GEORGIA'S K-12
MATHEMATICS STANDARDS
2021

Advanced Algebra (Algebra II):

Concepts and Connections

(HS Course 3)

MATHEMATICS
KEY COMPETENCIES &
COURSE STANDARDS

WITH
LEARNING OBJECTIVES
IN PROGRESSION ORDER

GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

Advanced Algebra: Concepts & Connections

Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Advanced Algebra: Concepts and Connections Course, which is the third course in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

Advanced Algebra: Concepts & Connections is the culminating course in a sequence of three high school courses designed to ensure career and college readiness. It is designed to prepare students for fourth course options relevant to their career pursuits. High school course content standards are listed by big ideas including Data and Statistical Reasoning, Probabilistic Reasoning, Functional and Graphical Reasoning, Patterning and Algebraic Reasoning, and Geometry Patterning and Spatial Reasoning.

This course is designed as the third course in a three-course series. This course enhances students' geometric, algebraic, graphical, and probabilistic reasoning skills. Students will apply their algebraic and geometric reasoning skills to make sense of problems involving geometry, trigonometry, algebra, probability, and statistics. Students will continue to enhance their analytical geometry and reasoning skills when analyzing and applying a deep understanding of polynomial expressions, proofs, constructions, rigid motions and transformations, similarity, congruence, circles, right triangle trigonometry, geometric measurement, and conditional probability.

Prerequisite:

This course is designed for students who have successfully completed *Geometry: Concepts & Connections*.

Georgia’s K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

Advanced Algebra: Concepts & Connections

The nine course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
AA.MP: **Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.**
AA.MM.1: **Apply mathematics to real-life situations; model real-life phenomena using mathematics.**
AA.DSR.2: **Communicate descriptive and inferential statistics by collecting, critiquing, analyzing, and interpreting real-world data.**
AA.FGR.3: **Explore and analyze structures and patterns for exponential and logarithmic functions and use exponential and logarithmic expressions, equations, and functions to model real-life phenomena.**
AA.FGR.4: **Explore and analyze structures and patterns for radical functions and use radical expressions, equations, and functions to model real-life phenomena.**
AA.FGR.5: **Extend exploration of quadratic solutions to include real and non-real numbers and explore how these numbers behave under familiar operations and within real-world situations; create polynomial expressions, solve polynomial equations, graph polynomial functions, and model real-world phenomena.**
AA.PAR.6: **Represent data with matrices, perform mathematical operations, and solve systems of linear equations leading to real-world linear programming applications.**
AA.GSR.7: **Develop an introductory understanding of the unit circle; solve trigonometric equations using the unit circle.**
AA.FGR.8: **Analyze the behaviors of rational functions to model applicable, mathematical problems.**

Advanced Algebra: Concepts & Connections

<table>
<tr><td colspan="4">MATHEMATICAL MODELING</td></tr>
<tr><td colspan="4">AA.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.</td></tr>
<tr><td colspan="2">Expectations</td><td colspan="2">Evidence of Student Learning
(not all inclusive; see Grade Level Overview for more details)</td></tr>
<tr><td>AA.MM.1.1</td><td>Explain applicable, mathematical problems using a mathematical model.</td><td colspan="2">Fundamentals
• Students should be provided with opportunities to learn mathematics in the context of culturally relevant problems.
• Mathematically applicable problems are problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (i.e., model with mathematics).</td></tr>
<tr><td>AA.MM.1.2</td><td>Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.</td><td colspan="2">Fundamentals
• Mathematically proficient students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.</td></tr>
<tr><td>AA.MM.1.3</td><td>Using abstract and quantitative reasoning, make decisions about information and data from a mathematical, applicable situation.</td><td>Fundamentals
• Students should be able to:
○ analyze functions, graphs, tables, and equations and make decisions about the real-life situations they describe based upon their understanding of mathematical functions.
○ analyze statistical results to decide the best course of action or approach to a problem.</td><td>Example
• Given a rectangle with length = $(x – 2)$ and width = $(2x + 3)$, a student could discover and articulate that the area = $(x – 2)(2x + 3) = 2x^2 – x – 6$. From the student's understanding of parabolas, a student would know that the parabola that represents all possible areas of this rectangle opens upwards and that there is no maximum area possible for this rectangle.</td></tr>
<tr><td>AA.MM.1.4</td><td>Use various mathematical representations and structures to represent and solve real-life problems.</td><td colspan="2">Fundamentals
• Students should be able to generate models, graphs, charts, and equations, to represent real-world phenomena in order to solve problems.
• Students should be provided opportunities to generate representations of real-world phenomena utilizing technology to show these phenomena and to solve problems.</td></tr>
</table>

