Georgia

Milestones

Assessment System

Study/Resource Guide for Students and Parents

Analytic Geometry

Study/Resource Guide

The Study/Resource Guides are intended to serve as a resource for parents and students. They contain practice questions and learning activities for the course. The standards identified in the Study/Resource Guides address a sampling of the state-mandated content standards.

For the purposes of day-to-day classroom instruction, teachers should consult the wide array of resources that can be found at www.georgiastandards.org.

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THE GEORGIA MILESTONES ASSESSMENT SYSTEM

Dear Student,

The **Georgia Milestones Analytic Geometry EOC Study/Resource Guide for Students and Parents** is intended as a resource for parents and students.

This guide contains information about the core content ideas and skills that are covered in the course. There are practice sample questions for every unit. The questions are fully explained and describe why each answer is either correct or incorrect. The explanations also help illustrate how each question connects to the Georgia state standards.

In addition, the guide includes activities that you can try to help you better understand the concepts taught in the course. The standards and additional instructional resources can be found on the Georgia Department of Education website, www.georgiastandards.org.

Get ready—open this guide—and get started!

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GEORGIA MILESTONES END-OF-COURSE (EOC) ASSESSMENTS

The EOC assessments serve as the final exam in certain courses. The courses are:

English Language Arts

- Ninth Grade Literature and Composition
- American Literature and Composition

Mathematics

- Algebra I
- Analytic Geometry
- Coordinate Algebra
- Geometry

Science

- Physical Science
- Biology

Social Studies

- United States History
- Economics/Business/Free Enterprise

All End-of-Course assessments accomplish the following:

- Ensure that students are learning
- Count as part of the course grade
- Provide data to teachers, schools, and school districts
- Identify instructional needs and help plan how to meet those needs
- Provide data for use in Georgia's accountability measures and reports

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HOW TO USE THIS GUIDE

Let's get started!

First, preview the entire guide. Learn what is discussed and where to find helpful information. Even though the focus of this guide is Analytic Geometry, you need to keep in mind your overall good reading habits.

- Start reading with a pencil or a highlighter in your hand and sticky notes nearby.
- Mark the important ideas, the things you might want to come back to, or the explanations you have questions about. On that last point, your teacher is your best resource.
- You will find some key ideas and important tips to help you prepare for the test.
- You can learn about the different types of items on the test.
- When you come to the additional practice items, don't just read them, *do* them. Think about strategies you can use for finding the right answer. Then read the analysis of the item to check your work. The reasoning behind the correct answer is explained for you. It will help you see any faulty reasoning in the ones you may have missed.
- Use the activities in this guide to get hands-on understanding of the concepts presented in each unit.
- With the Depth of Knowledge (DOK) information, you can gauge just how complex the item is. You will see that some items ask you to recall information and others ask you to infer or go beyond simple recall. The assessment will require all levels of thinking.
- Plan your studying and schedule your time.
- Proper preparation will help you do your best!

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OVERVIEW OF THE ANALYTIC GEOMETRY EOC ASSESSMENT

ITEM TYPES

The Analytic Geometry EOC assessment consists of **selected-response** and **technology-enhanced** items.

A **selected-response** item, sometimes called a multiple-choice item, is a question, problem, or statement that is followed by four answer choices. These questions are worth one point.

A **technology-enhanced (TE)** item has a question, problem, or statement. These types of items are worth one or two points. Partial credit may be awarded on two-point items if you select some but not all of the correct answers or if you get one part of the question correct but not the other part.

- In multi-select items, you will be asked to select more than one right answer.
- In multi-part items, the items will have more than one part. You will need to provide an answer in each part.
- In drag-and-drop items, you will be asked to use a mouse, touchpad, or touchscreen to move responses to designated areas on the screen.
- In drop-down menu items, you will be asked to use a mouse, touchpad, or touchscreen to open a drop-down menu and select an option from the menu. A drop-down item may have multiple drop-down menus.
- In keypad-input items, you will be asked to use a physical keyboard or the pop-up keyboard on a touchscreen to type to type a number, expression, or equation into an answer box.
- In coordinate-graph items, you will be asked to use a mouse, touchpad, or touchscreen to draw lines and/or plot points on a coordinate grid on the screen.
- In line-plot items, you will be asked to use a mouse, touchpad, or touchscreen to place Xs above a number line to create a line plot.
- In bar-graph items, you will be asked to use a mouse, touchpad, or touchscreen to select the height of each bar to create a bar graph.
- In number-line items, you will be asked to use a mouse, touchpad, or touchscreen to plot a point and/or represent inequalities.
- Since some technology-enhanced items in this guide were designed to be used in an online, interactive-delivery format, some of the item-level directions will not appear to be applicable when working within the format presented in this document (for example, “Move the clocks into the graph” or “Create a scatter plot”).
- This icon ➲ identifies special directions that will help you answer technology-enhanced items as shown in the format presented within this guide. These directions do not appear in the online version of the test but explain information about how the item works that would be easily identifiable if you were completing the item in an online environment.

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To practice using technology-enhanced items in an online environment very similar to how they will appear on the online test, visit "Experience Online Testing Georgia."

1. Go to the website "Welcome to Experience Online Testing Georgia" (http://gaexperienceonline.com/).
2. Select "Test Practice."
3. Under "End-of-Course (EOC) All Assessments," select "Online Tools Training."
4. Select "EOC Test Practice."
5. Select "Technology Enhanced Items."
6. You will be taken to a login screen. Use the username and password provided on the screen to log in and practice navigating technology-enhanced items online.

Please note that Google Chrome is the only supported browser for this public version of the online testing environment.

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DEPTH OF KNOWLEDGE DESCRIPTORS

Items found on the Georgia Milestones assessments, including the Analytic Geometry EOC assessment, are developed with a particular emphasis on the kinds of thinking required to answer questions. In current educational terms, this is referred to as Depth of Knowledge (DOK). DOK is measured on a scale of 1 to 4 and refers to the level of cognitive demand (different kinds of thinking) required to complete a task, or in this case, an assessment item. The following table shows the expectations of the four DOK levels in detail.

The DOK table lists the skills addressed in each level as well as common question cues. These question cues not only demonstrate how well you understand each skill but also relate to the expectations that are part of the state standards.

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Level 1—Recall of Information	
Level 1 generally requires that you identify, list, or define. This level usually asks you to recall facts, terms, concepts, and trends and may ask you to identify specific information contained in documents, maps, charts, tables, graphs, or illustrations. Items that require you to "describe" and/or "explain" could be classified as Level 1 or Level 2. A Level 1 item requires that you just recall, recite, or reproduce information.	
Skills Demonstrated	**Question Cues**
• Make observations • Recall information • Recognize formulas, properties, patterns, processes • Know vocabulary, definitions • Know basic concepts • Perform one-step processes • Translate from one representation to another • Identify relationships	• Find • List • Define • Identify; label; name • Choose; select • Compute; estimate • Express • Read from data displays • Order

Level 2—Basic Reasoning	
Level 2 includes the engagement (use) of some mental processing beyond recalling or reproducing a response. A Level 2 "describe" and/or "explain" item would require that you go beyond a description or explanation of recalled information to describe and/or explain a result or "how" or "why."	
Skills Demonstrated	**Question Cues**
• Apply learned information to abstract and real-life situations • Use methods, concepts, and theories in abstract and real-life situations • Perform multi-step processes • Solve problems using required skills or knowledge (requires more than habitual response) • Make a decision about how to proceed • Identify and organize components of a whole • Extend patterns • Identify/describe cause and effect • Recognize unstated assumptions; make inferences • Interpret facts • Compare or contrast simple concepts/ideas	• Apply • Calculate; solve • Complete • Describe • Explain how; demonstrate • Construct data displays • Construct; draw • Analyze • Extend • Connect • Classify • Arrange • Compare; contrast

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Level 3—Complex Reasoning Level 3 requires reasoning, using evidence, and thinking on a higher and more abstract level than Level 1 and Level 2. You will go beyond explaining or describing "how and why" to justifying the "how and why" through application and evidence. Level 3 items often involve making connections across time and place to explain a concept or a "big idea."	
Skills Demonstrated	**Question Cues**
• Solve an open-ended problem with more than one correct answer • Create a pattern • Relate knowledge from several sources • Draw conclusions • Make predictions • Translate knowledge into new contexts • Compare and discriminate between ideas • Assess value of methods, concepts, theories, processes, and formulas • Make choices based on a reasoned argument • Verify the value of evidence, information, numbers, and data	• Plan; prepare • Predict • Create; design • Generalize • Justify; explain why; support; convince • Assess • Rank; grade • Test; judge • Recommend • Select • Conclude

Level 4—Extended Reasoning Level 4 requires the complex reasoning of Level 3 with the addition of planning, investigating, applying significant conceptual understanding, and/or developing that will most likely require an extended period of time. You may be required to connect and relate ideas and concepts *within* the content area or *among* content areas in order to be at this highest level. The Level 4 items would be a show of evidence, through a task, a product, or an extended response, that the cognitive demands have been met.	
Skills Demonstrated	**Question Cues**
• Analyze and synthesize information from multiple sources • Apply mathematical models to illuminate a problem or situation • Design a mathematical model to inform and solve a practical or abstract situation • Combine and synthesize ideas into new concepts	• Design • Connect • Synthesize • Apply concepts • Critique • Analyze • Create • Prove

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DEPTH OF KNOWLEDGE EXAMPLE ITEMS

Example items that represent the applicable DOK levels across various Analytic Geometry content domains are provided on the following pages.

All example and sample items contained in this guide are the property of the Georgia Department of Education.

Example Item 1

Selected-Response

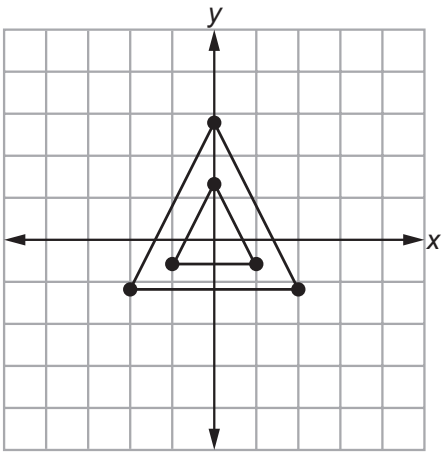
DOK Level 1: This is a DOK Level 1 item because it requires the student to demonstrate an understanding of dilations and determining the scale factor.

