

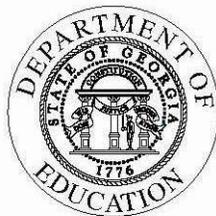


# **CRCT Content Descriptions**

**Based on the State-Mandated Content Standards**

## **Mathematics**

**Grades 3–8**



***Dr. John D. Barge, State School Superintendent***

**Updated August 2013**

**Criterion-Referenced Competency Tests (CRCT)**

## Content Descriptions Mathematics

Georgia law, as amended by the A+ Education Reform Act of 2000, requires that all students in grades one through eight take the CRCT in the content areas of Reading, English/Language Arts, and Mathematics. Students in grades three through eight are also assessed in Science and Social Studies. The CRCT are administered in the late spring of each year. These tests are designed to measure student achievement of the state-mandated content standards for Reading, English/Language Arts, Mathematics, Science, and Social Studies.

*Implementation of the CRCT program is subject to appropriation by the General Assembly. Due to budget constraints, the CRCT will not be administered in grades one and two in spring 2014.*

### Program Purpose

The CRCT are designed to measure student acquisition and understanding of the knowledge, concepts, and skills set forth in the state-mandated content standards. The assessment program serves as a measure of the quality of education in the state. Reports yielding information on academic achievement at the student, class, school, system, and state levels are produced annually.

In accordance with Georgia law and State Board rule, CRCT in specified grades and subjects are used for promotion criteria. Third grade students are required to show proficiency on the Reading CRCT in order to be considered for promotion to the next grade level. Fifth grade and eighth grade students are required to show proficiency on both the Reading and Mathematics CRCT in order to be promoted to the next grade level. Students who do not achieve proficiency must be offered remediation and a retest.

### CRCT Content Descriptions

The CRCT Content Descriptions are provided to acquaint Georgia educators with the content coverage of the CRCT. Only the knowledge, concepts, and skills reflected in the state-mandated content standards will be assessed on the CRCT. Committees of Georgia educators reviewed the standards and provided guidance for the assessment program.

It is important to note that some standards are better suited for classroom or individual assessment rather than large-scale paper-pencil assessment. While those standards designed for classroom/individual assessment are not included in the CRCT Content Descriptions, the knowledge, concepts, and skills outlined are often required for the mastery of the standards that are assessed. Therefore, the CRCT Content Descriptions are in *no way* intended to substitute for the state-mandated content standards; they are provided to help educators better understand how the standards will be assessed. Further, the CRCT Content Descriptions *by no means* suggest *when* concepts and skills should be introduced in the instructional sequence; rather, their purpose is to communicate when concepts and skills will be assessed on the CRCT. Georgia law requires educators to teach the standards set forth in the state-mandated content standards. The state-mandated content standards are located at: <http://www.georgiastandards.org>.

## **Mathematics Content Domains**

To provide reliable measures, as well as structure to the assessment program, the state-mandated content standards are grouped into content domains. Each domain is comprised of standards with similar content characteristics. The domains for Mathematics are:

Grades 3–5

Number and Operations

Algebra

Measurement and Data Analysis

Geometry

Grade 6

Number and Operations

Algebra

Geometry and Measurement

Data Analysis and Probability

Grades 7–8

Number and Operations

Algebra

Geometry

Data Analysis and Probability

The state-mandated content standards in mathematics require that mathematical concepts, as much as possible, be taught in the context of real-world phenomena. The Standards for Mathematical Practice require, among other things, students to solve single and multi-step routine and non-routine word problems while implementing a variety of problem-solving strategies. The mathematical practices ensure that concepts and skills are taught and applied within context rather than merely following a prescribed algorithm. The processes and proficiencies inherent in the mathematical practices are integrated in items across the content domains.

## **Using the Mathematics CRCT Content Descriptions**

The Mathematics CRCT Content Descriptions provide information about the content and skills assessed by the CRCT. The documents are organized by grade and content domain. The standards assessed in each domain are provided, as are the related concepts, skills, and abilities assessed. The state-mandated content standards are conceptual in nature, requiring instruction to be integrated; the concepts, knowledge, skills, and abilities described in this document should not be viewed as discrete or taught in isolation. Deep understanding by students, resulting in higher achievement, is best achieved when all the content standards are taught in an integrated, conceptual fashion.

## Mathematics

### Grade: 3

#### Standards for Mathematical Practice

The Standards for Mathematical Practice are integrated across the domains.

*Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to the mathematical content in instruction.*

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

#### Standards for Mathematical Practice

##### Associated Concepts, Skills, and Abilities

- **Make sense of problems and persevere in solving them.**

In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

- **Reason abstractly and quantitatively.**

Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.

