawing pictures, diagrams, tables, or using objects to make sense of the problem (TKES 3) • Discussing the meaning of the problem with classmates (TKES 4) • Making choices about which solution path to take (TKES 5) • Trying out potential solution paths and making changes as needed (TKES 8) • Checking answers and making sure solutions are reasonable and make sense (TKES 6) • Exploring other ways to solve the problem (TKES 8) • Persisting in efforts to solve challenging problems, even after reaching a point of frustration. (TKES 8)	**Standard 1 is part of my learning framework—I am allowed to try different ways to solve problems and can discuss my answers with my classmates. I am allowed to correct my errors when solving problems and will keep trying to find the answer. (I can solve problems without giving up.)** **Always Almost Always Sometimes Rarely**
2. Reason abstractly and quantitatively. *Students are:* • Using mathematical symbols to represent situations (TKES 3) • Taking quantities out of context to work with them (decontextualizing) (TKES 3) • Putting quantities back in context to see if they make sense (contextualizing) (TKES 3) • Considering units when determining if the answer makes sense in terms of the situation (TKES 3)	**Standard 2 is part of my learning framework—I can decide if my answers to questions make sense based on a given situation.** **Always Almost Always Sometimes Rarely**
3. Construct viable arguments and critique the reasoning of others. *Students are:* • Making and testing conjectures (TKES 8) • Explaining and justifying their thinking using words, objects, and drawings (TKES 6) • Listening to the ideas of others and deciding if they make sense (TKES 4) • Asking useful questions (TKES 3) • Identifying flaws in logic when responding to the arguments of others (TKES 4) • Elaborating with a second sentence (spontaneously or prompted by the teacher or another student) to explain their thinking and connect it to their first sentence. (TKES 8) • Talking about and asking questions about each other's thinking, in order to clarify or improve their own mathematical understanding. (TKES 4) • Revising their work based upon the justification and explanations of others. (TKES 8)	**Standard 3 is part of my learning framework—I am able to support the answers to problems using objects, drawings, diagrams, and explanations.** **Always Almost Always Sometimes Rarely**
4. Model with mathematics. *Students are:* • Using mathematical models (i.e. formulas, equations, symbols) to solve problems in the world (TKES 3) • Using appropriate tools such as objects, drawings, and tables to create mathematical models (TKES 3)	**Standard 4 is part of my learning framework—I can apply mathematics in the context of real world situations and identify those relationships.** **Always Almost Always Sometimes Rarely**

Standards for Mathematical Practice: Student Behaviors	Please Circle Your Current Learning Practices
• Making connections between different mathematical representations (concrete, verbal, algebraic, numerical, graphical, pictorial, etc.) (TKES 8) • Checking to see if an answer makes sense within the context of a situation and changing the model as needed (TKES 8)	

5. Use appropriate tools strategically.
Students are:
• Using technological tools to explore and deepen understanding of concepts (TKES 3)
• Deciding which tool will best help solve the problem. Examples may include: (TKES 3)
  ○ Calculator
  ○ Concrete models
  ○ Digital Technology
  ○ Pencil/paper
  ○ Ruler, compass, protractor
• Estimating solutions before using a tool (TKES 3)
• Comparing estimates to solutions to see if the tool was effective (TKES 3) How could you use manipulatives or a drawing to show your thinking?
• Which tool/manipulative would be best for this problem? What other resources could help you solve this problem? (TKES 3)
	Standard 5 is part of my learning framework—I am familiar with appropriate mathematical tools (calculators, technology, and geometry tools) and can use them to solve problems.
	
	Always **Almost Always** **Sometimes** **Rarely**

6. Attend to precision. *Students are:* • Communicating precisely using clear language and accurate mathematics vocabulary (TKES 1) • Deciding when to estimate or give an exact answer (TKES 1) • Calculating accurately and efficiently, expressing answers with an appropriate degree of precision (TKES 1) • Using appropriate units; appropriately labeling diagrams and graphs (TKES 1)	**Standard 6 is part of my learning framework—I can communicate using mathematical vocabulary precisely and can check my work.** **Always** **Almost Always** **Sometimes** **Rarely**
7. Look for and make use of structure. *Students are:* • Finding structure and patterns in numbers (TKES 1) • Finding structure and patterns in diagrams and graphs (TKES 1) • Using patterns to make rules about math (TKES 1) • Using these math rules to help them solve problems (TKES 1)	**Standard 7 is part of my learning framework—I can use what I know about math to solve new problems.** **Always** **Almost Always** **Sometimes** **Rarely**

