GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

History of Mathematics

MATHEMATICS KEY COMPETENCIES & COURSE STANDARDS

WITH LEARNING OBJECTIVES IN PROGRESSION ORDER

GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

History of Mathematics

Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School History of Mathematics course, which is a fourth mathematics course option in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

History of Mathematics is a two-semester elective course option for students who have completed AP Calculus or are taking AP Calculus concurrently. It traces the development of major branches of mathematics throughout history, specifically algebra, geometry, number theory, and methods of proofs, how that development was influenced by the needs of various world cultures, and how the mathematics in turn influenced world culture.

Instruction and assessment should include appropriate use of technology and manipulatives. Concepts should be introduced and used in an appropriate historical context.

Prerequisite:

This course is designed for students who have successfully *completed AP Calculus or are taking AP Calculus concurrently*.

Georgia's K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL (QUANTITATIVE) REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)
LOGICAL, MATHEMATICAL & INVESTIGATIVE REASONING (LMIR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

History of Mathematics

The eight course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
HM.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.
HM.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
HM.NR.2: Explore and use historical number systems and computational methods.
HM.LMIR.3: Engage in the mathematical and cultural accomplishments of the ancient Greeks in order to grasp the foundational aspects of modern mathematics.
HM.LMIR.4: Engage in the mathematical and cultural accomplishments of the world's societies in the 5th century through the 15th century in order to grasp the foundational aspects of modern mathematics.
HM.LMIR.5: Engage in the mathematical accomplishments of Europe in the 15th century through the early 17th century in order to grasp the foundational aspects of modern mathematics.
HM.LMIR.6: Engage in the mathematical and cultural accomplishments of the world's societies in the late 17th century through the early 20th century in order to grasp the foundational aspects of modern mathematics.
HM.LMIR.7: Investigate and describe modern mathematicians and their contributions to mathematics.

History of Mathematics

MATHEMATICAL MODELING		
HM.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
HM.MM.1.1	Explain contextual, mathematical problems using a mathematical model.	***Fundamentals*** • Students should be provided with opportunities to learn mathematics in the context of real-life problems. • Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).
HM.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.	***Fundamentals*** • Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.
HM.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation.	
HM.MM.1.4	Use various mathematical representations and structures with this information to represent and solve real-life problems.	

***NUMERICAL REASONING** – Origins of Mathematics*		
HM.NR.2: Explore and use historical number systems and computational methods.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Historical computation methods		
HM.NR.2.1	Use historical number systems to represent quantities.	***Examples*** • Babylonian, Roman, Egyptian (hieratic and hieroglyphic), Chinese, Yoruba, and Greek
HM.NR.2.2	Use historical multiplication and division algorithms.	***Examples*** • Duplation and mediation
HM.NR.2.3	Decompose fractions of the form 2/(pq) using the Egyptian method as recorded by Ahmes (Ahmose) in the Rhind Papyrus.	
HM.NR.2.4	Compute lengths, areas, and volumes according to historical formulas	***Example*** • Find the volume of a truncated pyramid using the Babylonian, Chinese, and Egyptian formulas.

HM.NR.2.5	Describe the limitations of the Babylonian, Roman, Egyptian (hieratic and hieroglyphic), Chinese, and Greek number systems as compared to Hindu-Arabic numerals	
HM.NR.2.6	Identify the number system and notation used by a society as an influence on the types of mathematics developed by that society.	
Use historical methods to solve equations.		
HM.NR.2.7	Solve linear equations using the method of false position.	
HM.NR.2.8	Translate ancient mathematical problems that involve linear, quadratic, or cubic equations into modern notation and solve them in a variety of ways.	