- **Construct viable arguments and critique the reasoning of others.**

In third grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others' thinking.

- Model with mathematics.**  
 Students experiment with representing problem situations in multiple ways, including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
- Use appropriate tools strategically.**  
 Third graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
- Attend to precision.**  
 As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
- Look for and make use of structure.**  
 In third grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
- Look for and express regularity in repeated reasoning.**  
 Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of  $7 \times 8$ , they might decompose 7 into 5 and 2 and then multiply  $5 \times 8$  and  $2 \times 8$  to arrive at  $40 + 16$ , or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

## Mathematics

Grade: 3

Domain: Number and Operations

### Domain Description

Number and Operations refers to students' skills in representing and solving problems involving multiplication and division; understanding properties of multiplication and the relationship between multiplication and division; multiplying and dividing within 100; solving problems involving the four operations; using place value understanding and properties of operations to perform multi-digit arithmetic; developing an understanding of fractions as numbers.

### Standards Associated with Domain

MCC3.OA.1

MCC3.OA.7

MCC3.NF.1

MCC3.OA.2

MCC3.OA.8

MCC3.NF.2

MCC3.OA.3

MCC3.NBT.1

MCC3.NF.3

MCC3.OA.5

MCC3.NBT.2

MCC3.OA.6

MCC3.NBT.3

### Associated Concepts, Skills, and Abilities

- Interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as  $5 \times 7$ .*
- Interpret whole-number quotients of whole numbers, e.g., interpret  $56 \div 8$  as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as  $56 \div 8$ .*
- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- Apply properties of operations as strategies to multiply and divide. *Examples: If  $6 \times 4 = 24$  is known, then  $4 \times 6 = 24$  is also known. (Commutative property of multiplication.)  $3 \times 5 \times 2$  can be found by  $3 \times 5 = 15$ , then  $15 \times 2 = 30$ , or by  $5 \times 2 = 10$ , then  $3 \times 10 = 30$ . (Associative property of multiplication.) Knowing that  $8 \times 5 = 40$  and  $8 \times 2 = 16$ , one can find  $8 \times 7$  as  $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ . (Distributive property.)*
- Understand division as an unknown-factor problem. *For example, find  $32 \div 8$  by finding the number that makes 32 when multiplied by 8.*
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that  $8 \times 5 = 40$ , one knows  $40 \div 5 = 8$ ) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
- Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- Use place value understanding to round whole numbers to the nearest 10 or 100.

- Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
- Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g.,  $9 \times 80$ ,  $5 \times 60$ ) using strategies based on place value and properties of operations.
- Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $a/b$  as the quantity formed by  $a$  parts of size  $1/b$ .
- Understand a fraction as a number on the number line; represent fractions on a number line diagram.
  - Represent a fraction  $1/b$  on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into  $b$  equal parts. Recognize that each part has size  $1/b$  and that the endpoint of the part based at 0 locates the number  $1/b$  on the number line.
  - Represent a fraction  $a/b$  on a number line diagram by marking off  $a$  lengths  $1/b$  from 0. Recognize that the resulting interval has size  $a/b$  and that its endpoint locates the number  $a/b$  on the number line.
- Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (Limited to fractions with denominators 2, 3, 4, 6, and 8.)
  - Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.
  - Recognize and generate simple equivalent fractions (e.g.,  $1/2 = 2/4$ ,  $4/6 = 2/3$ ). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
  - Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form  $3 = 3/1$ ; recognize that  $6/1 = 6$ ; locate  $4/4$  and 1 at the same point of a number line diagram.*
  - Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

## Mathematics

Grade: 3

Domain: Algebra

### Domain Description

Algebra refers to students' skills in representing and solving problems involving multiplication and division; solving problems involving the four operations; identifying and explaining patterns in arithmetic.

### Standards Associated with Domain

MCC3.OA.3

MCC3.OA.8

MCC3.OA.4

MCC3.OA.9

### Associated Concepts, Skills, and Abilities

- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations:  $8 \times [box] = 48$ ,  $5 = [box] \div 3$ ,  $6 \times 6 = [box]$ .*
- Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity.
- Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

## Mathematics

### Grade: 3

#### Domain: Measurement and Data Analysis

#### Domain Description

Note that these two domains have been combined into one reporting category for the CRCT.

Measurement and Data Analysis refer to students' skills in solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects; representing and interpreting data; understanding concepts of area and relating area to multiplication and to addition; recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures.

