Advanced Mathematical Decision Making

MATHEMATICS KEY COMPETENCIES & COURSE STANDARDS

WITH LEARNING OBJECTIVES IN PROGRESSION ORDER

GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

Advanced Mathematical Decision Making

Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Advanced Mathematical Decision Making Course, which is a fourth mathematics course option in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

Advanced Mathematical Decision Making (AMDM) is designed to follow the completion of Algebra II, Advanced Algebra, Accelerated Geometry B/Algebra II or Accelerated Analytic Geometry B/Advanced Algebra. The course will give students further experiences with statistical information and summaries, methods of designing and conducting statistical studies, an opportunity to analyze various voting processes, modeling of data, basic financial decisions, and use network models for making informed decisions.

Instruction and assessment should include the appropriate use of manipulatives and technology. Topics should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used, where appropriate, in the context of realistic phenomena.

Prerequisite:

This course is designed for students who have successfully completed *Advanced Algebra / Algebra II, Accelerated Geometry B/Algebra II* ***or*** *Accelerated Analytic Geometry B/Advanced Algebra*.

Georgia's K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL (QUANTITATIVE) REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)
QUANTITATIVE & PROPORTIONAL REASONING (QPR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

Advanced Mathematical Decision Making

The thirteen course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
AMDM.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.
AMDM.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
AMDM.QPR.2: Make decisions and solve problems using ratios, rates, and percents in a variety of real-world applications.
AMDM.QPR.3: Make predictions by analyzing averages and indices of large data sets through investigations of real-world contexts.
AMDM.PAR.4: Develop methods or algorithms to analyze discrete situations.
AMDM.PR.5: Analyze the chances for success or failure in order to make decisions.
AMDM.PR.6: Model strategic interaction among rational decision-makers.
AMDM.DSR.7: Conduct investigative research to solve real-life problems and answer statistical investigative questions involved in business and financial decision-making.
AMDM.PAR.8: Create and analyze mathematical models to make decisions related to earning, investing, spending, and borrowing money.
AMDM.FGR.9: Use functions to model problem situations in both discrete and continuous relationships.
AMDM.GSR.10: Use functions to model problem situations in both discrete and continuous relationships.
AMDM.PAR.11: Use functions to model problem situations in both discrete and continuous relationships.
AMDM.PAR.12: Make informed decisions and solve problems with a variety of network models in quantitative situations.

Advanced Mathematical Decision Making

MATHEMATICAL MODELING		
AMDM.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AMDM.MM.1.1	Explain contextual, mathematical problems using a mathematical model.	***Fundamentals*** • Students should be provided with opportunities to learn mathematics in the context of real-life problems. • Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).
AMDM.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.	***Fundamentals*** • Students should be able to use the content learned in this course to create mathematical models to explain real-life phenomena.
AMDM.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation.	
AMDM.MM.1.4	Use relevant information to create various mathematical representations and structures to solve real-life problems.	***Strategies and Methods*** • Students should be provided opportunities to use various mathematical representations and structures to illustrate and solve culturally relevant problems.

QUANTITATIVE & PROPORTIONAL REASONING *– Ratios, Rates, & Percents*		
AMDM.QPR.2: Make decisions and solve problems using ratios, rates, and percents in a variety of real-world applications.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AMDM.QPR.2.1	Apply proportions, ratios, rates, and percentages to various settings, including business, media, and consumerism.	***Examples*** • Aspect ratios – TV, films, and special effects
AMDM.QPR.2.2	Solve problems involving ratios in mechanical and agricultural contexts.	***Example*** • Determine the effect of tire size on speedometer.
AMDM.QPR.2.3	Use proportions to solve problems involving large quantities that are not easily measured.	***Examples*** • Estimate the size of a crowd at indoor and outdoor events. • Estimate the number of gumballs in a container. • Determine license plate number options.

QUANTITATIVE & PROPORTIONAL REASONING *– Averages & Indices*			
AMDM.QPR.3: Make predictions by analyzing averages and indices of large data sets through investigations of real-world contexts.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
AMDM.QPR.3.1	Use averages and weighted averages to make decisions.	***Examples*** • Batting averages • Slugging percentage • Class grades.	
AMDM.QPR.3.2	Calculate and interpret indices.	***Example*** • Fan cost index	***Terminology*** • The fan cost index determines the cost to take a family to an event.

