



# Content Descriptions

Based on the Georgia Performance Standards

## GPS Geometry



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*"Making Education Work for All Georgians"*

## **Introduction**

The State Board of Education is required by Georgia law (A+ Educational Reform Act of 2000, O.C.G.A. §20-2-281) to adopt End-of-Course Tests (EOCT) designed to measure student achievement in core subjects in grades nine through twelve. With educator input and State Board of Education approval, eight content areas were designated in 2001 to be tested. The Georgia Performance Standards (GPS) in mathematics were adopted by the State Board of Education in July 2004, and the GPS Geometry EOCT was developed based on these standards.

## **Program Purpose**

The EOCT are designed to improve student achievement by assessing student performance on the standards specific to each course tested. Student performance on each EOCT is provided to schools for diagnostic and remedial use. These results are used to help make instruction more effective and to ensure that all Georgia students have access to a rigorous curriculum that meets high academic standards. These results are also used for student accountability and to gauge the quality of education in the state. The EOCT are the final exams for each EOCT course. For students in grade 10 or above beginning the 2011-2012 school year, the final grade in each course is calculated by weighing the course grade 85% and the EOCT score 15%. For students in grade 9 beginning the 2011-2012 school year and later, the final grade in each course is calculated by weighing the course grade 80% and the EOCT score 20% (State Board Rule 160-4-2-.13). The student must have a final grade of at least 70 to pass the course and to earn credit toward graduation.

## **EOCT Content Descriptions**

The EOCT Content Descriptions are provided to acquaint Georgia educators with the content coverage of the EOCT. Only the knowledge, concepts, and skills addressed in the GPS are assessed on the EOCT. Committees of Georgia educators reviewed the curriculum and provided guidance for the assessment program.

It is important to note that some curricular standards are better suited for classroom or individual assessment rather than large-scale, summative assessment. While those curricular standards designed for classroom/individual assessment are not included in the Content Descriptions, the knowledge, concepts, and skills outlined are often required for the mastery of the standards that are assessed. Therefore, the EOCT Content Descriptions are in *no way* intended to substitute for the GPS; they are provided to help educators better understand how the curriculum will be assessed. Further, the EOCT Content Descriptions *by no means* suggest *when* concepts and skills should be introduced in the instructional sequence; rather, their purpose is to communicate when concepts and skills will be assessed on the EOCT. Georgia law requires educators to teach the standards set forth in the state-adopted curriculum (i.e., the GPS). The GPS are located at <http://www.georgiastandards.org>.

## **GPS Geometry Domains**

To provide reliable measures of student achievement and to give structure to the assessment program, the content standards contained in the GPS were grouped into content domains. Each domain was created by combining standards that share similar content characteristics. Three domains were identified for GPS Geometry.

- **Geometry**  
*Students will demonstrate the ability to explore, understand, and use the formal language of reasoning and justification in both algebraic and geometric contexts; apply properties of polygons; determine distances and points of concurrence; understand and apply properties of right triangles and right-triangle trigonometry; understand and apply properties of circles and spheres and use them in determining related measures.*
- **Algebra**  
*Students will demonstrate the ability to investigate exponential functions using numerical, analytical, and graphical approaches, focusing on the use of these functions in problem-solving situations; solve equations and inequalities related to these functions; explore the inverses of functions.*
- **Data Analysis and Probability**  
*Students will demonstrate the ability to pose questions to be answered by collecting data; organize, represent, investigate, interpret, and make inferences from data; compare data for two different samples and/or populations using measures of central tendency and measures of spread, including standard deviation.*

## **Process Standards**

The GPS in mathematics require content to be taught in conjunction with process skills identified as the process standards. These process standards are necessary for students to master each of the mathematics content standards. Problem solving, reasoning, representation, connections, and communication are the critical dimensions of mathematical proficiency that all students need.

The concepts and skills inherent in the process standards are integrated in items across the three content domains.

### **Overview of the Process Standards**

- Students will solve problems (using appropriate technology).
- Students will reason and evaluate mathematical arguments.
- Students will communicate mathematically.
- Students will make connections among mathematical ideas and to other disciplines.
- Students will represent mathematics in multiple ways.