#### Standards Associated with Domain

MCC3.MD.1

MCC3.MD.4

MCC3.MD.7

MCC3.MD.2

MCC3.MD.5

MCC3.MD.8

MCC3.MD.3

MCC3.MD.6

#### Associated Concepts, Skills, and Abilities

- Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
- Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Excludes compound units, such as  $\text{cm}^3$  and finding the geometric volume of a container. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems (problems involving notions of “times as much”).
- Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
- Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.
- Recognize area as an attribute of plane figures and understand concepts of area measurement.
  - A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
  - A plane figure which can be covered without gaps or overlaps by  $n$  unit squares is said to have an area of  $n$  square units.
- Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

- Relate area to the operations of multiplication and addition.
  - Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
  - Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
  - Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths  $a$  and  $b + c$  is the sum of  $a \times b$  and  $a \times c$ . Use area models to represent the distributive property in mathematical reasoning.
  - Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.
- Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

**Mathematics****Grade: 3****Domain: Geometry****Domain Description**

Geometry refers to students' skills in reasoning with shapes and their attributes.

**Standards Associated with Domain**

MCC3.G.1

MCC3.G.2

**Associated Concepts, Skills, and Abilities**

- Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
- Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as  $\frac{1}{4}$  of the area of the shape.*

## Mathematics

Grade: 4

### Standards for Mathematical Practice

The Standards for Mathematical Practice are integrated across the domains.

*Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to the mathematical content in instruction.*

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

### Standards for Mathematical Practice

#### Associated Concepts, Skills, and Abilities

- **Make sense of problems and persevere in solving them.**

In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

- **Reason abstractly and quantitatively.**

Fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.

- **Construct viable arguments and critique the reasoning of others.**

In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.

- **Model with mathematics.**  
 Students experiment with representing problem situations in multiple ways, including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
- **Use appropriate tools strategically.**  
 Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
- **Attend to precision.**  
 As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
- **Look for and make use of structure.**  
 In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems, such as tree diagrams and arrays, to the multiplication principle of counting. They generate number or shape patterns that follow a given rule.
- **Look for and express regularity in repeated reasoning.**  
 Students in fourth grade should notice repetitive actions in computation to make generalizations. Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

## Mathematics

Grade: 4

Domain: Number and Operations

### Domain Description

Number and Operations refers to students' skills in using the four operations with whole numbers to solve problems; gaining familiarity with factors and multiples; generalizing place value understanding for multi-digit whole numbers; using place value understanding and properties of operations to perform multi-digit arithmetic; extending understanding of fraction equivalence and ordering; building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers; understanding decimal notation for fractions, and comparing decimal fractions. Expectations in this domain are limited to whole numbers less than or equal to 1,000,000 and fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

### Standards Associated with Domain

|            |            |           |
|------------|------------|-----------|
| MCC4.OA.1  | MCC4.NBT.4 | MCC4.NF.4 |
| MCC4.OA.3  | MCC4.NBT.5 | MCC4.NF.5 |
| MCC4.OA.4  | MCC4.NBT.6 | MCC4.NF.6 |
| MCC4.NBT.1 | MCC4.NF.1  | MCC4.NF.7 |
| MCC4.NBT.2 | MCC4.NF.2  |           |
| MCC4.NBT.3 | MCC4.NF.3  |           |

### Associated Concepts, Skills, and Abilities

- Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
- Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
- Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that  $700 \div 70 = 10$  by applying concepts of place value and division.*
- Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons.
- Use place value understanding to round multi-digit whole numbers to any place.
- Fluently add and subtract multi-digit whole numbers using the standard algorithm.

- Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
- Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.
- Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ :
  - Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
  - Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:*  $3/8 = 1/8 + 1/8 + 1/8$ ;  $3/8 = 1/8 + 2/8$ ;  $2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$ .
  - Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
  - Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem
- Apply and extend previous understandings of multiplication to multiply a fraction by a whole number:
  - Understand a fraction  $a/b$  as a multiple of  $1/b$ . *For example, use a visual fraction model to represent  $5/4$  as the product of  $5 \times (1/4)$ , recording the conclusion by the equation  $5/4 = 5 \times (1/4)$ .*
  - Understand a multiple of  $a/b$  as a multiple of  $1/b$ , and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express  $3 \times (2/5)$  as  $6 \times (1/5)$ , recognizing this product as  $6/5$ . (In general,  $n \times (a/b) = (n \times a)/b$ .)*
  - Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat  $3/8$  of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*
- Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express  $3/10$  as  $30/100$ , and add  $3/10 + 4/100 = 34/100$ .*

- Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
- Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual model.