DATA & STATISTICAL REASONING – *descriptive and inferential statistics*				
AA.DSR.2: Communicate descriptive and inferential statistics by collecting, critiquing, analyzing, and interpreting real-world data.				
Expectations		**Evidence of Student Learning** **(not all inclusive; see Grade Level Overview for more details)**		
AA.DSR.2.1	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Distinguish between primary and secondary data and how it affects the types of conclusions that can be drawn.	***Strategies and Methods*** • Students should be provided opportunities to collect data of their own design (primary) and/or use data that already exists (secondary). • Students should be able to critique studies of different design types and explain how randomization relates to each style of investigation.	***Example*** • Students might design and carry out a study with a recognition of error in the design of the study. • Students might evaluate a research study and critique the investigative measures and/or conclusions drawn from the data.	
AA.DSR.2.2	When collecting and considering data, critically evaluate ethics, privacy, potential bias, and confounding variables along with their implications for interpretation in answering a statistical investigative question. Implement strategies for organizing and preparing big data sets.	***Fundamentals*** • Students should be able to question how data were collected, rationale for the study, positionality of the researcher, subjectivity of human decision making, etc. • Students should be able to recognize bias and describe its potential effects. They do not need to memorize definitions of types of bias.	***Examples*** • Students might be provided opportunities to search for data on the internet and prepare it by implementing strategies for dealing with messy data. • Students might be provided opportunities to search for data on the internet and then provide a critical evaluation of the methods used to collect, organize and communicate that data to the public.”	***Terminology*** • Messy data includes missing values, incorrect inputs, lack of representativeness, difficult formatting, etc.
AA.DSR.2.3	Distinguish between population distributions, sample data distributions, and sampling distributions. Use sample statistics to make inferences about population parameters based on a random sample from that population and to communicate conclusions using appropriate statistical language.	***Fundamentals*** • Students should recognize that it is most often not feasible to study an entire population distribution. Therefore, students should have opportunities to explore representative samples from the population to make inferences concerning the population. • Students should demonstrate understanding of how sampling distributions developed through simulation are used to describe the sample-to-sample variability of sample statistics. • Students should summarize results from statistical analyses using appropriate statistical justifications that indicate an understanding of the statistics.	***Strategies and Methods*** • Students should have many opportunities to communicate quantitative information using statistical language in oral, written, and graphical form to build data fluency.	

AA.DSR.2.4	Calculate and interpret z-scores as a measure of relative standing and as a method of standardizing units.	***Fundamentals*** • Students should understand that z-scores are a statistical tool that allows someone to compare samples with differing units. Students should have opportunities to use z-scores to make decisions when analyzing real-world data. • Students should understand that z-scores can be used with all distributions, regardless of shape. • Students should use technology tools to calculate standard deviation when necessary to determine z-scores.	***Example*** • Students might compare performance on SAT versus ACT despite the different scoring scales by using z-scores.	
AA.DSR.2.5	Given a normally distributed population, estimate percentages using the Empirical Rule, z-scores, and technology.	***Fundamentals*** • Students should understand that there are data sets for which such a procedure is not appropriate because it is not normally distributed.	***Strategies and Methods*** • Students should be encouraged to use tools such as calculators, spreadsheets, or tables to estimate areas under a normal curve.	
AA.DSR.2.6	Model sample-to-sample variability in sampling distributions of a statistic using simulations taken from a given population.	***Fundamentals*** • Students should be able to use simulations to decide if a specified model accurately reflects real outcomes. • Students should be able to consider the sample-to-sample variability by using statistics from repeated samples of the same size.	***Example*** • Students could involve a simulated sampling distribution for a sample mean or a sample population to decide if a specified model accurately reflects real outcomes.	
AA.DSR.2.7	Given a margin of error, develop and compare confidence intervals of different models to make conclusions about reliability.	***Fundamentals*** • Students should be able to apply the margin of error to make conclusions about the reliability of statistical results. • Students do not have to calculate the margin of error.	***Strategies and Methods*** • Students might be provided opportunities to develop confidence intervals using simulations and technology, such as statistical applets.	***Examples*** • Students might compare exit poll data with two different margins of error to determine if the results are conclusive. • Students might explore questions such as: "In a favorability poll, if a politician has a 52% approval rating ±5 points, can they claim that most people approve?"
AA.DSR.2.8	Summarize and evaluate reports based on data for appropriateness of study design, analysis methods, and statistical measures used.	***Fundamentals*** • Students should be able to communicate statistical information using written and oral reports.		