### **Associated GPS Standards**

MM2P1 through MM2P5

### **Associated GPS Concepts and Skills**

- Building new mathematical knowledge through problem solving.
- Solving problems that arise in mathematics and in other contexts.
- Applying and adapting a variety of appropriate strategies to solve problems.
- Monitoring and reflecting on the process of mathematical problem solving.
- Recognizing reasoning and proof as fundamental aspects of mathematics.
- Making and investigating mathematical conjectures.
- Developing and evaluating mathematical arguments and proofs.
- Selecting and using various types of reasoning and methods of proof.
- Organizing and consolidating mathematical thinking through communication.
- Communicating mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyzing and evaluating mathematical thinking and strategies of others.
- Using the language of mathematics to express mathematical ideas precisely.
- Recognizing and using connections among mathematical ideas.
- Understanding how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognizing and applying mathematics in contexts outside of mathematics.
- Creating and using representations to organize, record, and communicate mathematical ideas.
- Selecting, applying, and translating among mathematical representations to solve problems.
- Using representations to model and interpret physical, social, and mathematical phenomena.

## GPS Geometry

### Domain: Geometry

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#### Overview of the Domain

- Students will investigate properties of geometric figures in a coordinate plane.
- Students will understand and use the language of mathematical argument and justification.
- Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
- Students will identify and use special right triangles.
- Students will define and apply sine, cosine, and tangent ratios to right triangles.
- Students will understand and apply the properties of circles and their associated segments and angles.
- Students will find and compare the measures of spheres.

#### Associated GPS Standards

MM1G1 MM1G2 MM1G3 MM2G1 MM2G2 MM2G3 MM2G4

#### Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- determine the distance between two points on a coordinate grid
  - find distances between two points on the same horizontal or vertical line
  - use various methods (such as the distance formula or Pythagorean theorem) to calculate the distance when given two points with coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$
- calculate the distance between a point and a line on a coordinate grid
  - understand that distance between a point and a line is measured along a perpendicular
  - explore and understand perpendicular lines
  - utilize the distance formula or other methods when appropriate
- calculate the midpoint of a segment
  - determine the midpoint of a horizontal or a vertical line
  - use various methods (such as the midpoint formula, similar triangles, averaging the endpoints, etc.) to locate the midpoint when given two points on a coordinate grid with coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$
  - find an endpoint of a line segment when given its other endpoint and midpoint
- understand the distance formula as an application of the Pythagorean theorem
  - explore how the distance formula is derived from the Pythagorean theorem
  - find the length of a hypotenuse or a leg of a triangle plotted on a coordinate grid
- use the coordinate plane to investigate properties of and verify conjectures related to triangles and quadrilaterals
  - use relationship properties of side measures, slopes, diagonals, etc., of triangles and quadrilaterals to determine unknown side lengths

- use side and angle theorems to prove triangles and quadrilaterals are similar and/or congruent
- understand the minimal information necessary to conclude that two triangles are congruent
- utilize properties of parallel and perpendicular lines and angle bisectors to construct or draw the missing measure of a polygon, given a known relationship to another triangle or quadrilateral
- utilize the distance formula to classify figures as triangles and quadrilaterals (e.g., squares, rectangles, trapezoids, kites, parallelograms, and rhombuses)
- determine missing vertices of a triangle or a quadrilateral by utilizing side and angle relationships of a given figure
- use conjecture, inductive reasoning, deductive reasoning, counterexamples, and indirect proof, as appropriate, in mathematical and real-world applications
  - utilize prior knowledge of quadrilateral relationships to prove or disprove classification of quadrilaterals
  - utilize paragraph proofs, flow proofs, two-column proofs, or any other method that relays clear communication to justify conclusions regarding polygon relationships
- explore and use the relationships among conditional statements
  - determine the hypothesis and conclusion of a conditional statement, in word or in mathematical form
  - write the converse of a conditional statement by exchanging the hypothesis and conclusion
  - realize that the inverse of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement
  - understand that the contrapositive of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement and then the interchange of the hypothesis and conclusion
  - utilize conditional statements to prove algebraic, geometric, and real-world concepts
- determine the sum of interior and exterior angles in a polygon
  - utilize angle relationships of a polygon to find a missing measure or the total interior angles measures of a specific polygon
  - utilize angle relationships, such as linear pairs and the exterior angle sum theorem, to determine an exterior angle of a polygon
- understand inequality theorems involving triangles
  - apply the triangle inequality theorem to determine if given side lengths form a triangle
  - utilize the side-angle inequality theorem to determine the largest and smallest angle or side in a triangle
  - use the exterior-angle inequality theorem, linear pairs, or the sum of the angles of a triangle adding to  $180^\circ$  to determine the measure of an exterior angle of a triangle when given two remote interior angles