***LOGICAL, MATHEMATICAL & INVESTIGATIVE REASONING** – Ancient Greek Mathematics*		
HM.LMIR.3: Engage in the mathematical and cultural accomplishments of the ancient Greeks in order to grasp the foundational aspects of modern mathematics.		
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)
Greek geometry		
HM.LMIR.3.1	Prove statements in a deductive system by using its definitions, postulates, and axioms	
HM.LMIR.3.2	Prove the first five propositions in Book I of Euclid's *Elements*.	
HM.LMIR.3.3	Construct a regular pentagon with a straight-edge and compass.	
HM.LMIR.3.4	Compute the areas of regular polygons by Heron's formulas.	
Greek algebra and number sense		
HM.LMIR.3.5	Translate Greek geometric algebra into modern algebraic notation.	***Example*** • Interpret *Elements* II.4, "If a straight line is cut at random, then the square on the whole equals the sum of the squares on the segments plus twice the rectangle contained by the segments", as $(x + y)^2 = x^2 + y^2 + 2xy$.
HM.LMIR.3.6	Find the first four perfect numbers using Euclid's formula.	
HM.LMIR.3.7	Justify statements concerning figurate numbers using both graphical (as in the manner of the Greeks) and algebraic methods.	***Example*** • Show that the nth pentagonal number is equal to the *n*th square number plus the $(n – 1)$th triangular number.
HM.LMIR.3.8	Solve systems of linear and nonlinear equations using Diophantus' method.	***Example*** • Two numbers sum to 20 and the sum of their squares is 208. What are the numbers?
Greek culture and society		
HM.LMIR.3.9	Explain the distinction made between number and magnitude, commensurable and incommensurable, and arithmetic and logistic,	

	the cultural factors inherent in this distinction, and the logical crisis that occurred concerning incommensurable (irrational) magnitudes.	
HM.LMIR.3.10	Describe the cultural aspects of Greek society that influenced the way mathematics developed in ancient Greece.	
HM.LMIR.3.11	Describe the theories for the rise of intellectual thought in ancient Greece and the factors involved in its collapse.	
HM.LMIR.3.12	Analyze factors involved in the rise and fall of ancient Greek society.	***Example*** • Describe the contributions of Hypatia.

LOGICAL, MATHEMATICAL & INVESTIGATIVE REASONING – *Mathematics in the Middles Ages*		
HM.LMIR.4: Engage in the mathematical and cultural accomplishments of the world's societies in the 5th century through the 15th century in order to grasp the foundational aspects of modern mathematics.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Non-European mathematics in the middle ages.		
HM.LMIR.4.1	Translate medieval mathematical problems that involve linear, quadratic, or cubic equations into modern notation and solve them in a variety of ways.	***Example*** • Brahmagupta's algebraic prose
HM.LMIR.4.2	Use Khayyam's geometric construction to find a solution to a cubic equation.	
HM.LMIR.4.3	Identify cyclic quadrilaterals and find associated lengths by Ptolemy's Theorem.	
HM.LMIR.4.4	Investigate the relationships among the sides and angles of a spherical triangle.	
HM.LMIR.4.5	Describe the algebraic and geometric contributions of Islamic mathematicians in the Middle Ages.	***Examples*** • Abu Kamil, Al-Khwarizmi, and Abu'l-Wafa
HM.LMIR.4.6	Describe the algebraic and geometric contributions of Chinese mathematicians in the Middle Ages.	***Examples*** • Liu Hui, Wang Xiatong, and Qin Jiushao
European mathematics emerges from the dark ages.		
HM.LMIR.4.7	Describe the transition of Hindu-Arabic numerals from regional use in the 10th century to wide-spread use in the 15th century.	***Example*** • Fibonacci's argument for the use of the numerals versus the Italian abacists' arguments against their use
HM.LMIR.4.8	Describe the transmission of ideas from the Greeks, through the Islamic peoples, to medieval Europe.	