FUNCTIONAL & GRAPHICAL REASONING – *exponential and logarithmic functions*			
AA.FGR.3: Explore and analyze structures and patterns for exponential and logarithmic functions and use exponential and logarithmic expressions, equations, and functions to model real-life phenomena.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Grade Level Overview for more details)**	
AA.FGR.3.1	Find the inverse of exponential and logarithmic functions using equations, tables, and graphs, limiting the domain of inverses where necessary to maintain functionality, and prove by composition or verify by inspection that one function is the inverse of another.	***Strategies and Methods*** • To verify by inspection, students can compare graphs of two relations and show that one is the reflection of the other across the line y = x. • To verify by inspection, students can show that one table or set of coordinates is the inverse of another because the y-values of the first are the x-values of the second and vice-versa. • To verify by inspection, students can show that a series of operations upon input values of one function are opposite and reversed in order for a second function. • Students should be able to prove by composition that two functions are inverses of each other.	***Terminology*** • To prove by composition means to determine if f(g(x)) = g(f(x)) = x.
AA.FGR.3.2	Analyze, graph, and compare exponential and logarithmic functions.	***Fundamentals*** • Students should be able to graph and identify key features of exponential and logarithmic functions, including domain, range, and x- and y-intercepts; roots, zeros, and solutions; asymptotes; interval(s) where the function is positive, and/or negative; non-symmetry; end behavior. • Students should be able to calculate the average rate of change for a given interval, including the estimated rate of change. • Students should have opportunities to gain an intuitive sense into what happens to the graph or model as a result of changes to the various key features of the function.	
AA.FGR.3.3	Use the definition of a logarithm, logarithmic properties, and the inverse relationship between exponential and logarithmic functions to solve problems in context.	***Fundamentals*** • Students should be given opportunities to solve real-life, culturally relevant problems involving the use of the common logarithm and the natural logarithm. • Students should be able to apply their knowledge of the inverse relationship between exponential and logarithmic functions to solve real-life problems.	
AA.FGR.3.4	Create exponential equations and use logarithms to solve mathematical, applicable problems for which only one variable is unknown.	***Fundamentals*** • Students should be able to solve problems involving exponential equations using the relationship with logarithmic functions to solve for the single unknown variable. • Given pertinent information (e.g., ambient temperature and time), students should be able to use exponential equations to solve real-life problems and interpret the solutions.	***Examples*** • Students can solve and interpret equations that have one unknown variable, such as: ○ Exponential growth ○ Compound interest ○ Newton's Law of Cooling: $T(t) = T_S + (T_0 - T_S)\, e^{-kt}$

AA.FGR.3.5	Create and interpret logarithmic equations in one variable and use them to solve problems.	***Fundamentals*** • Given pertinent information, students should be able to use logarithmic equations to solve real-life problems and interpret the solutions.	***Example*** • Students can create and interpret equations involving *pH*, such as $pH = -log(H^+)$, to define the acidity or alkalinity of a substance.
AA.FGR.3.6	Create, interpret, and solve exponential equations to represent relationships between quantities and analyze the relationships numerically with tables, algebraically, and graphically.	***Fundamentals*** • Students should be able to analyze what is happening in the relationships between quantities. • Students should discuss the characteristics of exponential functions in context, including domain and range, zeros, intercepts, average rate of change, asymptote, and other relevant key features. • Students should be able to solve real-life problems that can be modeled by exponential equations. • Students should be encouraged to explore multiple solution pathways, which might include graphing with various tools, interpreting key features, and evaluating equations.	***Examples*** • Students can create, interpret and solve equations that have two unknown variables, such as: ○ Half-Life ○ Exponential growth ○ Exponential decay ○ Compound interest ○ Newton's Law of Cooling: $T(t) = T_S + (T_0 - T_S) e^{-kt}$
AA.FGR.3.7	Create, interpret, and solve logarithmic equations in two or more variables to represent relationships between quantities.	***Fundamentals*** • Students should be able to analyze and interpret logarithmic equations presented in mathematical, applicable situations. • Students should discuss the characteristics of logarithmic functions in context, including domain and range, zeros, intercepts, average rate of change, asymptote, and other relevant key features. • Students should be able to solve problems that can be modeled by logarithmic equations. • Students should be encouraged to explore multiple solution pathways, which might include graphing with various tools, interpreting key features, and evaluating equations.	***Example*** • Students are able to create and interpret equations involving logarithms such as the equation for the magnitude of an earthquakes $M = log_{10} (I/S)$.