## Mathematics

Grade: 4

Domain: Algebra

### Domain Description

Algebra refers to students' skills in using the four operations with whole numbers to solve problems; generating and analyzing patterns.

### Standards Associated with Domain

MCC4.OA.2

MCC4.OA.3

MCC4.OA.5

### Associated Concepts, Skills, and Abilities

- Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
- Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

## Mathematics

Grade: 4

Domain: Measurement and Data Analysis

### Domain Description

Note that these two domains have been combined into one reporting category for the CRCT.

Measurement and Data Analysis refer to students' skills in solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit; representing and interpreting data; understanding concepts of angle and measure angles.

### Standards Associated with Domain

MCC4.MD.1

MCC4.MD.4

MCC4.MD.6

MCC4.MD.2

MCC4.MD.5

MCC4.MD.7

MCC4.MD.3

### Associated Concepts, Skills, and Abilities

- Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4-ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36)...*
- Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. Situations should include those that require decomposing and composing units of measure.
- Apply the area and perimeter formulas for rectangles in real real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
- Make a line plot to display a data set of measurements in fractions of a unit ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
- Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
  - An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through  $\frac{1}{360}$  of a circle is called a “one-degree angle,” and can be used to measure angles
  - An angle that turns through  $n$  one-degree angles is said to have an angle measure of  $n$  degrees.

- Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
- Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

## **Mathematics**

**Grade:** 4

**Domain:** Geometry

### **Domain Description**

Geometry refers to students' skills in drawing and identifying lines and angles, and classifying shapes by properties of their lines and angles.

### **Standards Associated with Domain**

MCC4.G.1

MCC4.G.2

MCC4.G.3

### **Associated Concepts, Skills, and Abilities**

- Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
- Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
- Recognize a line of symmetry for a two-dimensional figure as a line across the figure, such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

## Mathematics

### Grade: 5

#### Standards for Mathematical Practice

The Standards for Mathematical Practice are integrated across the domains.

*Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to the mathematical content in instruction.*

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

#### Standards for Mathematical Practice

##### Associated Concepts, Skills, and Abilities

- **Make sense of problems and persevere in solving them.**

Students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?” and “Does this make sense?” and “Can I solve the problem in a different way?”

- **Reason abstractly and quantitatively.**

In grade 5, students should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

- Construct viable arguments and critique the reasoning of others.**  
 In grade 5, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.
- Model with mathematics.**  
 In grade 5, students experiment with representing problem situations in multiple ways, including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.
- Use appropriate tools strategically.**  
 In grade 5, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real-world data.
- Attend to precision.**  
 Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
- Look for and make use of structure.**  
 In fifth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply, and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.
- Look for and express regularity in repeated reasoning.**  
 In grade 5, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

## Mathematics

Grade: 5

Domain: Number and Operations

### Domain Description

Number and Operations refers to students' skills in understanding the place value system; performing operations with multi-digit whole numbers and with decimals to hundredths; using equivalent fractions as a strategy to add and subtract fractions; applying and extending previous understandings of multiplication and division to multiply and divide fractions.

### Standards Associated with Domain

MCC5.NBT.1

MCC5.NBT.2

MCC5.NBT.3

MCC5.NBT.4

MCC5.NBT.5

MCC5.NBT.6

MCC5.NBT.7

MCC5.NF.1

MCC5.NF.2

MCC5.NF.3

MCC5.NF.4

MCC5.NF.5

MCC5.NF.6

MCC5.NF.7

### Associated Concepts, Skills, and Abilities

- Recognize that in a multi-digit number, a digit in the one place represents 10 times as much as it represents in the place to its right and  $1/10$  of what it represents in the place to its left.
- Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
- Read, write, and compare decimals to the thousandths.
  - Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g.,  $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$ .
  - Compare two decimals to thousandths based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons.
- Use place value understanding to round decimals to any place.
- Fluently multiply multi-digit whole numbers using the standard algorithm.
- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
- Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,  $2/3 + 5/4 = 8/12 + 15/12 = 23/12$ . (In general,  $a/b + c/d = (ad + bc)/bd$ .)*

- Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result  $2/5 + 1/2 = 3/7$ , by observing that  $3/7 < 1/2$ .*
- Interpret a fraction as division of the numerator by the denominator ( $a/b = a \div b$ ). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret  $3/4$  as the result of dividing 3 by 4, noting that  $3/4$  multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size  $3/4$ . If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
- Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
  - Interpret the product  $(a/b) \times q$  as a parts of a partition of  $q$  into  $b$  equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ . *For example, use a visual fraction model to show  $(2/3) \times 4 = 8/3$ , and create a story context for this equation. Do the same with  $(2/3) \times (4/5) = 8/15$ . (In general,  $(a/b) \times (c/d) = ac/bd$ .)*
  - Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
- Interpret multiplication as scaling (resizing) by:
  - Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
  - Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence  $a/b = (n \times a)/(n \times b)$  to the effect of multiplying  $a/b$  by 1.
- Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
- Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.
  - Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for  $(1/3) \div 4$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $(1/3) \div 4 = 1/12$  because  $(1/12) \times 4 = 1/3$ .*
  - Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for  $4 \div (1/5)$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $4 \div (1/5) = 20$  because  $20 \times (1/5) = 4$ .*

- Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share  $\frac{1}{2}$  lb of chocolate equally? How many  $\frac{1}{3}$ -cup servings are in 2 cups of raisins?*

## Mathematics

Grade: 5

Domain: Algebra

### Domain Description

Algebra refers to students' skills in writing and interpreting numerical expression; analyzing patterns and relationships.

### Standards Associated with Domain

MCC5.OA.1

MCC5.OA.2

MCC5.OA.3

### Associated Concepts, Skills, and Abilities

- Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as  $2 \times (8 + 7)$ . Recognize that  $3 \times (18932 + 921)$  is three times as large as  $18932 + 921$ , without having to calculate the indicated sum or product.
- Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

## Mathematics

Grade: 5

Domain: Measurement and Data Analysis

### Domain Description

Measurement and Data Analysis refers to students' skills in converting like measurement units within a given measurement system; representing and interpreting data; understanding concepts of volume and relate volume to multiplication and addition.

### Standards Associated with Domain

MCC5.MD.1

MCC5.MD.3

MCC5.MD.5

MCC5.MD.2

MCC5.MD.4

### Associated Concepts, Skills, and Abilities

- Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.
- Make a line plot to display a data set of measurements in fractions of a unit ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
  - A cube with side length 1 unit, called a “unit cube,” is said to have a “one cubic unit” of volume, and can be used to measure volume.
  - A solid figure which can be packed without gaps or overlaps using  $n$  unit cubes is said to have a volume of  $n$  cubic units.
- Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- Relate volumes to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.
  - Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
  - Apply the formulas  $\text{Volume} = \text{length} \times \text{width} \times \text{height}$  and  $\text{Volume} = \text{area of the base} \times \text{height}$  for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.
  - Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

## Mathematics

Grade: 5

Domain: Geometry

### Domain Description

Geometry refers to students' skills in graphing points on the coordinate plane to solve real-world and mathematical problems; classifying two-dimensional figures into categories based on their properties.

### Standards Associated with Domain

MCC5.G.1

MCC5.G.3

MCC5.G.2

MCC5.G.4

### Associated Concepts, Skills, and Abilities

- Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g.,  $x$ -axis and  $x$ -coordinate,  $y$ -axis and  $y$ -coordinate).
- Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
- Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.
- Classify two-dimensional figures in a hierarchy based on properties.

## Mathematics

### Grade: 6

#### Standards for Mathematical Practice

The Standards for Mathematical Practice are integrated across the domains.

*Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to the mathematical content in instruction.*

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

#### Standards for Mathematical Practice

##### Associated Concepts, Skills, and Abilities

- **Make sense of problems and persevere in solving them.**

In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?” and “Does this make sense?” and “Can I solve the problem in a different way?”

- **Reason abstractly and quantitatively.**

In grade 6, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

- **Construct viable arguments and critique the reasoning of others.**

In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like, “How did you get that?” and “Why is that true?” and “Does that always work?” They explain their thinking to others and respond to others’ thinking.

- Model with mathematics.**

In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e., box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
- Use appropriate tools strategically.**

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
- Attend to precision.**

In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations, or inequalities.
- Look for and make use of structure.**

Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e.,  $6 + 2x = 3(2 + x)$  by distributive property) and solve equations (i.e.,  $2c + 3 = 15$ ,  $2c = 12$  by subtraction property of equality),  $c = 6$  by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real-world problems involving area and volume.
- Look for and express regularity in repeated reasoning.**

In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that  $a/b \div c/d = ad/bc$  and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

## Mathematics

### Grade: 6

### Domain: Number and Operations

#### Domain Description

Number and Operations refer to students' skills in applying and extending previous understandings of multiplication and division to divide fractions by fractions; computing fluently with multi-digit numbers and finding common factors and multiples; applying and extending previous understandings of numbers to the system of rational numbers; applying and extending previous understanding of arithmetic to algebraic expressions; understanding ratio concepts and using ratio reasoning to solve problems.