Analytic Geometry Content Domain: Congruence and Similarity

Standard: MGSE9-12.G.SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor.

b. The dilation of a line segment is longer or shorter according to the ratio given by the scale factor.

The smaller triangle is transformed to create the larger triangle. Which of these is the scale factor of the dilation centered at the point (0, 0)?



A. 4

B. 2

C. 1

D. $\frac{1}{2}$

Correct Answer: B

Explanation of Correct Answer: The correct answer is choice (B) 2. Since the length of each segment has doubled, the scale factor is 2. Choice (A) is incorrect because a scale factor of 4 would make the horizontal side of the image a length of 8. Choice (C) is incorrect because a scale factor of 1 does not change the size of the pre-image. Choice (D) is incorrect because it represents the scale factor when the pre-image and image are reversed.

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Example Item 2

Keypad-Input Multi-Part Technology-Enhanced

DOK Level 2: This is a DOK level 2 item because it requires students to determine conditional probabilities of independent events.

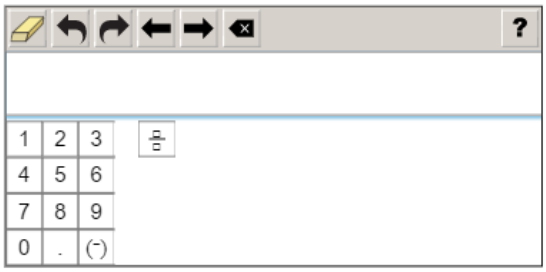
Analytic Geometry Content Domain: Statistics and Probability

Standard: MGSE9-12.S.CP.2. Understand that if two events A and B are independent, the probability of A and B occurring together is the product of their probabilities, and that if the probability of two events A and B occurring together is the product of their probabilities, the two events are independent.

Part A

Emily's bus is scheduled to arrive at 6:48 A.M. The probability that Emily arrives at the bus stop after 6:48 A.M. is 0.06. The probability that the bus arrives at the bus stop after 6:48 A.M. is 0.04. The time that Emily arrives at the bus stop and the time that the bus arrives at the bus stop are independent.

Part A What is the probability that the bus arrives at the bus stop after 6:48 A.M. given that Emily arrives at the bus stop after 6:48 A.M.?



Use a mouse, touchpad, or touchscreen to enter a response.

Go on to the next page to finish example item 2.

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Example Item 2. *Continued*.

Part B

Emily's bus is scheduled to arrive at 6:48 A.M. The probability that Emily arrives at the bus stop after 6:48 A.M. is 0.06. The probability that the bus arrives at the bus stop after 6:48 A.M. is 0.04. The time that Emily arrives at the bus stop and the time that the bus arrives at the bus stop are independent.

Part B What is the probability that both Emily and the bus will arrive at the bus stop after 6:48 A.M. on a given day?

Use a mouse, touchpad, or touchscreen to enter a response.

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Example Item 2. *Continued*.

Scoring Rubric

Points	Description
2	The student correctly answers both Part A and Part B.
1	The student correctly answers either Part A OR Part B.
0	The student does not correctly answer either part.

Exemplar Response

Part A

The correct response is shown below.

This is the correct response because the independence of the events means that the probability of the bus arriving after 6:48 given that Emily arrives after 6:48 is the same as the probability of the bus arriving after 6:48.

Part B

The correct response is shown below.

This is the correct response because the probability of two independent events occurring together is found by multiplying the probabilities of the two events.

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Example Item 3

Drop-Down Multi-Part Technology-Enhanced

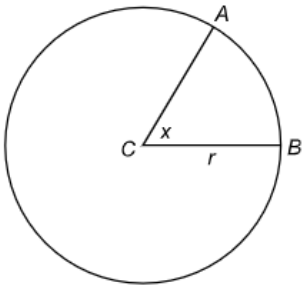
DOK Level 3: This is a DOK level 3 item because it requires students to determine the effects of changing measures in a circle with unknown dimensions.

Analytic Geometry Content Domain: Circles

Standard: MGSE9-12.G.C.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Part A

Circle *C*, with radius, *r*, and central angle measure, *x*, is shown.

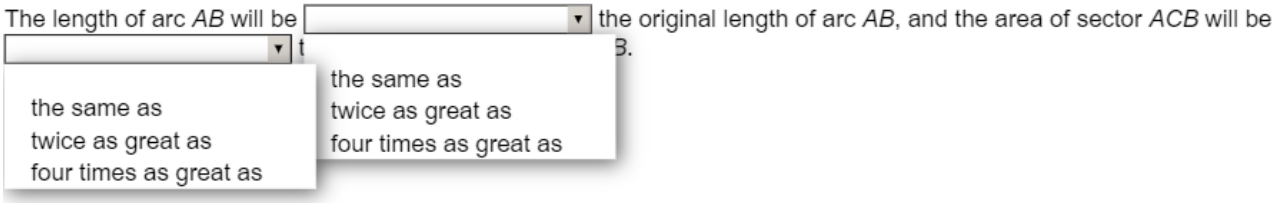


Part A What will be the effect of doubling the measure of the central angle, *x*, on the length of arc *AB* and the area of sector *ACB*?

Use the drop-down menus to complete the statement.

The length of arc *AB* will be [ ] the original length of arc *AB*, and the area of sector *ACB* will be [ ] the original area of sector *ACB*.

Use a mouse, touchpad, or touchscreen to click the arrow beside each of the two blank boxes. When you click the arrow, a drop-down menu will appear, showing you all the possible options for that blank box. Each drop-down menu with its options is shown below.



Go on to the next page to finish example item 3.

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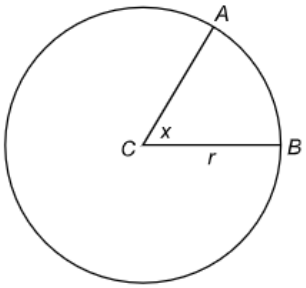
Example Item 3. *Continued*.

Part B

Circle *C*, with radius, *r*, and central angle measure, *x*, is shown.

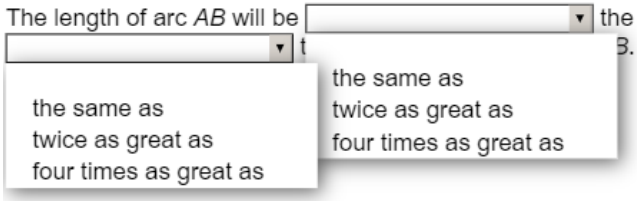


Part B What will be the effect of doubling the length of the radius, *r*, on the length of arc *AB* and the area of sector *ACB*?

Use the drop-down menus to complete the statement.

The length of arc *AB* will be [ ] the original length of arc *AB*, and the area of sector *ACB* will be [ ] the original area of sector *ACB*.

Use a mouse, touchpad, or touchscreen to click the arrow beside each of the two blank boxes. When you click the arrow, a drop-down menu will appear, showing you all the possible options for that blank box. Each drop-down menu with its options is shown below.



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Example Item 3. *Continued*.

Scoring Rubric

Points	Description
2	The student correctly completes both statements.
1	The student correctly completes one of the statements.
0	The student does not correctly complete either of the statements.

Exemplar Response

Part A

The correct response is shown below.

The length of arc *AB* will be [twice as great as] the original length of arc *AB*, and the area of sector *ACB* will be [twice as great as] the original area of sector *ACB*.

"Twice as great as" is correct for both drop-down menus because the length of an arc and the area of a sector both have a direct relationship to the measure of the central angle.

Part B

The correct response is shown below.

The length of arc *AB* will be [twice as great as] the original length of arc *AB*, and the area of sector *ACB* will be [four times as great as] the original area of sector *ACB*.

"Twice as great as" is correct for the first drop-down menu because the length of an arc has a direct relationship to the radius of the circle. "Four times as great as" is correct for the second drop-down because the area of a sector has a direct relationship to the square of the radius of the circle.

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DESCRIPTION OF TEST FORMAT AND ORGANIZATION

The Georgia Milestones Analytic Geometry EOC assessment consists of a total of 55 items. You will be asked to respond to selected-response (multiple-choice) and technology-enhanced items.

The test will be given in two sections.

- You may have up to 65 minutes per section to complete Sections 1 and 2.
- The total estimated testing time for the Analytic Geometry EOC assessment ranges from approximately 60 to 130 minutes. Total testing time describes the amount of time you have to complete the assessment. It does not take into account the time required for the test examiner to complete pre-administration and post-administration activities (such as reading the standardized directions to students).
- Sections 1 and 2 may be administered on the same day or across two consecutive days, based on the district's testing protocols for the EOC measures (in keeping with state guidance).
- During the Analytic Geometry EOC assessment, a formula sheet will be available for you to use. Another feature of the Analytic Geometry assessment is that you may use a graphing calculator in calculator-approved sections.

Effect on Course Grade

It is important that you take this course and the EOC assessment very seriously.

- For students in Grade 10 or above beginning with the 2011–2012 school year, the final grade in each course is calculated by weighing the course grade 85% and the EOC score 15%.
- For students in Grade 9 beginning with the 2011–2012 school year and later, the final grade in each course is calculated by weighing the course grade 80% and the EOC score 20%.
- A student must have a final grade of at least 70% to pass the course and to earn credit toward graduation.

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PREPARING FOR THE ANALYTIC GEOMETRY EOC ASSESSMENT

STUDY SKILLS

As you prepare for this test, ask yourself the following questions:

- How would you describe yourself as a student?
- What are your study skills strengths and/or weaknesses?
- How do you typically prepare for a classroom test?
- What study methods do you find particularly helpful?
- What is an ideal study situation or environment for you?
- How would you describe your actual study environment?
- How can you change the way you study to make your study time more productive?

ORGANIZATION—OR TAKING CONTROL OF YOUR WORLD

- Establish a study area that has minimal distractions.
- Gather your materials in advance.
- Develop and implement your study plan.

ACTIVE PARTICIPATION

The most important element in your preparation is *you*. You and your actions are the key ingredient. Your active studying helps you stay alert and be more productive. In short, you need to interact with the course content. Here's how you do it.