<table>
<tr><th colspan="5">FUNCTIONAL & GRAPHICAL REASONING – radical functions</th></tr>
<tr><th colspan="5">AA.FGR.4: Explore and analyze structures and patterns for radical functions and use radical expressions, equations, and functions to model real-life phenomena.</th></tr>
<tr><th colspan="2">Expectations</th><th colspan="3">Evidence of Student Learning
(not all inclusive; see Grade Level Overview for more details)</th></tr>
<tr><td>AA.FGR.4.1</td><td>Rewrite radical expressions as expressions with rational exponents. Extend the properties of integer exponents to rational exponents.</td><td colspan="3">Fundamentals
• In previous grades, students should have developed an understanding of the properties of integer exponents.
• Students should be able to convert back and forth between radical expressions and expressions with rational exponents.
• Students should be able to recognize that
$\sqrt[x]{b^n} = (\sqrt[x]{b})^n = b^{\frac{n}{x}}$
• Students will utilize the product rule, quotient rule, and power rule to work with expressions with rational exponents.</td></tr>
<tr><td>AA.FGR.4.2</td><td>Solve radical equations in one variable, and give examples showing how extraneous solutions may arise.</td><td>Fundamentals
• Students should be able to convert between radical expressions and expressions with rational exponents to solve equations.
• Students should understand how to use substitution to check answers to radical equations to ensure that solutions are not extraneous.</td><td>Strategies and Methods
• Students should have opportunities to use technology and tools to solve radical equations by graphing.
• Students should have opportunities to use technology and tools to explore and solve radical equations to strengthen conceptual understanding.</td><td>Example
• Given the volume of a sphere, students could determine the radius of the sphere by writing an equation for the radius, r, and solving for r.</td></tr>
<tr><td>AA.FGR.4.3</td><td>Analyze and graph radical functions.</td><td colspan="3">Fundamentals
• Students should be able to graph and identify key features of a radical function including: domain, range, and x and y-intercepts; roots, zeros, and solutions; intervals where the function is increasing, decreasing, positive, and/or negative; maximum and minimum values, including endpoint extrema; non-symmetry; end behavior.
• Students should be able to calculate the slope of average rate of change for a given interval, including the estimated rate of change.
• Students should be able to relate the key features of a model (i.e., graph, equation, table) to the real-world situation which the model represents.</td></tr>
<tr><td>AA.FGR.4.4</td><td>Create, interpret and solve radical equations with one unknown value and use them to solve problems that model real-world situations.</td><td>Fundamentals
• Students should be able to analyze and interpret radical equations presented in mathematical, applicable situations.</td><td>Strategies and Methods
• Students should have opportunities to use technology and tools to solve radical equations to</td><td>Example
• Students can create a radical equation using the distance formula, for</td></tr>
</table>

		• Students should discuss the characteristics of radical functions in context, including domain and range, zeros, intercepts, and other relevant key features. • Students should be able to solve problems that can be modeled by radical equations.	strengthen conceptual understanding. • Students should be encouraged to explore multiple solution pathways, which might include graphing with various tools, interpreting key features, and evaluating radical equations.	which the distance and three of the four coordinate values are known, and one is unknown.
AA.FGR.4.5	Create, interpret, and solve radical equations in two or more variables to represent relationships between quantities.	***Strategies and Methods*** • Less time should be devoted to the mechanics of solving radical equations and more time should be devoted to building students' capacity for interpreting radical functions within context.	***Example*** • Students can create and interpret problems involving radical equations in which two of the variables are unknown, such as problems involving velocity.	

FUNCTIONAL & GRAPHICAL REASONING – *polynomial functions*		
AA.FGR.5: Extend exploration of quadratic solutions to include real and non-real numbers and explore how these numbers behave under familiar operations and within real-world situations; create polynomial expressions, solve polynomial equations, graph polynomial functions, and model real-world phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Grade Level Overview for more details)**
AA.FGR.5.1	Graph and analyze quadratic functions in contextual situations and include analysis of data sets with regressions.	***Fundamentals*** • As an introduction to polynomial functions, students should be able to use quadratic functions in standard, factored, and vertex forms to graph and identify key features in context in order to answer questions about real-life phenomena. • Key features of quadratic functions should include x and y-intercepts, roots, zeros, and solutions; domain, range, and intervals where the function is increasing, decreasing, positive, and/or negative (using inequality and interval notations); vertex, extreme value, and axis of symmetry; end behavior, using technology where appropriate. • Students should be able to calculate the slope of average rate of change for a given interval, including the estimated rate of change. • Through contextual exploration, students should recognize that there are data sets for which a quadratic function is not the best model, and therefore, explore other types of polynomial regression. • Analysis of data sets with regressions should be done informally with verbal descriptions and with the use of technology.