Standards for Mathematical Practice: Student Behaviors	Please Circle Your Current Learning Practices
8. Look for and express regularity in repeated reasoning. *Students are:* • Looking for patterns when working with numbers, diagrams, tables, and graphs (TKES 1) • Observing when calculations are repeated (TKES 8) • Using observations from repeated calculations to take shortcuts(TKES 8)	**Standard 8 is part of my learning framework— I look closely for patterns and structure in problems, graphs, or diagrams in order to solve similar problems.** **Always** **Almost Always** **Sometimes** **Rarely**

(*Created by Dr. Pamela Kirkpatrick, Director, North Georgia Learning Resource System)*

APPENDIX B: STUDENT INTERVIEW QUESTIONS

The purpose of the student interview protocol is to gather formative data from students about their expectations and experiences for learning mathematics. The sample questions may be used to obtain information needed to follow the progress of students in the Foundation of Algebra course. Feel free to make up your own. You do not need to ask every sample question. Allow the interview to flow naturally similar to a conversation. Look for opportunities to ask follow-up questions when appropriate.

Considerations before starting:

- Select a sample size that is representative of the diversity within the school from the Foundations of Algebra course.
- Ask students to bring student work products.
- Take anecdotal notes during the interview.

Classroom Environment

1. What do you like the most about the Foundations of Algebra class? Why?
2. What do you least like about the Foundations of Algebra class? Why?
3. Do you feel the class will help you be successful in high school? How?
4. If you could make one improvement to the class, what would you do? Why?

Curriculum

1. How are the learning goals communicated in the Foundations of Algebra class?
2. Does your teacher explain the learning goals on a daily basis?
3. Are you being challenged in your classes? How?

Assessment

1. What kind of tests do you take? Beside tests, how do you know when you are doing well in the class?
2. Does your teacher ever change the way he/she teaches when students are having problems with learning?
3. Does your teacher ask you to explain your answer? If yes, are you able to do this?
4. Does your teacher know when you understand something and when you do not? How does he/she know?

Instruction

1. Do your teachers have different ways to teach things so that you understand? What are some examples?
2. What happens when you need extra help in your class?
3. What are different ways that you can get help when you do not understand?
4. Does your teacher introduce a new topic by connecting it to things you already know? Can you give an example?
5. Does your teacher explain why it is important for you to learn a lesson? What are some of the reasons that your teacher gives?

Formative Data

1. Does your teacher give you helpful feedback on the work that you turn in? If yes, how?
2. Does your teacher check for understanding? If so, how?

(*Adapted from the Georgia Assessment of Performance on School Standards Analysis: Focus Group Interview Questions-Students)*

APPENDIX C: STANDARDS FOR MATHEMATICAL PRACTICE LOOK FORS

Student Behaviors
1. Make sense of problems and persevere in solving them. *Students are:* • Working and reading rich problems carefully (TKES 3) • Drawing pictures, diagrams, tables, or using objects to make sense of the problem (TKES 3) • Discussing the meaning of the problem with classmates (TKES 4) • Making choices about which solution path to take (TKES 5) • Trying out potential solution paths and making changes as needed (TKES 8) • Checking answers and making sure solutions are reasonable and make sense (TKES 6) • Exploring other ways to solve the problem (TKES 8) • Persisting in efforts to solve challenging problems, even after reaching a point of frustration. (TKES 8)
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4. Model with mathematics. *Students are:* • Using mathematical models (i.e. formulas, equations, symbols) to solve problems in the world (TKES 3) • Using appropriate tools such as objects, drawings, and tables to create mathematical models (TKES 3) • Making connections between different mathematical representations (concrete, verbal, algebraic, numerical, graphical, pictorial, etc.) (TKES 8) • Checking to see if an answer makes sense within the context of a situation and changing the model as needed (TKES 8)

5. Use appropriate tools strategically.
Students are:
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  ○ Calculator
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• Estimating solutions before using a tool (TKES 3)
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7. Look for and make use of structure. *Teachers are:* • Providing sense making experiences for all students (TKES 2) • Allowing students to do the work of using structure to find the patterns for themselves rather than doing this work for students (TKES 8) *Teachers ask:* • Why does this happen? How is ____ related to ____? Why is this important to the problem? • What do you know about ____ that you can apply to this situation? How can you use what you know to explain why this works? • What patterns do you see?
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(Created by Dr. Pamela Seda)