- Carefully read the information and then DO something with it. Mark the important material with a highlighter, circle it with a pen, write notes on it, or summarize the information in your own words.
- Ask questions. As you study, questions often come into your mind. Write them down and actively seek the answers.
- Create sample test questions and answer them.
- Find a friend who is also planning to take the test and quiz each other.

TEST-TAKING STRATEGIES

Part of preparing for a test is having a set of strategies you can draw from. Include these strategies in your plan:

- Read and understand the directions completely. If you are not sure, ask a teacher.
- Read each question and all of the answer choices carefully.
- If you use scratch paper, make sure you copy your work to your test accurately.
- Underline important parts of each task. Make sure that your answer goes on the answer sheet.
- Be aware of time. If a question is taking too much time, come back to it later.
- Answer all questions. Check your answers for accuracy.
- Stay calm and do the best you can.

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PREPARING FOR THE ANALYTIC GEOMETRY EOC ASSESSMENT

Read this guide to help prepare for the Analytic Geometry EOC assessment.

The section of the guide titled “Content of the Analytic Geometry EOC Assessment” provides a snapshot of the Analytic Geometry course. In addition to reading this guide, do the following to prepare to take the assessment:

- Read your resources and other materials.
- Think about what you learned, ask yourself questions, and answer them.
- Read and become familiar with the way questions are asked on the assessment.
- Answer some practice Analytic Geometry questions.
- There are additional items to practice your skills available online. Ask your teacher about online practice sites that are available for your use.

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CONTENT OF THE ANALYTIC GEOMETRY EOC ASSESSMENT

Up to this point in the guide, you have been learning how to prepare for taking the EOC assessment. Now you will learn about the topics and standards that are assessed in the Analytic Geometry EOC assessment and will see some sample items.

- The first part of this section focuses on what will be tested. It also includes sample items that will let you apply what you have learned in your classes and from this guide.
- The second part contains additional items to practice your skills.
- The third part contains a table that shows the standard assessed for each item, the DOK level, the correct answer (key), and a rationale/explanation of the right and wrong answers.
- You can use the additional practice items to familiarize yourself with the item formats found on the assessment.

All example and sample items contained in this guide are the property of the Georgia Department of Education.

The Analytic Geometry EOC assessment will assess the Analytic Geometry standards documented at www.georgiastandards.org.

The content of the assessment is organized into five groupings, or domains, of standards for the purpose of providing feedback on student performance.

- A content domain is a reporting category that *broadly* describes and defines the content of the course, as measured by the EOC assessment.
- On the actual test the standards for Analytic Geometry are grouped into five domains that follow your classwork: Congruence and Similarity; Circles; Equations and Measurement; Expressions, Equations, and Functions (*including Number*); and Statistics and Probability.
- Each domain was created by organizing standards that share similar content characteristics.
- The content standards describe the level of understanding each student is expected to achieve. They include the knowledge, concepts, and skills assessed on the EOC assessment, and they are used to plan instruction throughout the course.

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SNAPSHOT OF THE COURSE

This section of the guide is organized into seven units that review the material taught within the five domains of the Analytic Geometry course. The material is presented by concept rather than by category or standard. In each unit you will find sample items similar to what you will see on the EOC assessment. The next section of the guide contains additional items to practice your skills followed by a table that shows for each item the standard assessed, the DOK level, the correct answer (key), and a rationale/explanation about the key and distractors.

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The more you understand about the concepts in each unit, the greater your chances of getting a good score on the EOC assessment.

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UNIT 1: SIMILARITY, CONGRUENCE, AND PROOFS

This unit introduces the concepts of similarity and congruence. The definition of similarity is explored through dilation transformations. The concept of scale factor with respect to dilations allows figures to be enlarged or reduced. Rigid motions lead to the definition of congruence. Once congruence is established, various congruence criteria (e.g., ASA, SSS, SAS) can be explored. Once similarity is established, various similarity criteria (e.g., AA) can be explored. These criteria, along with other postulates and definitions, provide a framework for solving various geometric proofs. In this unit, various geometric figures are constructed. These topics allow students a deeper understanding of formal reasoning, which will be beneficial throughout the remainder of Geometry. Students are asked to prove theorems about parallelograms. Theorems include opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and, conversely, rectangles are parallelograms with congruent diagonals. The method for proving is not specified, so it could be done by using knowledge of congruency and establishing a formalized proof, it could be proven by constructions, or it could be proved algebraically by using the coordinate plane.

1.1 Understand Similarity in Terms of Similarity Transformations

MGSE9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor.

a. The dilation of a line not passing through the center of the dilation results in a parallel line and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter according to the ratio given by the scale factor.

MGSE9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain, using similarity transformations, the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

MGSE9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

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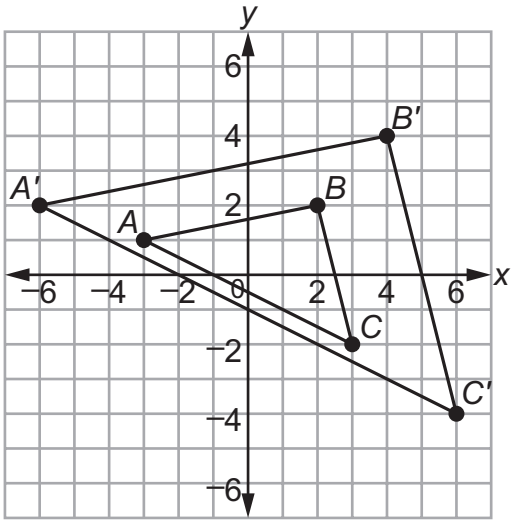
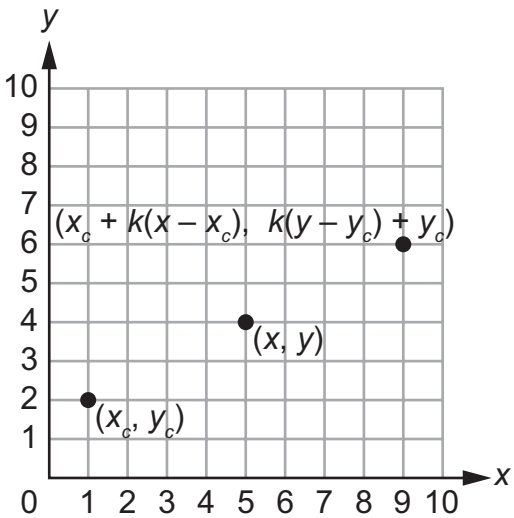
KEY IDEAS

A ***dilation*** is a transformation that changes the size of a figure, but not the shape, based on a ratio given by a ***scale factor*** with respect to a fixed point called the ***center***. When the scale factor is greater than 1, the figure is made larger. When the scale factor is between 0 and 1, the figure is made smaller. When the scale factor is 1, the figure does not change. When the center of dilation is the origin, you can multiply each coordinate of the original figure, or ***pre-image***, by the scale factor to find the coordinates of the dilated figure, or ***image***.

Example: The diagram below shows $\triangle ABC$ dilated about the origin with a scale factor of 2 to create $\triangle A'B'C'$.



When the center of dilation is not the origin, you can use a rule that is derived from shifting the center of dilation, multiplying the shifted coordinates by the scale factor, and then shifting the center of dilation back to its original location. For a point (x, y) and a center of dilation (x_c, y_c), the rule for finding the coordinates of the dilated point with a scale factor of k is $(x_c + k(x - x_c), k(y - y_c) + y_c)$.



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When a figure is transformed under a dilation, the ***corresponding angles*** of the pre-image and the image have equal measures.

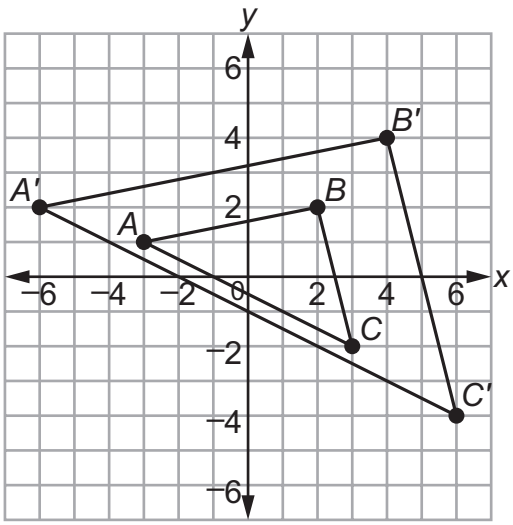
For $\triangle ABC$ and $\triangle A'B'C'$ below, $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\angle C \cong \angle C'$.

When a figure is transformed under a dilation, the ***corresponding sides*** of the pre-image and the image are proportional.

For $\triangle ABC$ and $\triangle A'B'C'$ on the coordinate grid below, $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}$.

So when a figure is under a dilation transformation, the pre-image and the image are ***similar***.

For $\triangle ABC$ and $\triangle A'B'C'$ below, $\triangle ABC \sim \triangle A'B'C'$.