PATTERNING & ALGEBRAIC REASONING *– Identification Numbers, Voting, & Algorithms*		
AMDM.PAR.4: Develop methods or algorithms to analyze discrete situations.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AMDM.PAR.4.1	Create and verify identification numbers.	***Examples*** • UPC codes • QR codes • Product keys • Credit card number
AMDM.PAR.4.2	Analyze and evaluate the mathematics behind various methods of voting and selection.	***Examples*** • Rank voting • Plurality • Borda count • Pairwise comparisons
AMDM.PAR.4.3	Evaluate various voting and selection processes to determine an appropriate method for a given situation.	
AMDM.PAR.4.4	Apply various ranking algorithms to determine an appropriate method for a given situation.	***Examples*** • Inheritance • Sports drafts • Sports pools

PROBABILISTIC REASONING *– Conditional Probabilities & Compound Events*		
AMDM.PR.5: Analyze the chances for success or failure in order to make decisions.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AMDM.PR.5.1	Determine conditional probabilities and probabilities of compound events to make decisions in problem situations.	***Examples*** • Clothing combinations • Food combinations • Travel routes
AMDM.PR.5.2	Use probabilities to make and justify decisions about risks in everyday life.	***Examples*** • Insurance policies • Gambling

PROBABILISTIC REASONING *– Mathematical Assumptions & Expected Value*			
AMDM.PR.6: Model strategic interaction among rational decision-makers.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
AMDM.PR.6.1	Calculate expected value to analyze mathematical fairness, payoff, and risk.	***Examples*** • The purchase of a lottery ticket • Amount of insurance needed • Replacement value	
AMDM.PR.6.2	Analyze real-life situations involving strategic interactions using the mathematics of zero-sum games.	***Terminology*** • A zero-sum game is a mathematical representation of a situation in which each participant's gain or loss of utility is exactly balanced by the losses or gains of the utility of the other participants.	***Examples*** • Viruses • Zombies • Prisoner's Dilemma
AMDM.PR.6.3	Construct a mathematical model of probabilistic situations to make mathematical assumptions.	***Example*** • Students could create their own carnival game of chance.	

DATA & STATISTICAL REASONING *– Investigative Research*		
AMDM.DSR.7: Conduct investigative research to solve real-life problems and answer statistical investigative questions involved in business and financial decision-making.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AMDM.DSR.7.1	Apply statistical methods to design, conduct, and analyze statistical studies. Identify a contextual, real-	***Strategies and Methods*** • Simple random sampling • Stratified random sampling

	life problem that can be answered using investigative research.	• Cluster sampling • Convenience sampling	
AMDM.DSR.7.2	Build the skills and vocabulary necessary to analyze and critique reported statistical information, summaries, and graphical displays. Develop statistical investigative questions that can help solve a real-life problem involved in business and financial decision-making.	***Strategies and Methods*** • Interpret and compare results of polls given margin of error. • Determine possible sources of statistical bias. • Determine appropriate sampling techniques. • Determine sources of variability of data, those that can and cannot be controlled. ***Examples*** • Trends – gas prices, birth rates, death rates, stock market • Financial portfolios, mutual funds, life insurance	
AMDM.DSR.7.3	Create a statistical study using sound methodology to answer statistical investigative questions and to solve the real-life problem.	***Fundamentals*** • Students should have experiences with both experimental and observational studies.	***Terminology*** • Sound methodology ensures the integrity, credibility, and generalizability of the evidence, analysis, and conclusions presented in a study based on the techniques and approaches used.
AMDM.DSR.7.4	Explain how the sample size impacts the precision with which estimates of the population parameters can be made (i.e., the larger the sample size the more precision).		
AMDM.DSR.7.5	Recognize that random selection from a population plays a different role than random assignment in an experiment.		
AMDM.DSR.7.6	Incorporate random designs in data collection.		
AMDM.DSR.7.7	Describe ways in which big data can be used to make decisions in various business enterprises and in the context of business and financial decision-making.	***Examples*** • How much house can you afford? • Loan application process • Credit worthiness • Investment potential	
AMDM.DSR.7.8	Use distributions to identify the key features of the data collected.		
AMDM.DSR.7.9	Interpret results and make connections to the original research question.	***Strategies and Methods*** • Analyze and report using histograms, boxplots, pie charts, and bar graphs.	