AA.FGR.5.2	Define complex numbers i such that $i^2 = -1$ and show that every complex number has the form $a + bi$ where a and b are real numbers and that the complex conjugate is $a - bi$.	***Fundamentals*** • Students should be able to identify the real part of a complex number and the imaginary part. • Students should convert any power of the imaginary unit, *i,* to an equivalent form and identify the pattern that emerges. • Students should have opportunities to identify the complex conjugate of any complex number and recognize that complex numbers always occur as pairs when they represent solutions to a polynomial function.			
AA.FGR.5.3	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	***Fundamentals*** • Students should be provided opportunities to solve real-life problems that require the addition, subtraction, or multiplication of complex numbers. • Division of complex numbers is beyond the scope of Advanced Algebra.			
AA.FGR.5.4	Use the structure of an expression to factor quadratics.	***Relevance and Application*** • Expressions should include special-case quadratics such as perfect-square trinomials and the difference of two perfect squares.			
AA.FGR.5.5	Write and solve quadratic equations and inequalities with real coefficients and use the solution to explain a mathematical, applicable situation.	***Relevance and Application*** • Equations and inequalities presented in real-life, mathematical problems should include quadratics with complex solutions.	***Fundamentals*** • In previous grades, students had opportunities to create and solve quadratic equations. Given a real-life scenario, students should be able to model the scenario using quadratic equations and inequalities in one variable. • Given a quadratic equation or inequality in one variable (model), students should be able to create a real-life scenario that matches the model. • Students should be able to connect the solutions of quadratic equations and inequalities to the graph of the corresponding quadratic function and use these to solve real-life problems that can be modeled by quadratic	***Strategies and Methods*** • Students should be able to solve quadratic equations and inequalities fluently (flexibly, accurately, efficiently) by inspection, taking square roots, factoring, completing the square, and applying the quadratic formula, as appropriate to the initial form of the equation. • Students should be provided opportunities to explore a variety of real-life problems modeled by	***Example*** • Students can create equations involving areas using unknown dimensions.

			equations and inequalities. • Students should be able to model real-life occurrences with quadratic equations or inequalities and use these to solve problems.	quadratic equations and inequalities.	
AA.FGR.5.6	Solve systems of quadratic and linear functions to determine points of intersection.	***Fundamentals*** • Students should be able to solve real-life problems modeled by systems of quadratic and linear functions using algebraic techniques (by hand) or using technology to identify the intersections of a parabola and a line.			
AA.FGR.5.7	Create and analyze quadratic equations to represent relationships between quantities as a model for contextual situations.	***Fundamentals*** • Students should be able to solve real-life problems modeled by quadratic equations and inequalities in two or more variables. • Given a real-life scenario, students should be able to model the scenario using quadratic equations and inequalities in two or more variables. • Given a quadratic equation or inequality in two or more variables (model), students should be able to create a real-life scenario that matches the model.			***Example*** • Students can create equations involving projectile motion.
AA.FGR.5.8	Identify the number of zeros that exist for any polynomial based upon the greatest degree of the polynomial and the end behavior of the polynomial by observing the sign of the leading coefficient.	***Fundamentals*** • Given a polynomial function, students should be able to apply the Fundamental Theorem of Algebra to describe the maximum number of times the function may cross the x-axis. • Given a polynomial function, students should be able to tell if the left and right sides are increasing as x approaches negative and positive infinity based upon the sign of the leading coefficient and whether the greatest exponent is even or odd. • Students should verify, using technology, if their predictions are correct for the number of zeros a polynomial has. When there are fewer zeros than the highest exponent led them to expect, students should understand		***Example*** • Given the polynomial function $f(x) = -2x^5 - 4x^4 + x^3 - 6x^2 + 9x + 4$, students should be able to say that there are five or fewer real zeros, and that as x approaches negative infinity, f(x) approaches infinity, and as x approaches infinity, f(x) approaches negative infinity. Using technology, students should be able to verify the number of zeroes and/or complex solutions of this polynomial.	

		that this is because there are complex solutions. • Students should understand that complex solutions always occur in pairs.	
AA.FGR.5.9	Identify zeros of polynomial functions using technology or pre-factored polynomials and use the zeros to construct a graph of the function defined by the polynomial function. Analyze identify key features of these polynomial functions.	***Fundamentals*** • Students should be able to graph and identify key features of polynomial functions to include x and y-intercepts, roots of multiplicity, zeros, and solutions; domain, range, and intervals where the function is increasing, decreasing, positive, and/or negative (using inequality and interval notations); vertex, extreme value, and axis of symmetry; end behavior, using technology where appropriate. • Students are not expected to graph polynomial functions presented in standard form, by hand. • Students should be able to identify key features of a polynomial equation to create a rough sketch of a graph, by hand. • When presented a polynomial function in standard form, students should be able to use technology to graph the function and identify key features, including the zeros of the function.	
AA.FGR.5.10	Use the structure of an expression to factor polynomials, including the sum of cubes, the difference of cubes, and higher-order polynomials that may be expressed as a quadratic within a quadratic.	***Fundamentals*** • Students should be able to rewrite polynomial expressions in various equivalent forms, based on the context of the problem.	***Example*** • $x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x + y)(x - y)$.
AA.FGR.5.11	Using all the zeros of a polynomial function, list all the factors and multiply to write a multiple of the polynomial function in standard form.	***Fundamentals*** • Students should be able to analyze a graph of a polynomial to identify where multiplicity exists due to a local maximum or minimum value that is situated on the x-axis and recognize that repeating that factor may be necessary. • Student should note that multiplying factors to write a polynomial generates one of many possibly polynomials with the same zeros, because the leading coefficient of a polynomial is not evident from factor alone.	***Example*** • Given the solutions of -2, 4, and 5 with a multiplicity of 2, students would be asked to write an equation that represents a multiple of the polynomial, *f(x)*, in standard form. The number of possible equations is endless, but answers could include: ○ $(x+2)(x-4)(x-5)(x-5) = x^4 - 12x^3 + 29x^2 + 30x - 200$, and ○ $2(x+2)(x-4)(x-5)(x-5) = 2x^4 - 24x^3 + 58x^2 + 60x - 400$