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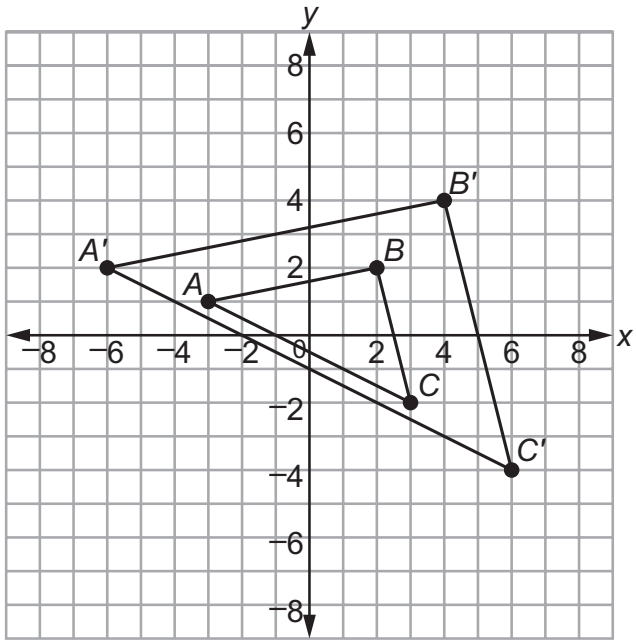
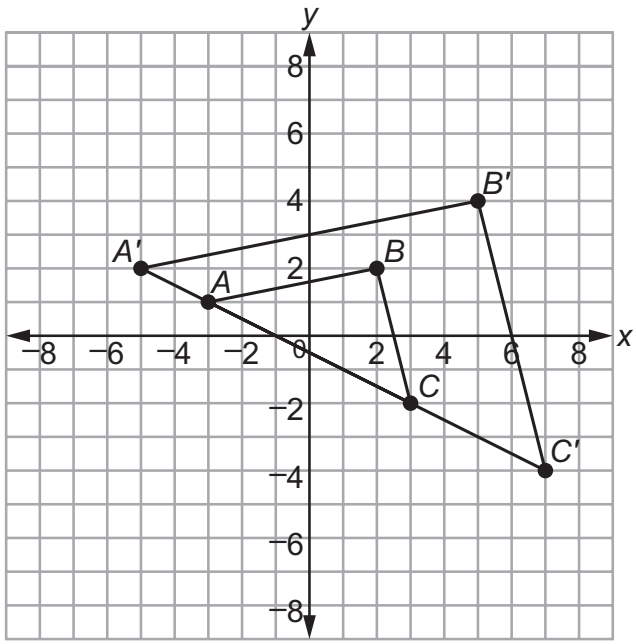
When a figure is dilated, a segment of the pre-image that does not pass through the center of dilation is parallel to its image. In the figure below, $\overline{AC} \parallel \overline{A'C'}$ since neither segment passes through the center of dilation. The same is true about $\overline{AB}$ and $\overline{A'B'}$ as well as $\overline{BC}$ and $\overline{B'C'}$.



center of dilation (0,0)
scale factor = 2

When the segment of a figure does pass through the center of dilation, the segment of the pre-image and image are on the same line. In the figure below, the center of dilation is on $\overline{AC}$, so $\overline{AC}$ and $\overline{A'C'}$ are on the same line.



center of dilation (–1,0)
scale factor = 2

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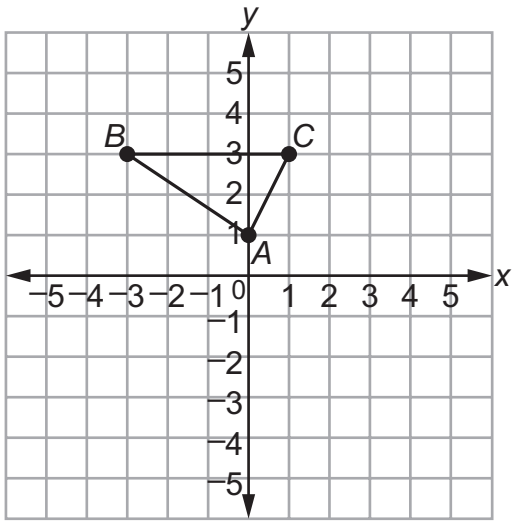
REVIEW EXAMPLES

- Draw a triangle with vertices at $A(0, 1)$, $B(-3, 3)$, and $C(1, 3)$. Dilate the triangle using a scale factor of 1.5 and a center of (0, 0). Sketch and name the dilated triangle $A'B'C'$.

  Solution:

  Plot points $A(0, 1)$, $B(-3, 3)$, and $C(1, 3)$. Draw $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$.



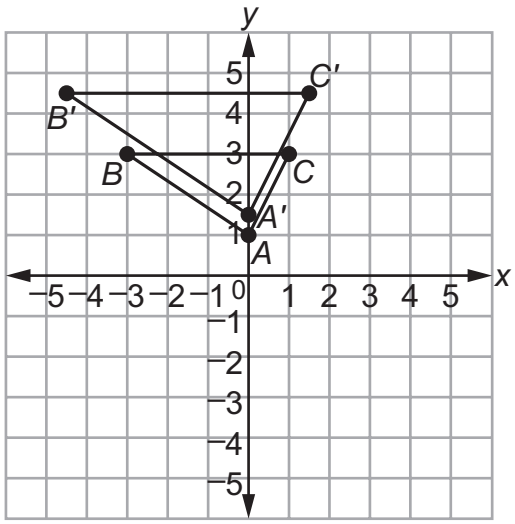
  The center of dilation is the origin, so to find the coordinates of the image, multiply the coordinates of the pre-image by the scale factor, 1.5.

  Point A': $(1.5 \cdot 0, 1.5 \cdot 1) = (0, 1.5)$

  Point B': $(1.5 \cdot (-3), 1.5 \cdot 3) = (-4.5, 4.5)$

  Point C': $(1.5 \cdot 1, 1.5 \cdot 3) = (1.5, 4.5)$

  Plot points $A'(0, 1.5)$, $B'(-4.5, 4.5)$, and $C'(1.5, 4.5)$. Draw $\overline{A'B'}$, $\overline{A'C'}$, and $\overline{B'C'}$.



  ***Note*:** Since no part of the pre-image passes through the center of dilation, $\overline{BC} \parallel \overline{B'C'}$, $\overline{AB} \parallel \overline{A'B'}$, and $\overline{AC} \parallel \overline{A'C'}$.

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- Line segment CD is 5 inches long. If line segment CD is dilated to form line segment $C'D'$ with a scale factor of 0.6, what is the length of line segment $C'D'$?

  Solution:

  The ratio of the length of the image and the pre-image is equal to the scale factor.

$$\frac{C'D'}{CD} = 0.6$$

  Substitute 5 for CD.

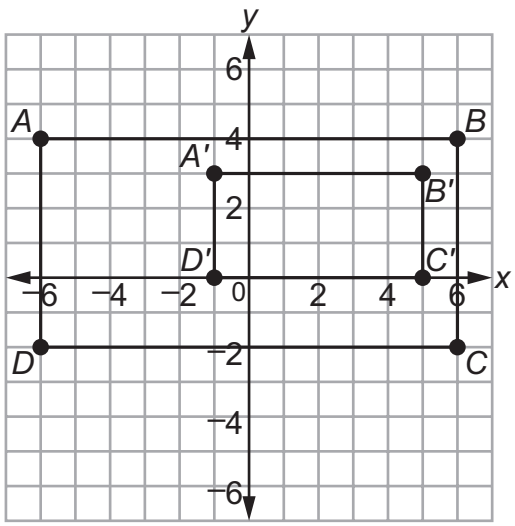
$$\frac{C'D'}{5} = 0.6$$

  Solve for $C'D'$.

$$C'D' = 0.6 \cdot 5$$
$$C'D' = 3$$

  The length of line segment $C'D'$ is 3 inches.

- Figure $A'B'C'D'$ is a dilation of figure $ABCD$.



  a. Determine the center of dilation.
  b. Determine the scale factor of the dilation.
  c. What is the relationship between the sides of the pre-image and the corresponding sides of the image?

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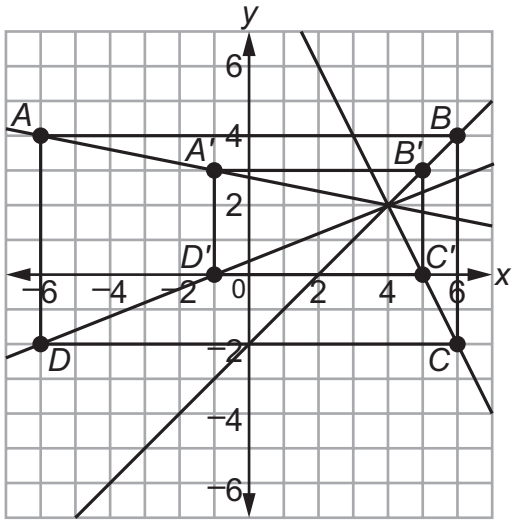
Solution:

a. To find the center of dilation, draw lines connecting each corresponding vertex from the pre-image to the image. The lines meet at the center of dilation.



The center of dilation is (4, 2).

b. Find the ratios of the lengths of the corresponding sides.

$$\frac{A'B'}{AB} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{B'C'}{BC} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{C'D'}{CD} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{A'D'}{AD} = \frac{3}{6} = \frac{1}{2}$$

The ratio for each pair of corresponding sides is $\frac{1}{2}$, so the scale factor is $\frac{1}{2}$.

c. Each side of the image is parallel to the corresponding side of its pre-image and is $\frac{1}{2}$ the length.

Note: Lines connecting corresponding points pass through the center of dilation.

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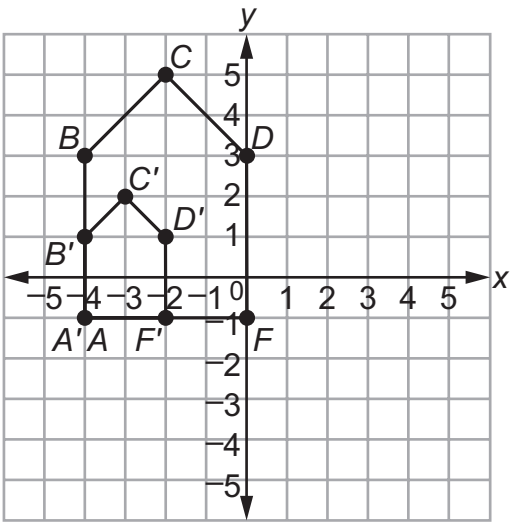
SAMPLE ITEMS

1. Figure $A'B'C'D'F'$ is a dilation of figure $ABCDF$ by a scale factor of $\frac{1}{2}$. The dilation is centered at (–4, –1).



Which statement is true?