HM.LMIR.4.9	Describe the influence of the Catholic Church and Charlemagne on the establishment of mathematics as one of the central pillars of education.	
HM.LMIR.4.10	Use historical multiplication and division algorithms.	***Example*** • Gelosia

LOGICAL, MATHEMATICAL & INVESTIGATIVE REASONING – *Mathematics of the Classical Era*		
HM.LMIR.5: Engage in the mathematical accomplishments of Europe in the 15th century through the early 17th century in order to grasp the foundational aspects of modern mathematics.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
HM.LMIR.5.1	Use historical multiplication and division algorithms.	***Example*** • Napier's rods
HM.LMIR.5.2	Use Cardano's cubic formula to find a solution to a cubic equation.	
HM.LMIR.5.3	Explain the cultural factors that encouraged the development of algebra in 15th century Italy, and how this development influenced mathematical thought throughout Europe.	
HM.LMIR.5.4	Identify the works of Galileo, Copernicus, and Kepler as a landmark in scientific thought, describe the conflict between their explanation of the workings of the solar system and then-current perspectives, and contrast their works to those of Aristotle.	
HM.LMIR.5.5	Describe the mathematical contributions of Fermat, Pascal, and Descartes.	

LOGICAL, MATHEMATICAL & INVESTIGATIVE REASONING – *Modern Mathematics*		
HM.LMIR.6: Engage in the mathematical and cultural accomplishments of the world's societies in the late 17th century through the early 20th century in order to grasp the foundational aspects of modern mathematics.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
The origins of calculus		
HM.LMIR.6.1	Determine tangents to quadratic curves using the algebraic techniques of Fermat, Barrow, and Newton.	
HM.LMIR.6.2	Describe the influence the French Revolution had on mathematics education.	***Examples*** • The establishment of the Ecole Normale and the Ecole Polytechnique under Monge, Lagrange, Legendre, Laplace served as a model of modern universities.

Non-Euclidean geometry		
HM.LMIR.6.3	Prove that the summit angles of an isosceles birectangle are congruent, but that it is impossible to prove they are right without referring to the parallel postulate or one of its consequences.	
HM.LMIR.6.4	Compare and contrast the hypotheses of the acute angle (Hyperbolic), the right angle (Euclidean), and the obtuse angle (Spherical).	
HM.LMIR.6.5	Prove that under the hypothesis of the acute angle, similarity implies congruence.	
HM.LMIR.6.6	Describe the societal factors that inhibited the development of non-Euclidean geometry.	
Abstract algebra and number theory		
HM.LMIR.6.7	Add, subtract, and multiply two quaternions.	
HM.LMIR.6.8	Investigate abstract algebra and group-theoretic concepts.	
HM.LMIR.6.9	Identify whether a given set with a binary operation is a group.	
HM.LMIR.6.10	Explain how the ancient Greek pattern of material axiomatics evolved into abstract axiomatics.	***Examples*** • Non-Euclidean geometry, non-commutative algebra
HM.LMIR.6.11	Solve simple linear congruences of the form $ax = b$ mod m.	
HM.LMIR.6.12	Use Fermat's Little Theorem and Euler's Theorem to simplify expressions of the form a^k mod m.	
HM.LMIR.6.13	Use Gauss' Law of Quadratic Reciprocity to determine quadratic residues of two odd primes; i.e., solve quadratic congruences of the form $x^2 = p$ mod q.	
HM.LMIR.6.14	Verify that the real primes which can be expressed as the sum of two squares are no longer prime in the field of Gaussian integers.	
The nature of mathematicians in the 17^{th}, 18^{th}, and 19^{th} centuries.		
HM.LMIR.6.15	Describe the mathematical contributions of Newton, Euler, and Gauss.	
HM.LMIR.6.16	Explore the history of African American mathematicians in the 17^{th}, 18^{th}, and 19^{th} centuries and describe their contributions to mathematics.	***Example*** • Benjamin Banneker
HM.LMIR.6.17	Explore the history of female mathematicians in the 17^{th}, 18^{th}, and 19^{th} centuries and describe their contributions to mathematics.	***Examples*** • Maria Gaetana Agnesi, Sophie Germain, Sonja Kovalevsky, Ada Lovelace