- understand congruence postulates and theorems for triangles
  - identify and use SSS, SAS, ASA, AAS, HL to prove/justify that given triangles are congruent through proofs including two-column, paragraph, and flow chart, or any other valid form of communication
  - understand that SSA and AAA are not valid methods to prove triangle congruency
- use and prove properties of and relationships among the following special quadrilaterals:
  - parallelograms—understand that the opposite sides are congruent, the opposite angles are congruent, the consecutive angles are supplementary, and the diagonals bisect each other
  - rectangles—understand that the diagonals are congruent and that rectangles have all the properties of a parallelogram
  - rhombuses—understand that the diagonals are perpendicular and bisect a pair of opposite angles and that rhombuses have all the properties of a parallelogram
  - squares—understand that the diagonals are perpendicular and congruent and that squares have all the properties of a parallelogram
  - isosceles trapezoids—understand that they have only one pair of parallel sides and congruent diagonals
  - kites—understand that two pairs of consecutive sides are congruent, the diagonals are perpendicular, one diagonal is bisected, and angles between non-congruent sides are congruent to each other
- use properties to identify and classify quadrilaterals
- use theorems to find unknown angle and side measures
- find and use points of concurrency, such as incenter, orthocenter, circumcenter, and centroid, in triangles
  - use bisectors, medians, and altitudes to find points of concurrency
  - locate centers of circles inscribed in or circumscribed about triangles
  - make decisions about which center best meets a given set of conditions
- determine the lengths of sides of  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles
  - use the fact that the length of the hypotenuse is twice the length of the shorter leg and the length of the longer leg is  $\sqrt{3}$  times the length of the shorter leg to determine the lengths of all three sides given any one of the three sides
  - solve problems that involve application of these side length relationships
- determine the lengths of sides of  $45^\circ$ - $45^\circ$ - $90^\circ$  triangles
  - use the fact that the length of the hypotenuse is  $\sqrt{2}$  times the length of each leg to determine the lengths of all sides of a triangle given the length of any one of the three sides
  - solve problems that involve application of these side length relationships
- understand and apply the basic trigonometric ratios for right triangles
  - apply trigonometric ratios to find unknown measures of sides; solve real-life problems involving trigonometric ratios of right triangles; solve for the length of any side of a right triangle given sufficient information; work

from a model presented graphically or verbally, including situations involving angles of elevation and/or depression

- explain the relationship between the trigonometric ratios of complementary angles
  - understand that  $\sin \theta = \cos (90 - \theta)$ ,  $\cos \theta = \sin (90 - \theta)$ , and if  $\tan \theta = \frac{x}{y}$ , then  $\tan (90 - \theta) = \frac{y}{x}$
- understand and use properties of and relationships among radii, chords, tangents, and secants of circles as an application of triangle similarity, e.g.,
  - relationship of tangent to radius to point of tangency
  - congruence of tangents from a given point outside the circle
  - products of lengths of segments created by intersecting chords
  - products of lengths of segments created by two secants or by a secant and a tangent from a given point outside the circle
- understand and use properties of and relationships among angles related to circles, such as central, inscribed, and related angles, e.g.,
  - relationship between arc measures and angle measures
  - relationship between measures of central angles and inscribed angles
- use the properties of circles to solve problems involving the length of an arc and the area of a sector
- justify measurements and relationships in circles using geometric and algebraic properties
- understand, use, and apply the surface area and volume of a sphere
  - calculate surface area and volume of a sphere
  - use the formulas for surface area and volume of a sphere to find other values, including:
    - radius of a sphere
    - diameter of a sphere
    - circumference of a great circle of a sphere
    - area of a great circle of a sphere
  - determine the effect on surface area and volume when changing the radius or diameter of a sphere or vice versa



## GPS Geometry

### Domain: Algebra

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#### Overview of the Domain

- Students will explore exponential functions.
- Students will explore the inverses of functions.