A. $\frac{AB}{A'B'} = \frac{B'C'}{BC}$

B. $\frac{AB}{A'B'} = \frac{BC}{B'C'}$

C. $\frac{AB}{A'B'} = \frac{BC}{D'F'}$

D. $\frac{AB}{A'B'} = \frac{D'F'}{BC}$

2. Which transformation results in a figure that is similar to the original figure but has a greater area?

A. a dilation of triangle *QRS* by a scale factor of 0.25
B. a dilation of triangle *QRS* by a scale factor of 0.5
C. a dilation of triangle *QRS* by a scale factor of 1
D. a dilation of triangle *QRS* by a scale factor of 2

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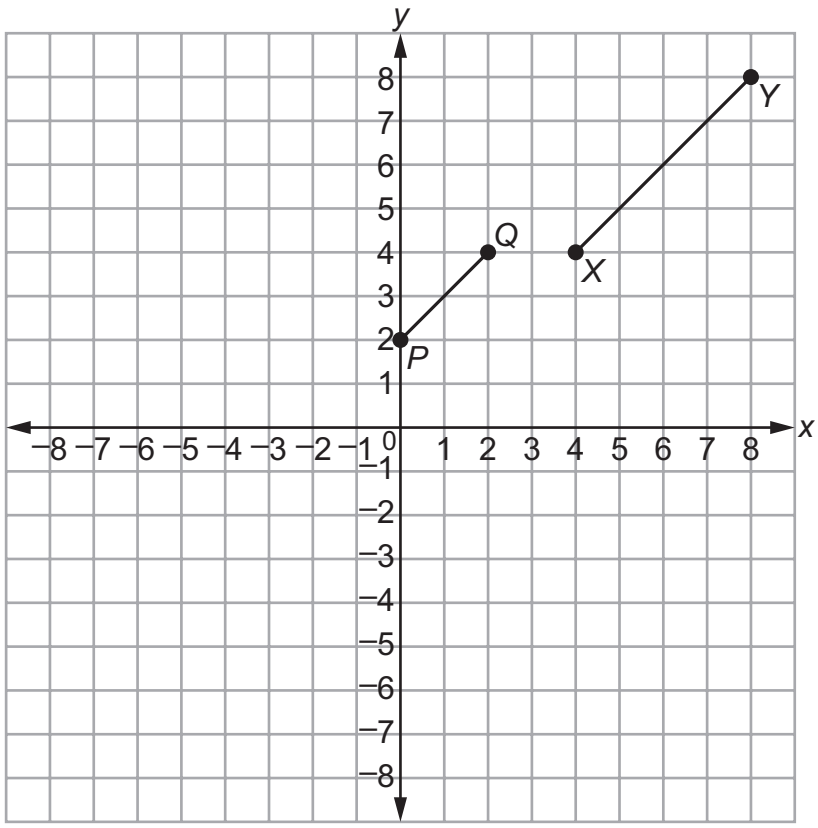
3. In the coordinate plane, segment $\overline{PQ}$ is the result of a dilation of segment $\overline{XY}$ by a scale factor of $\frac{1}{2}$.



Which point is the center of dilation?

A. (–4, 0)
B. (0, –4)
C. (0, 4)
D. (4, 0)

Note: Draw lines connecting corresponding points to determine the point of intersection (center of dilation).

Answers to Unit 1.1 Sample Items

1. B 2. D 3. A

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1.2 Prove Theorems Involving Similarity

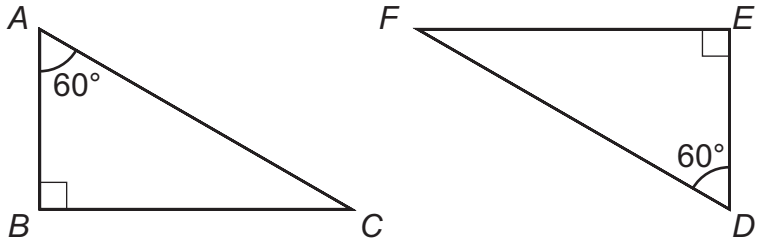
MGSE9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, (and its converse); the Pythagorean Theorem using triangle similarity.

MGSE9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

KEY IDEAS

When proving that two triangles are similar, it is sufficient to show that two pairs of corresponding angles of the triangles are congruent. This is called ***Angle-Angle* (*AA*) *Similarity***.

Example: The triangles below are similar by AA Similarity because each triangle has a 60° angle and a 90° angle. The similarity statement is written as $\triangle ABC \sim \triangle DEF$, and the order in which the vertices are written indicates which angles/sides correspond to each other.



When a triangle is dilated, the pre-image and the image are similar triangles. There are three cases of triangles being dilated:

- The image is congruent to the pre-image (scale factor of 1).
- The image is smaller than the pre-image (scale factor between 0 and 1).
- The image is larger than the pre-image (scale factor greater than 1).

When two triangles are **similar**, all corresponding pairs of angles are congruent.

When two triangles are **similar**, all corresponding pairs of sides are proportional.

When two triangles are **congruent**, the triangles are also similar.

A ***two-column proof*** is a series of statements and reasons often displayed in a chart that works from given information to the statement that needs to be proven. Reasons can be given information, can be based on definitions, or can be based on postulates or theorems.

A ***paragraph proof*** also uses a series of statements and reasons that work from given information to the statement that needs to be proven, but the information is presented as running text in paragraph form.

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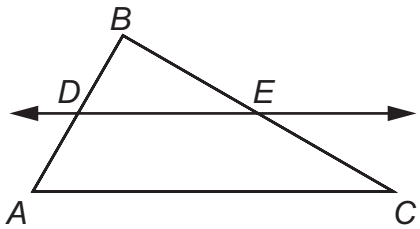
REVIEW EXAMPLES

♦ In the triangle shown, $\overline{AC} \parallel \overleftrightarrow{DE}$.



Prove that $\overleftrightarrow{DE}$ divides $\overline{AB}$ and $\overline{CB}$ proportionally.

Solution:

Step	Statement	Reason
1	$\overline{AC} \parallel \overleftrightarrow{DE}$	Given
2	$\angle BDE \cong \angle BAC$	If two parallel lines are cut by a transversal, then corresponding angles are congruent.
3	$\angle DBE \cong \angle ABC$	Reflexive Property of Congruence because they are the same angle
4	$\triangle DBE \sim \triangle ABC$	Angle-Angle (AA) Similarity
5	$\frac{BA}{BD} = \frac{BC}{BE}$	Corresponding sides of similar triangles are proportional.
6	$BD + DA = BA$ $BE + EC = BC$	Segment Addition Postulate
7	$\frac{BD + DA}{BD} = \frac{BE + EC}{BE}$	Substitution
8	$\frac{BD}{BD} + \frac{DA}{BD} = \frac{BE}{BE} + \frac{EC}{BE}$	Rewrite each fraction as a sum of two fractions.
9	$1 + \frac{DA}{BD} = 1 + \frac{EC}{BE}$	Simplify.
10	$\frac{DA}{BD} = \frac{EC}{BE}$	Subtraction Property of Equality
11	$\overleftrightarrow{DE}$ divides $\overline{AB}$ and $\overline{CB}$ proportionally.	Definition of proportionality

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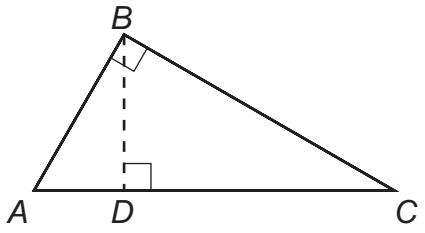
♦ Gale is trying to prove the Pythagorean Theorem using similar triangles. Part of her proof is shown below.



Step	Statement	Reason
1	$\angle ABC \cong \angle BDC$	All right angles are congruent.
2	$\angle ACB \cong \angle BCD$	Reflexive Property of Congruence
3	$\triangle ABC \sim \triangle BDC$	Angle-Angle (AA) Similarity
4	$\frac{BC}{DC} = \frac{AC}{BC}$	Corresponding sides of similar triangles are proportional.
5	$BC^2 = AC \bullet DC$	In a proportion, the product of the means equals the product of the extremes.
6	$\angle ABC \cong \angle ADB$	All right angles are congruent.
7	$\angle BAC \cong \angle DAB$	Reflexive Property of Congruence
8	$\triangle ABC \sim \triangle ADB$	Angle-Angle (AA) Similarity
9	$\frac{AB}{AD} = \frac{AC}{AB}$	Corresponding sides of similar triangles are proportional.
10	$AB^2 = AC \bullet AD$	In a proportion, the product of the means equals the product of the extremes.

What should Gale do to finish her proof?

Solution:

Step	Statement	Reason
11	$AB^2 + BC^2 = AC \bullet AD + AC \bullet DC$	Addition Property of Equality
12	$AB^2 + BC^2 = AC(AD + DC)$	Distributive Property
13	$AC = AD + DC$	Segment Addition Postulate
14	$AB^2 + BC^2 = AC \bullet AC$	Substitution
15	$AB^2 + BC^2 = AC^2$	Definition of exponent

$AB^2 + BC^2 = AC^2$ is a statement of the Pythagorean Theorem, so Gale's proof is complete.

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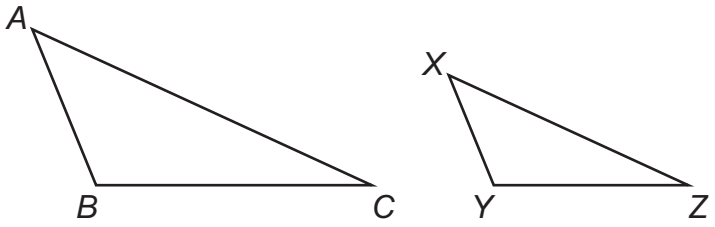
SAMPLE ITEMS

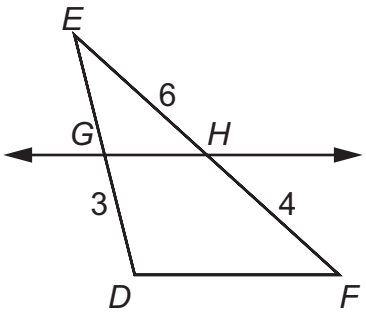
1. In the triangles shown, $\triangle ABC$ is dilated by a factor of $\frac{2}{3}$ to form $\triangle XYZ$.



Given that $m\angle A = 50°$ and $m\angle B = 100°$, what is $m\angle Z$?

A. 15°
B. 25°
C. 30°
D. 50°

2. In the triangle shown, $\overleftrightarrow{GH} \parallel \overline{DF}$.



What is the length of $\overline{GE}$?

A. 2.0
B. 4.5
C. 7.5
D. 8.0

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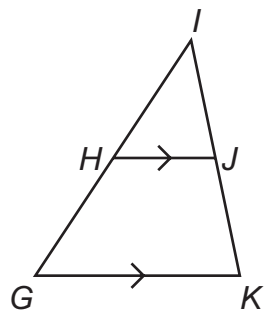
3. Use this triangle to answer the question.