PATTERNING & ALGEBRAIC REASONING *– Financial Models*		
AMDM.PAR.8: Create and analyze mathematical models to make decisions related to earning, investing, spending, and borrowing money.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AMDM.PAR.8.1	Use exponential functions to model change in a variety of financial situations.	***Examples*** • Simple interest • Compound interest • Compound continuous
AMDM.PAR.8.2	Determine, represent, and analyze mathematical models for income, expenditures, and various types of loans and investments.	***Terminology*** • Time value of money, budget, growth potential, wealth building, debt snowfall

FUNCTIONAL & GRAPHICAL REASONING *– Modeling with Functions*		
AMDM.FGR.9: Use functions to model problem situations in both discrete and continuous relationships.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AMDM.FGR.9.1	Determine whether a problem situation involving two quantities is best modeled by a discrete or continuous relationship.	***Examples*** • Pattern identification (discrete) • Population growth (discrete) • Compound interest (discrete) • Medication dosage (continuous) • Climate change (continuous) • Bone decay (continuous)
AMDM.FGR.9.2	Use linear, exponential, logistic, and piecewise functions to construct a model.	

GEOMETRIC & SPATIAL REASONING *– Deterministic Models*			
AMDM.GSR.10: Use functions to model problem situations in both discrete and continuous relationships.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
AMDM.GSR.10.1	Create and use two-dimensional and three-dimensional representations to model authentic situations.	***Example*** • Roofing and construction	
AMDM.GSR.10.2	Solve problems involving inaccessible distances using basic trigonometric principles including extensions of right triangle trigonometry.	***Example*** • Cell phone triangulation	***Strategies and Methods*** • Law of Sines, Law of Cosines, Unit Circle

PATTERNING & ALGEBRAIC REASONING *– Vectors & Matrices*		
AMDM.PAR.11: Use functions to model problem situations in both discrete and continuous relationships.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AMDM.PAR.11.1	Represent situations and solve problems using vectors. in areas such as transportation, computer graphics, and the physics of force and motion.	***Example*** • Transportation • Computer Graphics • Physics of force and motion
AMDM.PAR.11.2	Represent geometric transformations and solve problems using matrices.	***Example*** • Computer animations • Food chains

PATTERNING & ALGEBRAIC REASONING *– Networks*		
AMDM.PAR.12: Make informed decisions and solve problems with a variety of network models in quantitative situations.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AMDM.PAR.12.1	Solve problems represented by a vertex-edge graphs.	***Example*** • Critical and Euler paths • Minimal spanning trees
AMDM.PAR.12.2	Construct, analyze, and interpret flow charts to develop an algorithm to describe processes such as quality control procedures.	***Example*** • Manufacturing • Restaurants
AMDM.PAR.12.3	Investigate the scheduling of projects using Program Evaluation Review Technique (PERT).	***Terminology*** • PERT is a project management planning tool used to calculate the amount of time it will take to realistically finish a project. PERT stands for Program Evaluation Review Technique. ***Strategies and Methods*** • Students should be able to use PERT charts as tools to plan tasks within a project - making it easier to schedule and coordinate team members accomplishing the work.
AMDM.PAR.12.4	Consider problems that can be resolved by coloring graphs.	

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

Mathematical Practices	
AMDM.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Evidence**
AMDM.MP.1	Make sense of problems and persevere in solving them.
AMDM.MP.2	Reason abstractly and quantitatively.
AMDM.MP.3	Construct viable arguments and critique the reasoning of others.
AMDM.MP.4	Model with mathematics.
AMDM.MP.5	Use appropriate tools strategically.
AMDM.MP.6	Attend to precision.
AMDM.MP.7	Look for and make use of structure.
AMDM.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