LOGICAL, MATHEMATICAL & INVESTIGATIVE REASONING – *20TH Century Mathematics*		
HM. LMIR.7: Investigate and describe modern mathematicians and their contributions to mathematics.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
The modern nature of mathematics		
HM.LMIR.7.1	Investigate the implications of infinite sets of real numbers.	
HM.LMIR.7.2	Compare and contrast denumerable and nondenumerable sets.	
HM.LMIR.7.3	Identify algebraic and transcendental numbers.	
HM.LMIR.7.4	Describe the mathematical contributions of Cantor.	***Examples*** • Transfinite numbers; the continuum hypothesis
HM.LMIR.7.5	Describe the implications of Klein's Erlangen Programme and Gödel's Incompleteness Theorem on the nature of mathematical discovery and proof.	
The modern nature of mathematicians		
HM.LMIR.7.6	Explore the history of 20th century African American mathematicians and describe their contributions to mathematics.	***Examples*** • David Blackwell, Euphemia Lofton Haynes, Katherine Johnson
HM.LMIR.7.7	Explore the history of 20th century female mathematicians and describe their contributions to mathematics.	***Examples*** • Emmy Noether, Julia Robinson
HM.LMIR.7.8	Explore the history of 20th century Indian, Asian, Hispanic, Latin American mathematicians and describe their contributions to mathematics.	

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

Mathematical Practices	
HM.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
HM.MP.1	Make sense of problems and persevere in solving them.
HM.MP.2	Reason abstractly and quantitatively.
HM.MP.3	Construct viable arguments and critique the reasoning of others.
HM.MP.4	Model with mathematics.
HM.MP.5	Use appropriate tools strategically.
HM.MP.6	Attend to precision.
HM.MP.7	Look for and make use of structure.
HM.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