<table>
<tr><td colspan="4">PATTERNING & ALGEBRAIC REASONING – linear algebra and matrices</td></tr>
<tr><td colspan="4">AA.PAR.6: Represent data with matrices, perform mathematical operations, and solve systems of linear equations leading to real-world linear programming applications.</td></tr>
<tr><td colspan="2">Expectations</td><td colspan="2">Evidence of Student Learning
(not all inclusive; see Grade Level Overview for more details)</td></tr>
<tr><td>AA.PAR.6.1</td><td>Use matrices to represent data, and perform mathematical operations with matrices and scalars, demonstrating that some properties of real numbers hold for matrices, but that others do not.</td><td>Fundamentals
• This is the first formal reference to matrices in the K-12 mathematics standards progression.
• Students should be able to perform operations with matrices that include the identity matrix and the zero matrix.
• Students should recognize that matrix multiplication is not commutative.</td><td>Strategies and Methods
• Students should be able to calculate the following matrix operations without the use of technology: scalar multiplication, addition, subtraction, multiplication of 2x2 matrices, calculate the determinant of a 2x2 matrix, and the inverse of an invertible 2x2 matrix. Students should have opportunities to utilize technology to perform the same calculations with matrices of greater dimension.
• Students may use technology in calculations with matrices of greater dimension than 2x2.</td></tr>
<tr><td>AA.PAR.6.2</td><td>Rewrite a system of linear equations using a matrix representation.</td><td>Relevance and Application
• A system of linear equations in standard form can be represented as an equation of a coefficient matrix multiplied by a variable matrix, equal to a constant matrix.</td><td>Example
• Given the system of linear equations,
$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$
students should represent it as:
$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$</td></tr>
<tr><td>AA.PAR.6.3</td><td>Use the inverse of an invertible matrix to solve systems of linear equations.</td><td colspan="2">Relevance and Application
• Students may use technology for matrices of dimension 2 x 2 or higher to calculate the inverse of an invertible matrix.</td></tr>
<tr><td>AA.PAR.6.4</td><td>Utilize linear programming to represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as solutions or non-solutions under the established constraints in real-world problems.</td><td colspan="2">Relevance and Application
• Food and Agriculture, Engineering, and Manufacturing optimization problems would be appropriate for this learning objective. Other contexts may be used, as well.</td></tr>
</table>

GEOMETRIC & SPATIAL REASONING – *Trigonometry and the Unit Circle*			
AA.GSR.7: Develop an introductory understanding of the unit circle; solve trigonometric equations using the unit circle.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Grade Level Overview for more details)**	
AA.GSR.7.1	Define the three basic trigonometric ratios in terms of x, y, and r using the unit circle centered at the origin of the coordinate plane.	***Fundamentals*** • This learning objective is applicable to all four quadrants of the unit circle. • Students should connect the parts of the right triangle in the first quadrant to the corresponding parts of the unit circle where the hypotenuse is the radius, the adjacent side is x, and the opposite side is y. • Students should be able to articulate the pattern associated with angle measures in all four quadrants of the unit circle, e.g., 150° as 180°-30°, 210° as 180°+30°, 330° as 360°-30°, etc. • Students should explore, interpret, and use radian measures based on conversions from degree measures, such as 150°, 210°, etc., and articulate the patterns associates with those radian measures, including the connection of $\frac{5\pi}{6} \approx 2.617$ radius units measured along the arc length of the circle. • Through explorations, students develop an understanding that a unit circle has a radius equal to 1. • This learning objective is limited to angle measures of 30° ($\frac{\pi}{6}$), 45° ($\frac{\pi}{4}$) and 60° ($\frac{\pi}{3}$), and their associated reflected angles within one counterclockwise revolution of the unit circle.	
AA.GSR.7.2	Apply understanding of the angle measures and coordinates of the unit circle to solve practical, real-life problems involving trigonometric equations.	***Fundamentals*** • This learning objective is limited to special right triangles with angle measures 30° ($\frac{\pi}{6}$), 45° ($\frac{\pi}{4}$), and 60° ($\frac{\pi}{3}$) and their associated reflected angles within one counterclockwise revolution of the unit circle.	***Relevance and Application*** • Students should find exact values from the unit circle to solve contextual problems such as a Ferris Wheel Rider's height above ground during a one revolution ride. • Students should have the opportunity to solve in situations like: If the tide height at a marina is modeled by y = 3cos(t) + 5.5 with y measured in feet and t measured in hours, at what time is the tide a height of 4 feet.