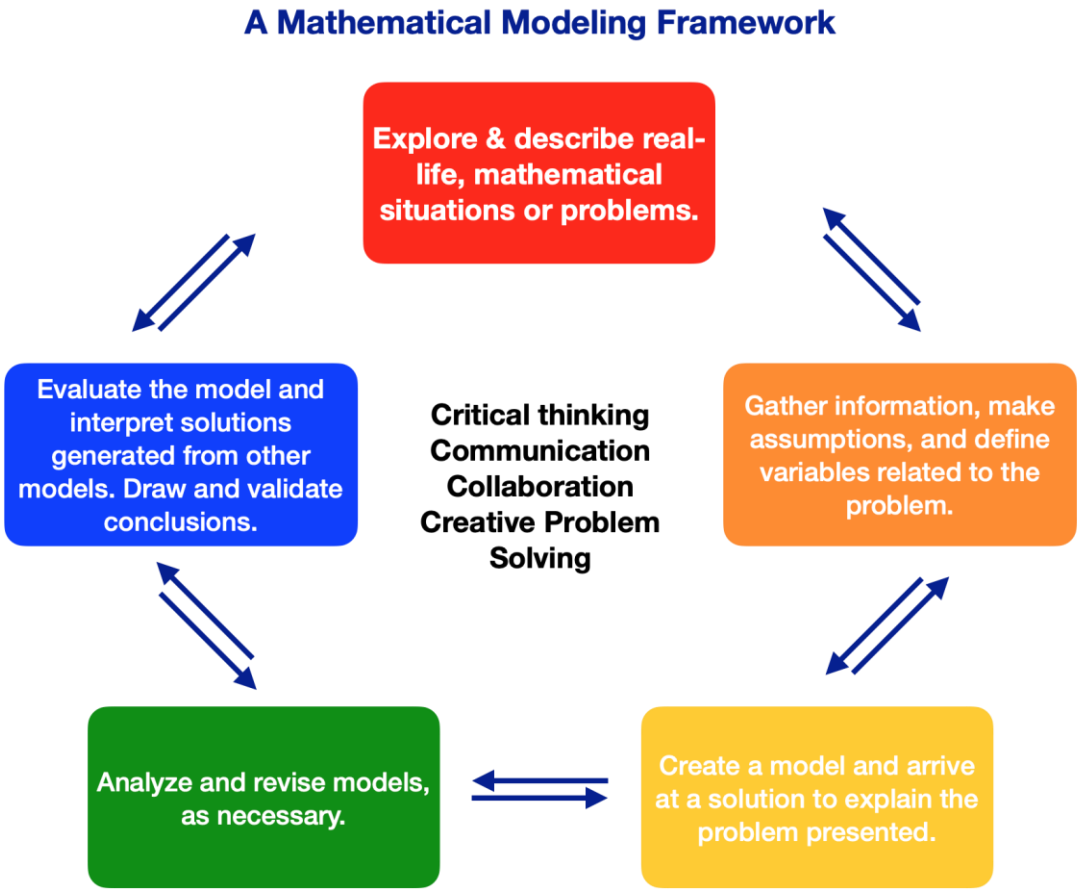

Image adapted from: Suh, Matson, Seshaiyer, 2017

FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

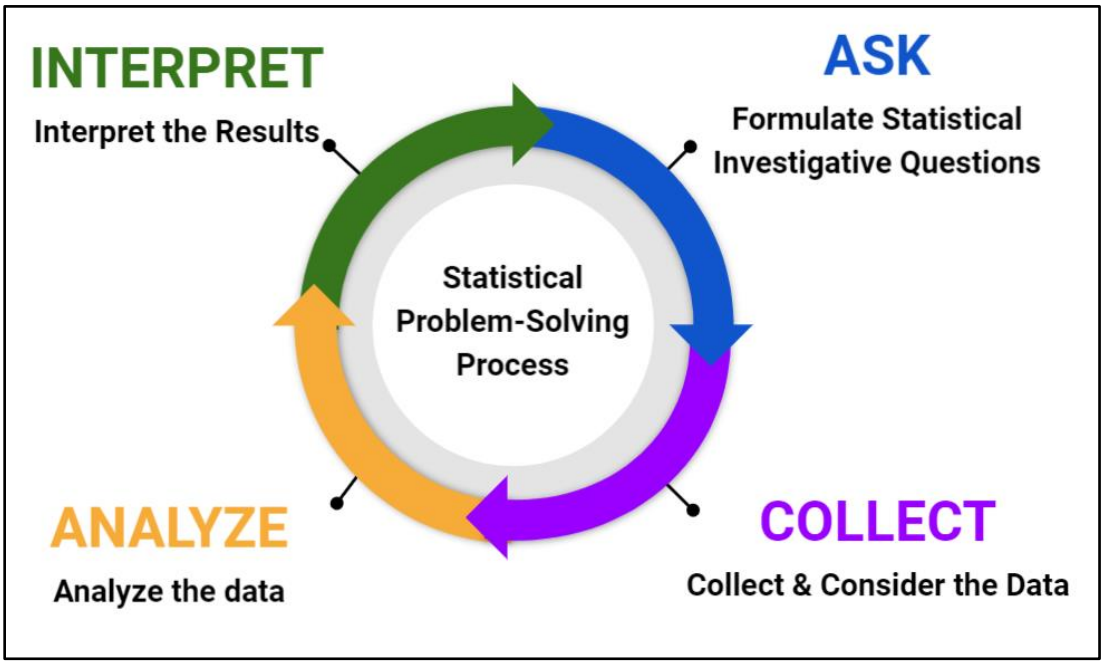

Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. Formulate Statistical Investigative Questions
Ask questions that anticipate variability.

II. Collect & Consider the Data
Ensure that data collection designs acknowledge variability.

III. Analyze the Data
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. Interpret the Results
Answer statistical investigative questions based on the collected data.