#### Associated GPS Standards

MM2A2      MM2A5

#### Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- extend properties of exponents to include all integer exponents and use expressions with integer exponents to model real-world functional relationships
  - apply product of powers, quotient of powers, power of a power, power of a product, and power of a quotient to simplify and/or evaluate expressions
  - understand that for any real number  $a$ ,  $a^0 = 1$  and  $a^{-n} = \frac{1}{a^n}$  and apply these properties
- investigate and explain characteristics of exponential functions; use these characteristics to model and solve real-world problems
  - identify domain and range
  - identify zeroes
  - find  $x$ - and  $y$ -intercepts
  - recognize and/or determine intervals where the value of a function is increasing or decreasing
  - find maximum and minimum values over a limited domain
  - investigate rates of change over intervals
  - recognize and explain behavior at extremes
- graph exponential functions as transformations of  $f(x) = a^x$  (transformations as listed under quadratic functions)
  - recognize and use transformations of  $f(x) = a^x$
  - use tables of values
- solve simple exponential equations and inequalities
  - by using algebraic and analytical methods
  - by creating and interpreting graphs
- understand basic exponential functions as models of real phenomena and use exponential functions to solve problems
  - apply simple and compound interest formulas
  - recognize exponential growth and decay functions in problem situations and in numerical, graphical, and algebraic representations
  - use exponential growth and decay models to solve problems; understand how the parameters of an exponential function relate to a situation modeled by that function

- understand and recognize geometric sequences as exponential functions whose domains are the sequence of natural (counting) numbers
  - interpret the constant ratio in a geometric sequence as the base of the associated exponential function
  - recognize and use concepts such as the common ratio and powers of the common ratio to solve real-world problems involving exponential growth and decay
- understand the relationship between a function and its inverse
  - read, interpret, and use function and inverse function notation
  - recognize and find the inverse of a function or relation using a variety of methods: interchanging the first and second coordinates of each ordered pair; graphing the function and its reflection across the line  $y = x$ ; using analytical (algebraic) techniques; using composition of functions, i.e., exploring the identity function,  $I(x) = x$ , and how it is related to a function and its inverse; and/or determining that two functions,  $f$  and  $g$ , are inverses by recognizing or demonstrating that  $(f \circ g)(x) = (g \circ f)(x) = I(x) = x$
  - understand that functions with inverses that are also functions are considered to be one-to-one; understand how one-to-oneness relates to a real-world functional relationship
  - recognize that the domain of the inverse is the range of the original relation and vice versa
  - understand how and why domain restrictions come into play with inverse functions and relate them to the behavior of the original function
  - understand and apply methods to characterize a relation as a function, including inspection, tables of values, and graphical methods such as the vertical line test

## **GPS Geometry**

### **Domain: Data Analysis and Probability**

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#### **Overview of the Domain**

- Students will use sample data to make informal inferences using population means and standard deviations.

#### **Associated GPS Standard**

MM2D1

#### **Associated GPS Concepts and Skills**

Assessment of this domain will focus on student ability to

- recognize an appropriate question given a research topic and populations of interest
- calculate the mean and standard deviation for a population or a sample; understand when the sample or population standard deviation formula is appropriate
- use means and standard deviations to compare data sets
  - understand and apply various strategies for estimating means and standard deviations for comparison purposes
  - understand various representations of data, including tables, graphs, line plots, stem-and-leaf plots, histograms, and box-and-whisker plots; know which information can be directly determined and which can only be estimated from a given representation
  - understand the role of  $n$  in comparing standard deviations of data sets, including recognizing when  $n$  is unknown
- compare the means and standard deviations of random samples drawn from a population with corresponding population parameters
  - understand that the distribution of the sample means has less variability than the population distribution
  - understand and apply the relationship between the number of samples, the sample variance, and the population variance
  - recognize that  $n$ , the sample size, can be determined from the values of the sample means