FUNCTIONAL & GRAPHICAL REASONING – rational functions			
AA.FGR.8: Analyze the behaviors of rational functions to model applicable, mathematical problems.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Grade Level Overview for more details)**	
AA.FGR.8.1	Rewrite simple rational expressions in equivalent forms.	***Fundamentals*** • Students should be given opportunities to explore culturally relevant situations and problems that can be represented with rational expressions. • Students should be able to rewrite rational expressions in various equivalent forms, based on the context of the problem, with the understanding that any factor of the numerator, over itself in the denominator, is equal to a factor of one.	***Example*** • $\frac{x^2+4x+4}{x^2-4}=\frac{(x+2)(x+2)}{(x+2)(x-2)}=1*\frac{(x+2)}{(x-2)}=\frac{x+2}{x-2}$
AA.FGR.8.2	Add, subtract, multiply and divide rational expressions, including problems in context and express rational expressions in irreducible form.	***Fundamentals*** • Limit operations with rational expressions to those that would occur within the context of real-life problems, e.g., uniform motion, work, mixtures. • Limit division to factorable expressions for which no remainder exists.	***Example*** • Mary spent the first 120 miles of her road trip in traffic. When the traffic cleared, she was able to drive twice as fast for the remaining 300 miles. Write an expression that represents the total time she drove in terms of her known distance and unknown rates. $\frac{120}{x}+\frac{300}{2x}=\frac{120}{x}+\frac{150}{x}=\frac{270}{x}$
AA.FGR.8.3	Graph rational functions, identifying key characteristics.	***Fundamentals*** • Students should be given graphs, or use technology to generate their own graphs, to identify characteristics of rational functions. • Students should be able to use technology to graph and identify key features of rational functions, to include x and y-intercepts, roots of multiplicity, zeros, and solutions; domain, range, and intervals where the function is increasing, decreasing, positive, and/or negative (using inequality and interval notations); vertex, extreme value, and axis of symmetry; end behavior, using technology where appropriate.	
AA.FGR.8.4	Solve simple rational equations in one variable, and give examples showing how extraneous solutions may arise.	***Fundamentals*** • Limit solving rational equations to those that would occur within the context of real-world problems, e.g., uniform motion, work, mixtures. • Students should be able to check for extraneous solutions.	***Strategies and Methods*** • Students should be encouraged to use technology and tools to solve rational equations in order to enhance conceptual understanding. • Less time should be devoted to the mechanics of solving rational equations and more time should be devoted to building students capacity for interpreting rational functions within context.

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

Mathematical Practices	
AA.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
AA.MP.1	Make sense of problems and persevere in solving them.
AA.MP.2	Reason abstractly and quantitatively.
AA.MP.3	Construct viable arguments and critique the reasoning of others.
AA.MP.4	Model with mathematics.
AA.MP.5	Use appropriate tools strategically.
AA.MP.6	Attend to precision.
AA.MP.7	Look for and make use of structure.
AA.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