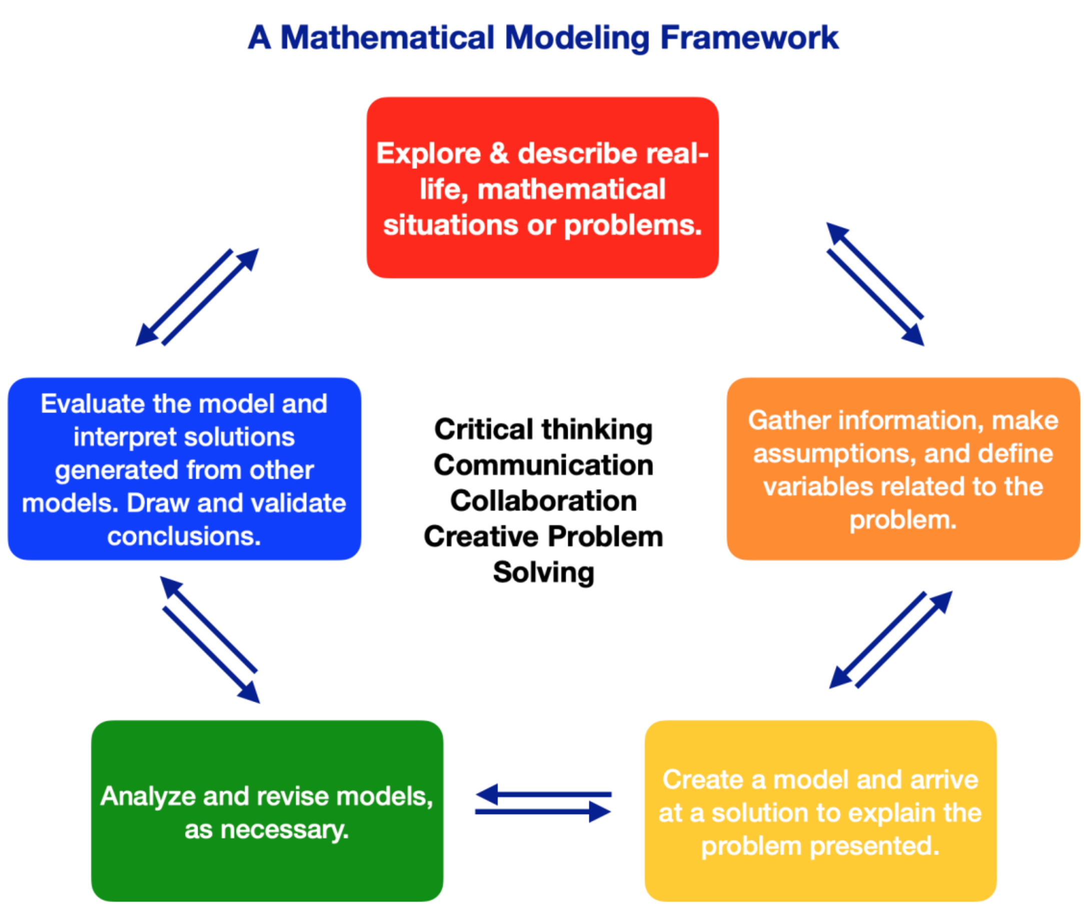

Image adapted from: Suh, Matson, Seshaiyer, 2017

FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

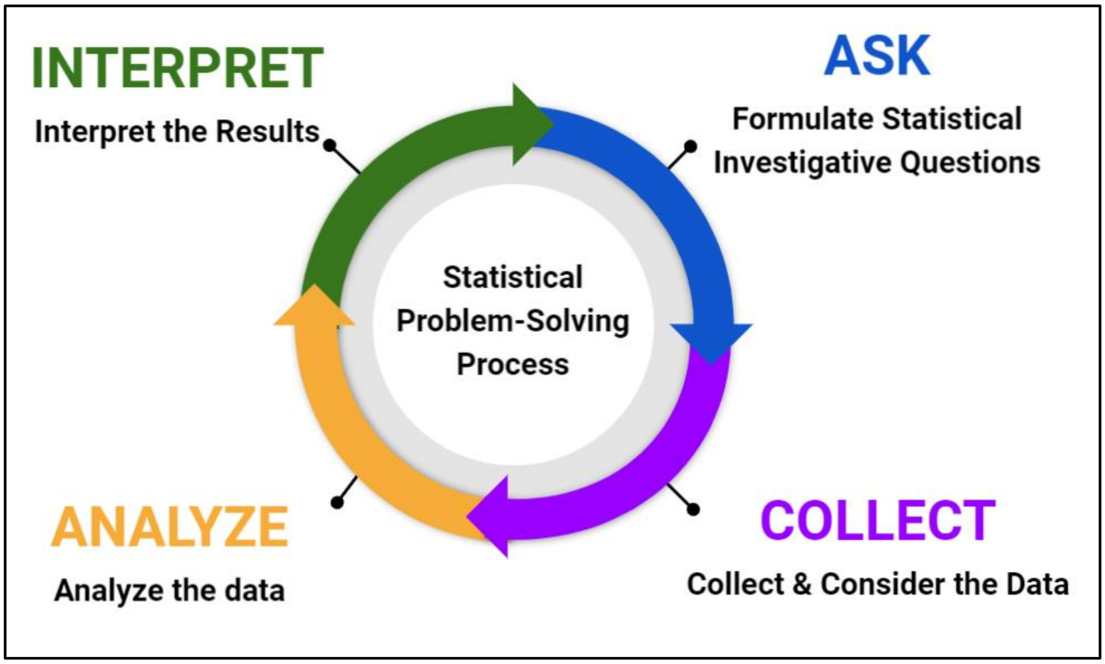

Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. **Formulate Statistical Investigative Questions**
Ask questions that anticipate variability.

II. **Collect & Consider the Data**
Ensure that data collection designs acknowledge variability.

III. **Analyze the Data**
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. **Interpret the Results**
Answer statistical investigative questions based on the collected data.