#### Standards Associated with Domain

MCC6.NS.1  
MCC6.NS.2  
MCC6.NS.3  
MCC6.NS.4

MCC6.NS.5  
MCC6.NS.6  
MCC6.NS.7  
MCC6.EE.1

MCC6.RP.3b  
MCC6.RP.3c

#### Associated Concepts, Skills, and Abilities

- Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for  $(2/3) \div (3/4)$  and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that  $(2/3) \div (3/4) = 8/9$  because  $3/4$  of  $8/9$  is  $2/3$ . (In general,  $(a/b) \div (c/d) = ad/bc$ .) How much chocolate will each person get if 3 people share  $1/2$  lb of chocolate equally? How many  $3/4$ -cup servings are in  $2/3$  of a cup of yogurt? How wide is a rectangular strip of land with length  $3/4$  mi and area  $1/2$  square mi?*
- Fluently divide multi-digit numbers using the standard algorithm.
- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
- Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express  $36 + 8$  as  $4(9 + 2)$ .*
- Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
  - Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g.,  $-(-3) = 3$ , and that 0 is its own opposite.

- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- Understand ordering and absolute value of rational numbers.
  - Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret  $-3 > -7$  as a statement that  $-3$  is located to the right of  $-7$  on a number line oriented from left to right.*
  - Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, interpret  $-3 > -7$  as a statement that  $-3$  is located to the right of  $-7$  on a number line oriented from left to right.*
  - Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of  $-30$  dollars, write  $|-30| = 30$  to describe the size of the debt in dollars.*
  - Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than  $-30$  dollars represents a debt greater than 30 dollars.*
- Write and evaluate numerical expressions involving whole-number exponents.
- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  - Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole given a part and the percent.

## Mathematics

Grade: 6

Domain: Algebra

### Domain Description

Algebra refers to students' skills in applying and extending previous understandings of arithmetic to algebraic expressions; reasoning about and solving one-variable equations and inequalities; representing and analyzing quantitative relationships between dependent and independent variables; understanding ratio concepts and using ratio reasoning to solve problems.

### Standards Associated with Domain

MCC6.EE.2

MCC6.EE.6

MCC6.NS.8

MCC6.EE.3

MCC6.EE.7

MCC6.RP.1

MCC6.EE.4

MCC6.EE.8

MCC6.RP.2

MCC6.EE.5

MCC6.EE.9

MCC6.RP.3a

### Associated Concepts, Skills, and Abilities

- Write, read, and evaluate expressions in which letters stand for numbers.
  - Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract  $y$  from 5" as  $5 - y$ .*
  - Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression  $2(8 + 7)$  as a product of two factors; view  $(8 + 7)$  as both a single entity and a sum of two terms.*
  - Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas  $V = s^3$  and  $A = 6s^2$  to find the volume and surface area of a cube with sides of length  $s = 1/2$ .*
- Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression  $3(2 + x)$  to produce the equivalent expression  $6 + 3x$ ; apply the distributive property to the expression  $24x + 18y$  to produce the equivalent expression  $6(4x + 3y)$ ; apply properties of operations to  $y + y + y$  to produce the equivalent expression  $3y$ .*
- Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions  $y + y + y$  and  $3y$  are equivalent because they name the same number regardless of which number  $y$  stands for.*
- Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

- Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  for cases in which  $p$ ,  $q$  and  $x$  are all nonnegative rational numbers.
- Write an inequality of the form  $x > c$  or  $x < c$  to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form  $x > c$  or  $x < c$  have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
- Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation  $d = 65t$  to represent the relationship between distance and time.*
- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
- Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*
- Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $3/4$  cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”*
- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

## Mathematics

Grade: 6

Domain: Geometry and Measurement

### Domain Description

Geometry and Measurement refers to students' skills in solving real-world and mathematical problems involving area, surface area, and volume; understanding ratio concepts and using ratio reasoning to solve problems.

### Standards Associated with Domain

MCC6.G.1

MCC6.G.3

MCC6.RP.3d

MCC6.G.2

MCC6.G.4

### Associated Concepts, Skills, and Abilities

- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas  $\text{Volume} = \text{length} \times \text{width} \times \text{height}$  and  $\text{Volume} = \text{area of the base} \times \text{height}$  to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
- Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

## Mathematics

Grade: 6

Domain: Data Analysis and Probability

### Domain Description

Data Analysis and Probability refer to students' skills in developing understanding of statistical variability; summarizing and describing distributions.