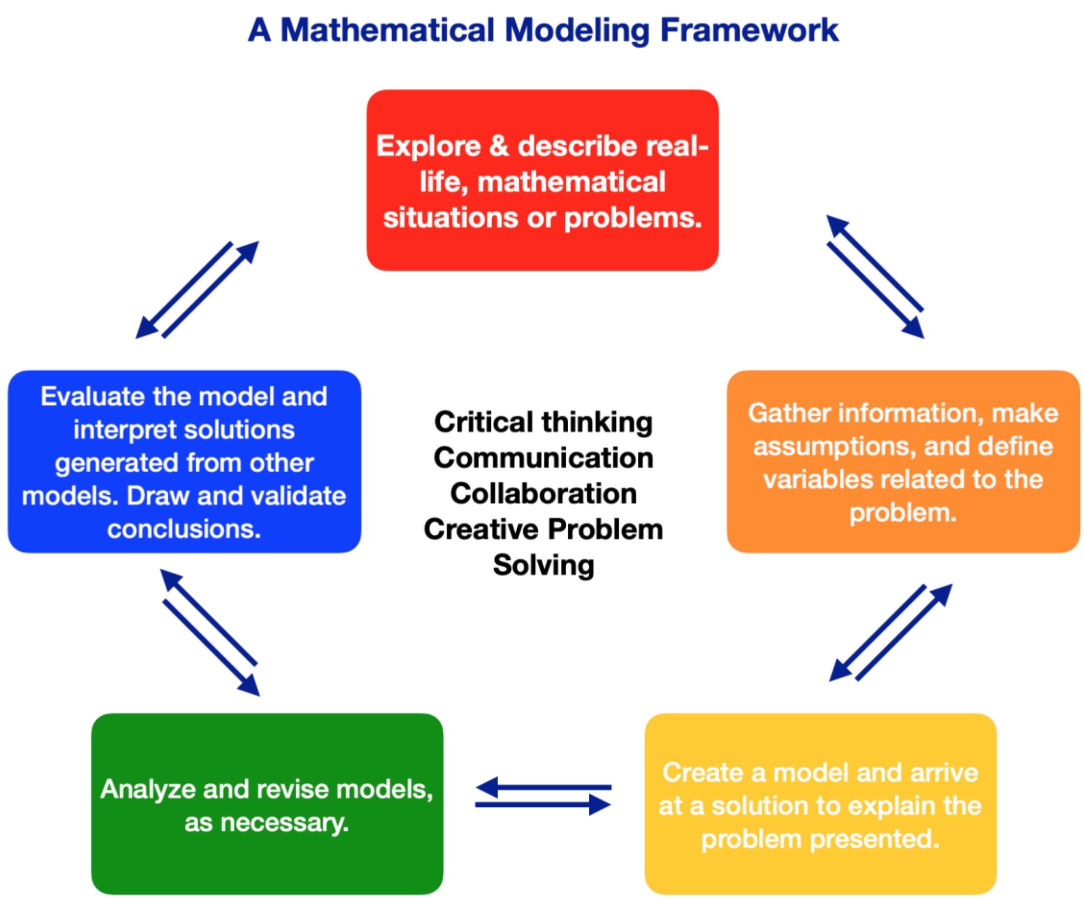

Image adapted from: Suh, Matson, Seshaiyer, 2017

IV.

FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

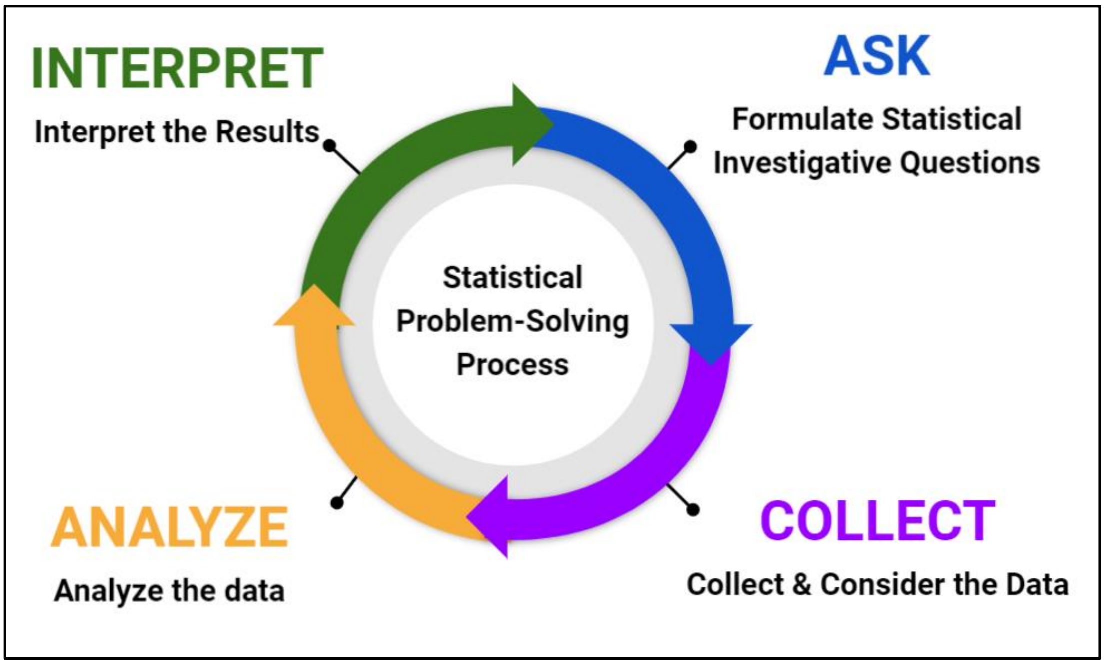

Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. Formulate Statistical Investigative Questions
Ask questions that anticipate variability.

II. Collect & Consider the Data
Ensure that data collection designs acknowledge variability.

III. Analyze the Data
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. Interpret the Results
Answer statistical investigative questions based on the collected data.