This is a proof of the statement "If a line is parallel to one side of a triangle and intersects the other two sides at distinct points, then it separates these sides into segments of proportional lengths."

Step	Statement	Reason
1	$\overline{GK}$ is parallel to $\overline{HJ}$.	Given
2	$\angle IGK \cong \angle IHJ$ $\angle IKG \cong \angle IJH$	**?**
3	$\triangle GIK \sim \triangle HIJ$	AA Similarity
4	$\frac{IG}{IH} = \frac{IK}{IJ}$	Corresponding sides of similar triangles are proportional.
5	$\frac{HG + IH}{IH} = \frac{JK + IJ}{IJ}$	Segment Addition Postulate
6	$\frac{HG}{IH} = \frac{JK}{IJ}$	Subtraction Property of Equality

Which reason justifies Step 2?

A. Alternate interior angles are congruent.
B. Alternate exterior angles are congruent.
C. Corresponding angles are congruent.
D. Vertical angles are congruent.

Answers to Unit 1.2 Sample Items

1. C 2. B 3. C

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1.3 Understand Congruence in Terms of Rigid Motions

MGSE9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

MGSE9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

MGSE9-12.G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (Extend to include HL and AAS.)

KEY IDEAS

A ***rigid motion*** is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations (in any order). This transformation leaves the size and shape of the original figure unchanged.

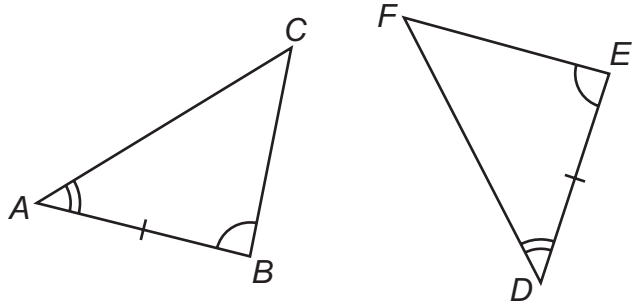
Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations). ***Congruent figures*** have the same corresponding side lengths and the same corresponding angle measures as each other.

Two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent. This is sometimes referred to as ***CPCTC***, which means **C**orresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent.

When given two congruent triangles, you can use a series of translations, reflections, and rotations to show the triangles are congruent.

You can use ***ASA (Angle-Side-Angle)*** to show two triangles are congruent. If two angles and the included side of a triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

$\triangle ABC \cong \triangle DEF$ by ASA.



You can use ***SSS (Side-Side-Side)*** to show two triangles are congruent. If three sides of a triangle are congruent to three sides of another triangle, then the triangles are congruent.

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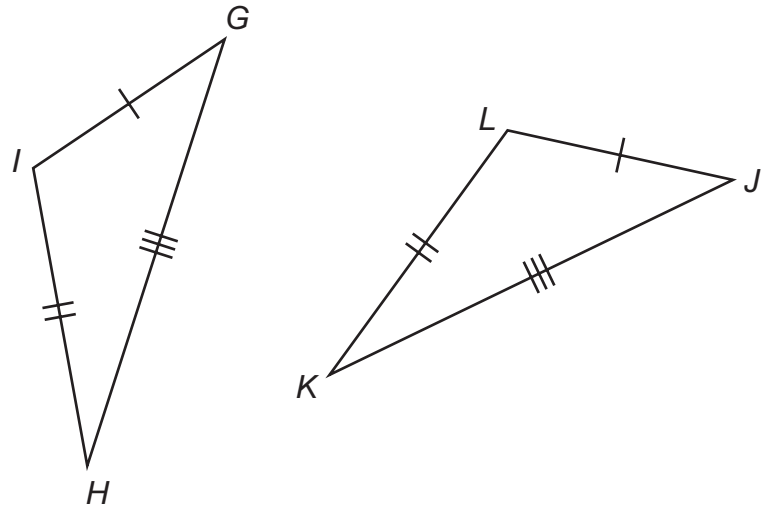
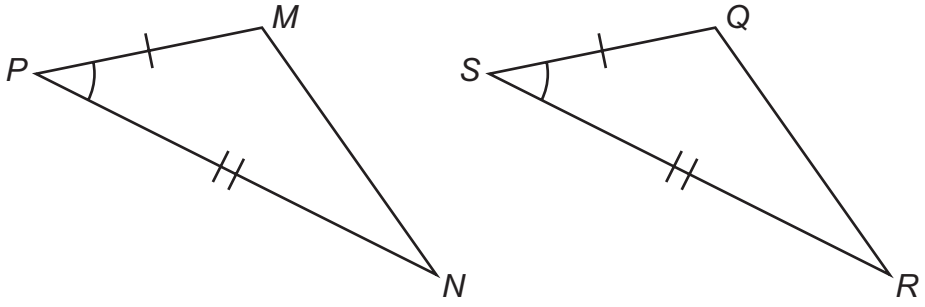
$\triangle GIH \cong \triangle JLK$ by SSS.



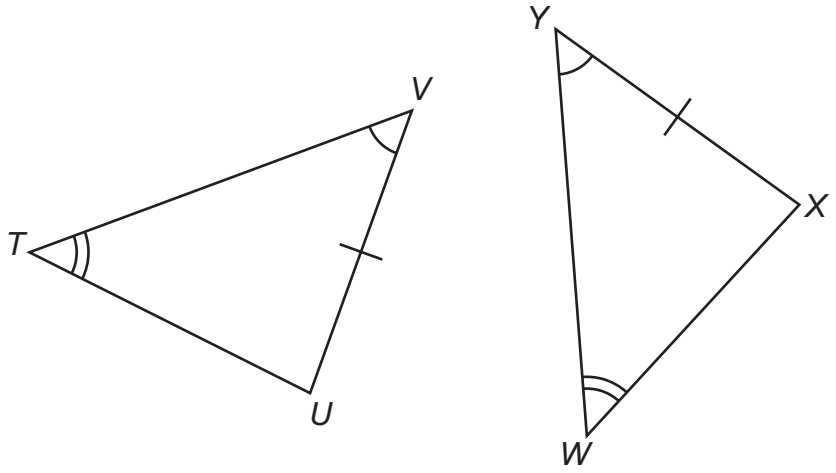
You can use ***SAS (Side-Angle-Side)*** to show two triangles are congruent. If two sides and the included angle of a triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

$\triangle MPN \cong \triangle QSR$ by SAS.



You can use ***AAS (Angle-Angle-Side)*** to show two triangles are congruent. If two angles and a non-included side of a triangle are congruent to two angles and the corresponding non-included side of another triangle, then the triangles are congruent.

$\triangle VTU \cong \triangle YWX$ by AAS.



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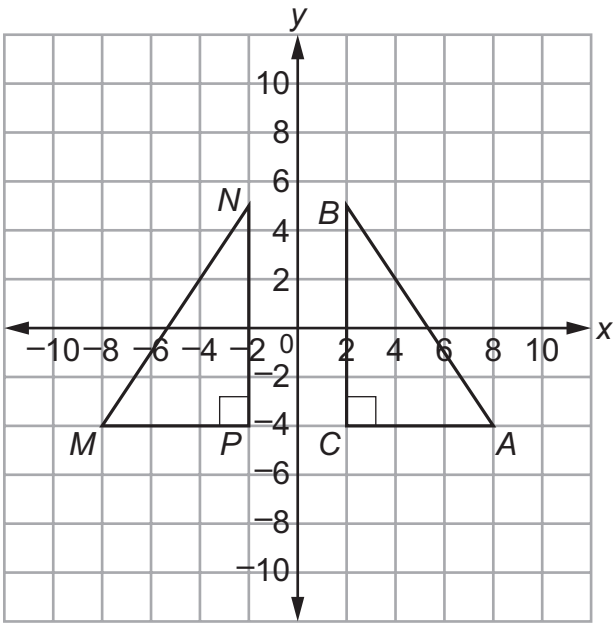
Important Tips

- If two sides and a ***non-included angle*** of one triangle are congruent to two sides and a non-included angle of a second triangle, the triangles are not necessarily congruent. Therefore, there is no way to show triangle congruency by Side-Side-Angle (SSA).
- If two triangles have all three angles congruent to each other, the triangles are similar, but not necessarily congruent. Thus, you can show similarity by Angle-Angle-Angle (AAA), but you cannot show congruence by AAA.

REVIEW EXAMPLES

- Is $\triangle ABC$ congruent to $\triangle MNP$? Explain.



(scale unit = 2)

Solution:

$\overline{AC}$ corresponds to $\overline{MP}$. Both segments are 6 units long. $\overline{BC}$ corresponds to $\overline{NP}$. Both segments are 9 units long. Angle C (the included angle of $\overline{AC}$ and $\overline{BC}$) corresponds to angle P (the included angle of $\overline{MP}$ and $\overline{NP}$). Both angles measure 90°. Because two sides and an included angle are congruent, the triangles are congruent by SAS.

Or, $\triangle ABC$ is a reflection of $\triangle MNP$ over the y-axis. This means that all of the corresponding sides and corresponding angles are congruent, so the triangles are congruent. (Reflections preserve angle measurement and lengths; therefore, corresponding angles and sides are congruent.)

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♦ Rectangle *WXYZ* has coordinates *W*(1, 2), *X*(3, 2), *Y*(3, –3), and *Z*(1, –3).

a. Graph the image of rectangle *WXYZ* after a rotation of 90° clockwise about the origin. Label the image *W′X′Y′Z′*.

b. Translate rectangle *W′X′Y′Z′* 2 units left and 3 units up. Label the image *W″X″Y″Z″*.

c. Is rectangle *WXYZ* congruent to rectangle *W″X″Y″Z″* ? Explain.