### Standards Associated with Domain

MCC6.SP.1

MCC6.SP.3

MCC6.SP.5

MCC6.SP.2

MCC6.SP.4

### Associated Concepts, Skills, and Abilities

- Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.*
- Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- Summarize numerical data sets in relation to their context, by:
  - Reporting the number of observations.
  - Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
  - Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
  - Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

## Mathematics

### Grade: 7

#### Standards for Mathematical Practice

The Standards for Mathematical Practice are integrated across the domains.

*Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to the mathematical content in instruction.*

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

#### Standards for Mathematical Practice

##### Associated Concepts, Skills, and Abilities

- **Make sense of problems and persevere in solving them.**

In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?” and “Does this make sense?” and “Can I solve the problem in a different way?”

- **Reason abstractly and quantitatively.**

In grade 7, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

- **Construct viable arguments and critique the reasoning of others.**

In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like, “How did you get that?” and “Why is that true?” and “Does that always work?” They explain their thinking to others and respond to others’ thinking.

- **Model with mathematics.**

In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e., box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

- **Use appropriate tools strategically.**

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.

- **Attend to precision.**

In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations, or inequalities.

- **Look for and make use of structure.**

Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e.,  $6 + 2x = 3(2 + x)$  by distributive property) and solve equations (i.e.,  $2c + 3 = 15$ ,  $2c = 12$  by subtraction property of equality),  $c = 6$  by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real-world problems involving area and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.

- **Look for and express regularity in repeated reasoning.**

In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that  $a/b \div c/d = ad/bc$  and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

## Mathematics

Grade: 7

Domain: Number and Operations

### Domain Description

Number and Operations refers to students' skills in applying and extending previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers; solving real-life and mathematical problems using numerical and algebraic expressions and equations.

### Standards Associated with Domain

MCC7.NS.1

MCC7.NS.3

MCC7.NS.2

MCC7.EE.3

### Associated Concepts, Skills, and Abilities

- Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
  - Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
  - Understand  $p + q$  as the number located a distance  $|q|$  from  $p$ , in the positive or negative direction depending on whether  $q$  is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
  - Understand subtraction of rational numbers as adding the additive inverse,  $p - q = p + (-q)$ . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
  - Apply properties of operations as strategies to add and subtract rational numbers.
- Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
  - Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as  $(-1)(-1) = 1$  and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
  - Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If  $p$  and  $q$  are integers, then  $-(p/q) = (-p)/q = p/(-q)$ . Interpret quotients of rational numbers by describing real-world contexts.
  - Apply properties of operations as strategies to multiply and divide rational numbers.
  - Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- Solve real-world and mathematical problems involving the four operations with rational numbers.
- Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as

appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional  $\frac{1}{10}$  of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar  $9\frac{3}{4}$  inches long in the center of a door that is  $27\frac{1}{2}$  inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

## Mathematics

Grade: 7

Domain: Algebra

### Domain Description

Algebra refers to students' skills in using properties of operations to generate equivalent expressions; solving real-life and mathematical problems using numerical and algebraic expressions and equations; analyzing proportional relationships and using them to solve real-world and mathematical problems.

### Standards Associated with Domain

MCC7.EE.1

MCC7.EE.4

MCC7.RP.2

MCC7.EE.2

MCC7.RP.1

MCC7.RP.3

### Associated Concepts, Skills, and Abilities

- Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,  $a + 0.05a = 1.05a$  means that “increase by 5%” is the same as “multiply by 1.05.”*
- Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
  - Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
  - Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*
- Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks  $\frac{1}{2}$  mile in each  $\frac{1}{4}$  hour, compute the unit rate as the complex fraction  $\frac{1/2}{1/4}$  miles per hour, equivalently 2 miles per hour.*
- Recognize and represent proportional relationships between quantities.
  - Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
  - Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

- Represent proportional relationships by equations. *For example, if total cost  $t$  is proportional to the number  $n$  of items purchased at a constant price  $p$ , the relationship between the total cost and the number of items can be expressed as  $t = pn$ .*
- Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.
- Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

## **Mathematics**

**Grade:** 7

**Domain:** Geometry

### **Domain Description**

Measurement and Geometry refers to students' skills in drawing, constructing, and describing geometrical figures and describing the relationships between them; solving real-life and mathematical problems involving angle measure, area, surface area, and volume.

### **Standard Associated with Domain**

MCC7.G.1

MCC7.G.3

MCC7.G.5

MCC7.G.2

MCC7.G.4

MCC7.G.6

### **Associated Concepts, Skills, and Abilities**

- Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
- Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
- Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

## Mathematics

Grade: 7

Domain: Data Analysis and Probability

### Domain Description

Data Analysis and Probability refer to students' skills in using random sampling to draw inferences about a population; drawing informal comparative inferences about two populations; investigating chance processes and developing, using, and evaluating probability models.

### Standards Associated with Domain

MCC7.SP.1

MCC7.SP.4

MCC7.SP.7

MCC7.SP.2

MCC7.SP.5

MCC7.SP.8

MCC7.SP.3

MCC7.SP.6

### Associated Concepts, Skills, and Abilities

- Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
- Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
- Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
- Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
- Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

- Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
  - Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
  - Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*
- Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
  - Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
  - Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
  - Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

## Mathematics

### Grade: 8

#### Standards for Mathematical Practice

The Standards for Mathematical Practice are integrated across the four domains.

*Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to the mathematical content in instruction.*

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

#### Standards for Mathematical Practice

##### Associated Concepts, Skills, and Abilities

- **Make sense of problems and persevere in solving them.**

In grade 8, students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?” and “Does this make sense?” and “Can I solve the problem in a different way?”

- **Reason abstractly and quantitatively.**

In grade 8, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

- **Construct viable arguments and critique the reasoning of others.**

In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like, “How did you get that?” and “Why is that true?” and “Does that always work?” They explain their thinking to others and respond to others’ thinking.

- Model with mathematics.**

In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
- Use appropriate tools strategically.**

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.
- Attend to precision.**

In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.
- Look for and make use of structure.**

Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
- Look for and express regularity in repeated reasoning.**

In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

## Mathematics

Grade: 8

Domain: Number and Operations

### Domain Description

Number and Operations refers to students' skills in knowing that there are numbers that are not rational, and approximating them by rational numbers; working with radicals and integer exponents.

### Standards Associated with Domain

MCC8.NS.1

MCC8.EE.1

MCC8.EE.3

MCC8.NS.2

MCC8.EE.2

MCC8.EE.4

### Associated Concepts, Skills, and Abilities

- Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.
- Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ). *For example, by truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
- Know and apply the properties of integer exponents to generate equivalent numerical expressions.
- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example,  $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ .*
- Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. Evaluate square roots or small perfect squares and cube roots of small perfect cubes. Know that the square root of 2 is irrational. *For example, estimate the population of the United States as  $3 \times 10^8$  and the population of the world as  $7 \times 10^9$ , and determine that the world population is more than 20 times larger.*
- Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

## Mathematics

Grade: 8

Domain: Algebra

### Domain Description

Algebra refers to students' skills in understanding the connections between proportional relationships, lines, and linear equations; analyzing and solving linear equations and pairs of simultaneous linear equations; defining, evaluating, and comparing functions; using functions to model relationships between quantities.

### Standards Associated with Domain

MCC8.EE.5

MCC8.EE.8

MCC8.F.3

MCC8.EE.6

MCC8.F.1

MCC8.F.4

MCC8.EE.7

MCC8.F.2

MCC8.F.5

### Associated Concepts, Skills, and Abilities

- Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
- Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .
- Solve linear equations in one variable.
  - Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form  $x = a$ ,  $a = a$ , or  $a = b$  results (where  $a$  and  $b$  are different numbers).
  - Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
- Analyze and solve pairs of simultaneous linear equations.
  - Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
  - Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,  $3x + 2y = 5$  and  $3x + 2y = 6$  have no solution because  $3x + 2y$  cannot simultaneously be 5 and 6.*
  - Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented*

*by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

- Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points  $(1,1)$ ,  $(2,4)$  and  $(3,9)$ , which are not on a straight line.*
- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

## Mathematics

Grade: 8

Domain: Geometry

### Domain Description

Geometry refers to students' skills in understanding congruence and similarity using physical models, transparencies, or geometry software; understanding and applying the Pythagorean Theorem; solving real-world and mathematical problems involving volume of cylinders, cones, and spheres.

### Standards Associated with Domain

MCC8.G.1

MCC8.G.2

MCC8.G.3

MCC8.G.4

MCC8.G.5

MCC8.G.6

MCC8.G.7

MCC8.G.8

MCC8.G.9

### Associated Concepts, Skills, and Abilities

- Verify experimentally the properties of rotations, reflections, and translations:
  - Lines are taken to lines, and line segments to line segments of the same length.
  - Angles are taken to angles of the same measure.
  - Parallel lines are taken to parallel lines.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
- Explain a proof of the Pythagorean Theorem and its converse.
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
- Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

## Mathematics

Grade: 8

Domain: Data Analysis and Probability

### Domain Description

Data Analysis and Probability refer to students' skills in investigating patterns of association in bivariate data.

### Standards Associated with Domain

MCC8.SP.1

MCC8.SP.3

MCC8.SP.2

MCC8.SP.4

### Associated Concepts, Skills, and Abilities

- Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
- Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*