Solution:

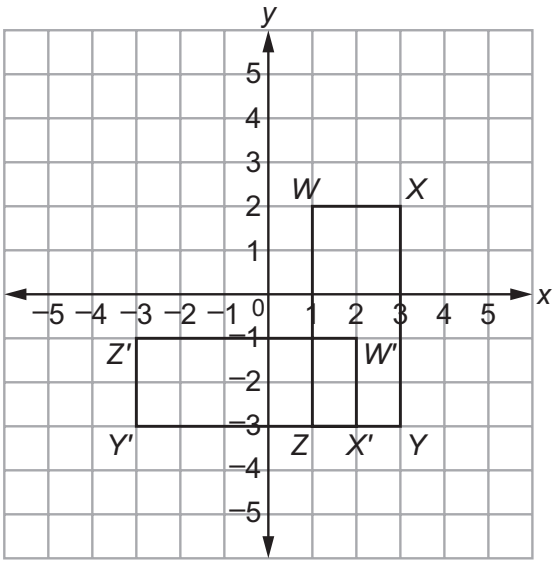
a. For a 90° clockwise rotation about the origin, use the rule $(x, y) \rightarrow (y, -x)$.

$W(1, 2) \rightarrow W'(2, -1)$

$X(3, 2) \rightarrow X'(2, -3)$

$Y(3, -3) \rightarrow Y'(-3, -3)$

$Z(1, -3) \rightarrow Z'(-3, -1)$



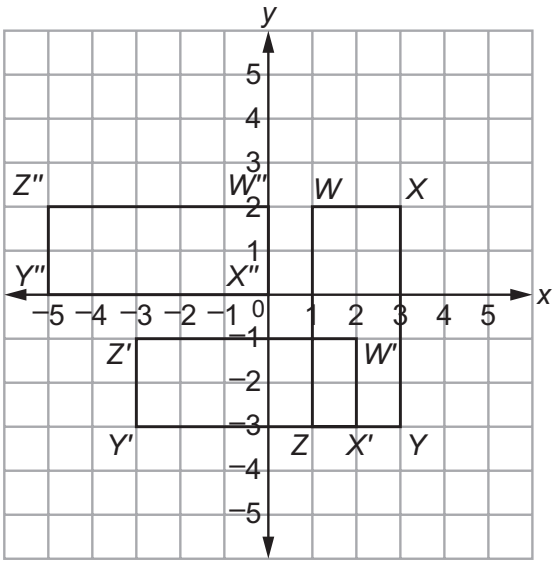
b. To translate rectangle *W′X′Y′Z′* 2 units left and 3 units up, use the rule $(x, y) \rightarrow (x - 2, y + 3)$.

$W'(2, -1) \rightarrow W''(0, 2)$

$X'(2, -3) \rightarrow X''(0, 0)$

$Y'(-3, -3) \rightarrow Y''(-5, 0)$

$Z'(-3, -1) \rightarrow Z''(-5, 2)$



c. Rectangle *W″X″Y″Z″* is the result of a rotation and a translation of rectangle *WXYZ*. These are both rigid transformations, so the shape and the size of the original figure are unchanged. All of the corresponding parts of *WXYZ* and *W″X″Y″Z″* are congruent, so *WXYZ* and *W″X″Y″Z″* are congruent.

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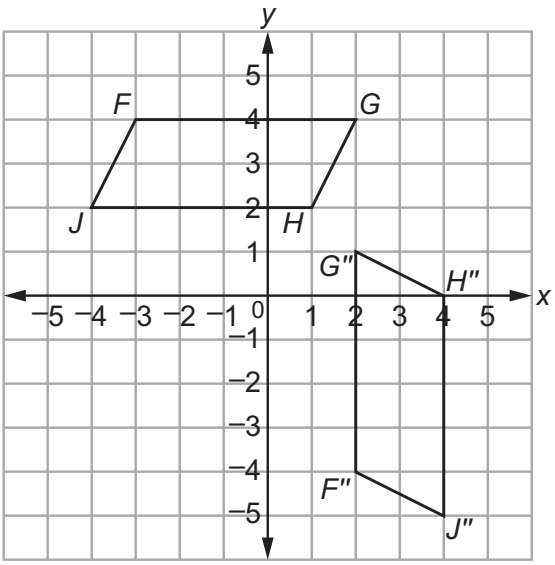
SAMPLE ITEMS

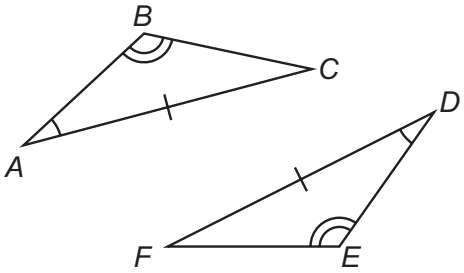
1. Parallelogram *FGHJ* was translated 3 units down to form parallelogram *F′G′H′J′*. Parallelogram *F′G′H′J′* was then rotated 90° counterclockwise about point *G′* to obtain parallelogram *F″G″H″J″*.



Which statement is true about parallelogram *FGHJ* and parallelogram *F″G″H″J″*?

A. The figures are both similar and congruent.
B. The figures are neither similar nor congruent.
C. The figures are similar but not congruent.
D. The figures are congruent but not similar.

2. Consider the triangles shown.



Which can be used to prove the triangles are congruent?

A. SSS
B. ASA
C. SAS
D. AAS

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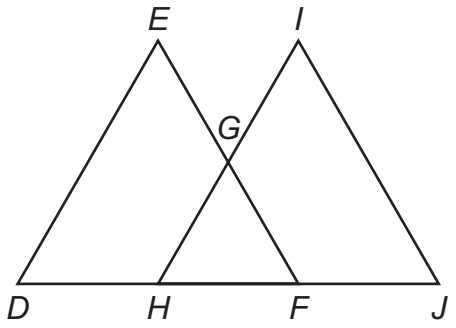
3. In this diagram, $\overline{DE} \cong \overline{JI}$ and $\angle D \cong \angle J$.



Which additional information is sufficient to prove that $\triangle DEF$ is congruent to $\triangle JIH$?

A. $\overline{ED} \cong \overline{IH}$

B. $\overline{DH} \cong \overline{JF}$

C. $\overline{HG} \cong \overline{GI}$

D. $\overline{HF} \cong \overline{JF}$

Answers to Unit 1.3 Sample Items

1. A 2. D 3. B

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1.4 Prove Geometric Theorems

MGSE9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

MGSE9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

MGSE9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

KEY IDEAS

A ***two-column proof*** is a series of statements and reasons often displayed in a chart that works from given information to the statement that needs to be proven. Reasons can be given information, can be based on definitions, or can be based on postulates or theorems.

A ***paragraph proof*** also uses a series of statements and reasons that work from given information to the statement that needs to be proven, but the information is presented as running text in paragraph form.

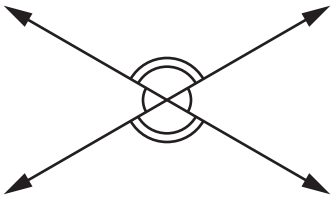
It is important to plan a geometric proof logically. Think through what needs to be proven and decide how to get to that statement from the given information. Often a diagram or a flow chart will help organize your thoughts.

An ***auxiliary line*** is a line drawn in a diagram that makes other figures, such as congruent triangles or angles formed by a transversal. Many times, an auxiliary line is needed to help complete a proof.

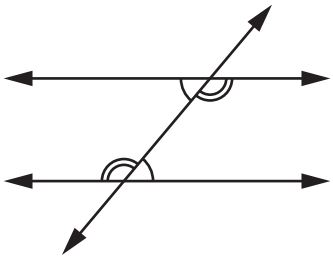
Once a theorem in geometry has been proven, that theorem can be used as a reason in future proofs.

Some important key ideas about lines and angles include the following:

- ***Vertical Angle Theorem***: Vertical angles are congruent.



- ***Alternate Interior Angles Theorem***: If two parallel lines are cut by a transversal, then alternate interior angles formed by the transversal are congruent.



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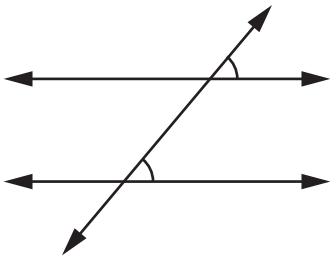
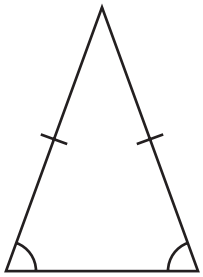
- ***Corresponding Angles Postulate***: If two parallel lines are cut by a transversal, then corresponding angles formed by the transversal are congruent.



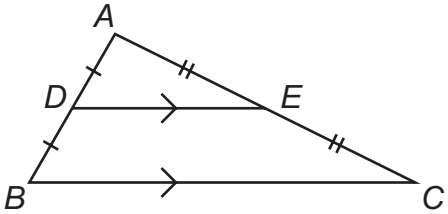
- Points on a perpendicular bisector of a line segment are equidistant from both of the segment's endpoints.

Some important key ideas about triangles include the following:

- ***Triangle Angle-Sum Theorem***: The sum of the measures of the angles of a triangle is 180°.
- ***Isosceles Triangle Theorem***: If two sides of a triangle are congruent, then the angles opposite those sides are also congruent.



- ***Triangle Midsegment Theorem***: If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and half its length.



$$m\overline{DE} = \frac{1}{2}m\overline{BC}$$

- ***Points of Concurrency***: The point where three or more lines intersect. There are 4 points of concurrency: incenter, centroid, orthocenter, and circumcenter.

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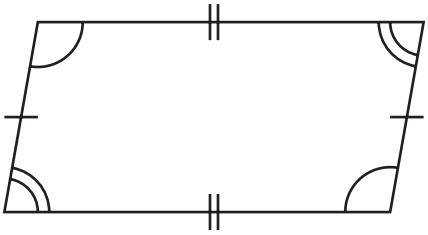
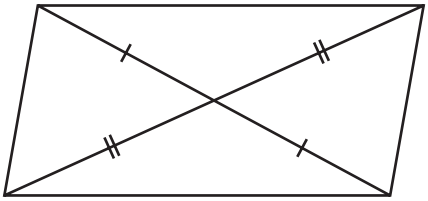
Some important key ideas about parallelograms include the following:

- Opposite sides are congruent and opposite angles are congruent.



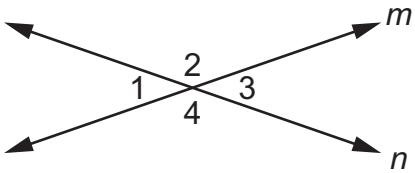
- The diagonals of a parallelogram bisect each other.



- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- A rectangle is a parallelogram with congruent diagonals.

REVIEW EXAMPLES

♦ In this diagram, line *m* intersects line *n*.



Write a two-column proof to show that the vertical angles ∠1 and ∠3 are congruent.

Solution:

Construct a proof using intersecting lines.

Step	Statement	Reason
1	Line *m* intersects line *n.*	Given
2	∠1 and ∠2 form a linear pair. ∠2 and ∠3 form a linear pair.	Definition of a linear pair
3	$m\angle 1 + m\angle 2 = 180°$ $m\angle 2 + m\angle 3 = 180°$	Angles that form a linear pair have measures that sum to 180°.
4	$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	Substitution
5	$m\angle 1 = m\angle 3$	Subtraction Property of Equality
6	$\angle 1 \cong \angle 3$	Definition of congruent angles

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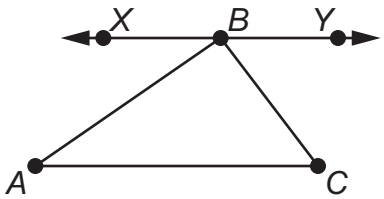
- In this diagram, $\overleftrightarrow{XY}$ is parallel to $\overline{AC}$, and point B lies on $\overleftrightarrow{XY}$.



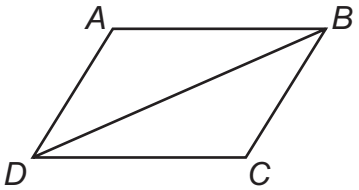
Write a paragraph to prove that the sum of the angles in a triangle is 180°.

Solution:

$\overline{AC}$ and $\overleftrightarrow{XY}$ are parallel, so $\overline{AB}$ is a transversal. The alternate interior angles formed by the transversal are congruent. So, $m\angle A = m\angle ABX$. Similarly, $\overline{BC}$ is a transversal, so $m\angle C = m\angle CBY$. The sum of the angle measures that make a straight line is 180°.

So, $m\angle ABX + m\angle ABC + m\angle CBY = 180°$. Now, substitute $m\angle A$ for $m\angle ABX$ and $m\angle C$ for $m\angle CBY$ to get $m\angle A + m\angle ABC + m\angle C = 180°$.

- In this diagram, $ABCD$ is a parallelogram and $\overline{BD}$ is a diagonal.



Write a two-column proof to show that $\overline{AB}$ and $\overline{CD}$ are congruent.

Solution:

Construct a proof using properties of the parallelogram and its diagonal.

Step	Statement	Reason
1	$ABCD$ is a parallelogram.	Given
2	$\overline{BD}$ is a diagonal.	Given
3	$\overline{AB}$ is parallel to $\overline{DC}$. $\overline{AD}$ is parallel to $\overline{BC}$.	Definition of a parallelogram
4	$\angle ABD \cong \angle CDB$ $\angle DBC \cong \angle BDA$	Alternate interior angles are congruent.
5	$\overline{BD} \cong \overline{BD}$	Reflexive Property of Congruence
6	$\triangle ADB \cong \triangle CBD$	ASA
7	$\overline{AB} \cong \overline{CD}$	CPCTC

Note: Corresponding parts of congruent triangles are congruent.

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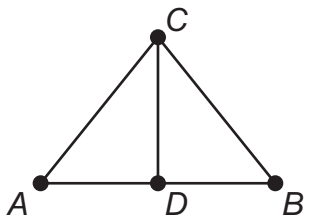
SAMPLE ITEMS

1. In this diagram, $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$. The two-column proof shows that $\overline{AC}$ is congruent to $\overline{BC}$.



Step	Statement	Reason
1	$\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.	Given
2	$\overline{AD} \cong \overline{BD}$	Definition of a bisector
3	$\overline{CD} \cong \overline{CD}$	Reflexive Property of Congruence
4	$\angle ADC$ and $\angle BDC$ are right angles.	Definition of perpendicular lines
5	$\angle ADC \cong \angle BDC$	All right angles are congruent.
6	$\triangle ADC \cong \triangle BDC$	**?**
7	$\overline{AC} \cong \overline{BC}$	CPCTC

Which of the following would justify Step 6?

A. AAS
B. ASA
C. SAS
D. SSS

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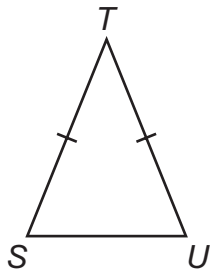
2. In this diagram, *STU* is an isosceles triangle where $\overline{ST}$ is congruent to $\overline{UT}$. The two-column proof shows that $\angle S$ is congruent to $\angle U$.



Step	Statement	Reason
1	$\overline{ST} \cong \overline{UT}$	Given
2	Construct $\overline{TV}$, the angle bisector for $\angle T$, where V is on $\overline{SU}$.	Every angle has a bisector.
3	$\angle STV \cong \angle UTV$	Definition of an angle bisector
4	$\overline{TV} \cong \overline{TV}$	Reflexive Property of Congruence
5	$\triangle STV \cong \triangle UTV$	SAS
6	$\angle S \cong \angle U$	**?**

Which reason is missing in the proof?

A. CPCTC
B. Reflexive Property of Congruence
C. Definition of right angles
D. Angle Congruence Postulate

Answers to Unit 1.4 Sample Items

1. C 2. A

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1.5 Make Geometric Constructions

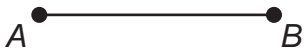
MGSE9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

MGSE9-12.G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon, each inscribed in a circle.

KEY IDEAS

To ***copy a segment***, follow the steps given:

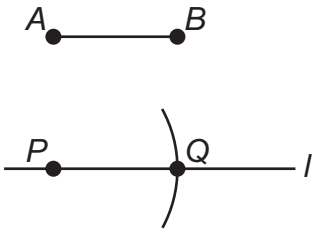
Given: $\overline{AB}$



Construct: $\overline{PQ}$ congruent to $\overline{AB}$

Procedure:

1. Use a straightedge to draw a line, l.
2. Choose a point on line l and label it point P.
3. Place the point of a compass on point A.
4. Adjust the compass width to the length of $\overline{AB}$.
5. Without changing the compass, place the compass point on point P and draw an arc intersecting line l. Label the point of intersection as point Q.
6. $\overline{PQ} \cong \overline{AB}$



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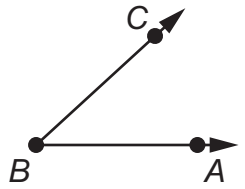
To ***copy an angle***, follow the steps given:

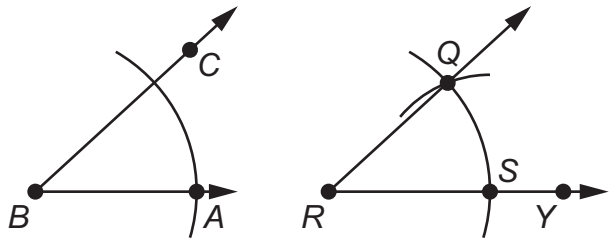


Given: $\angle ABC$

Construct: $\angle QRY$ congruent to $\angle ABC$

Procedure:

1. Draw a point, R, that will be the vertex of the new angle.
2. From point R, use a straightedge to draw $\overrightarrow{RY}$, which will become one side of the new angle.
3. Place the point of a compass on vertex B and draw an arc through point A.
4. Without changing the compass, place the compass point on point R, draw an arc intersecting $\overrightarrow{RY}$, and label the point of intersection point S.
5. Place the compass point on point A and adjust its width to where the arc intersects $\overrightarrow{BC}$.
6. Without changing the compass width, place the compass point on point S and draw another arc across the first arc. Label the point where both arcs intersect as point Q.
7. Use a straightedge to draw $\overrightarrow{RQ}$.
8. $\angle QRY \cong \angle ABC$



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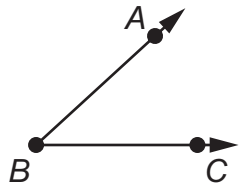
To ***bisect an angle***, follow the steps given:

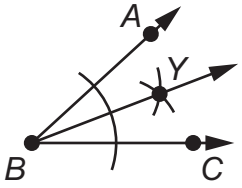


Given: $\angle ABC$

Construct: $\overrightarrow{BY}$, the bisector of $\angle ABC$

Procedure:

1. Place the point of a compass on vertex B.
2. Open the compass and draw an arc that crosses both sides of the angle.
3. Set the compass width to more than half the distance from point B to where the arc crosses $\overrightarrow{BA}$. Place the compass point where the arc crosses $\overrightarrow{BA}$ and draw an arc in the angle's interior.
4. Without changing the compass width, place the compass point where the arc crosses $\overrightarrow{BC}$ and draw an arc so that it crosses the previous arc. Label the intersection point Y.
5. Using a straightedge, draw a ray from vertex B through point Y.
6. $\overrightarrow{BY}$ is the bisector of $\angle ABC$, and $\angle ABY \cong \angle YBC$.



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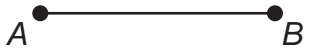
To ***construct a perpendicular bisector of a line segment***, follow the steps given:

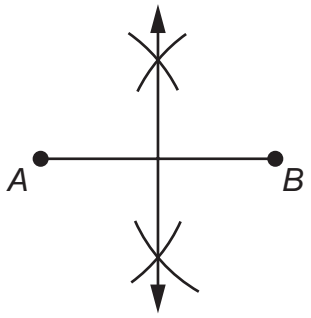
Given: $\overline{AB}$



Construct: The perpendicular bisector of $\overline{AB}$

Procedure:

1. Adjust a compass to a width greater than half the length of $\overline{AB}$.
2. Place the compass point on point *A* and draw an arc passing above $\overline{AB}$ and an arc passing below $\overline{AB}$.
3. Without changing the compass width, place the compass point on point *B* and draw an arc passing above $\overline{AB}$ and an arc passing below $\overline{AB}$.
4. Use a straightedge to draw a line through the points of intersection of these arcs.
5. The segment is the perpendicular bisector of $\overline{AB}$.



Note: To bisect $\overline{AB}$, follow the same steps listed above to construct the perpendicular bisector. The point where the perpendicular bisector intersects $\overline{AB}$ is the midpoint of $\overline{AB}$.

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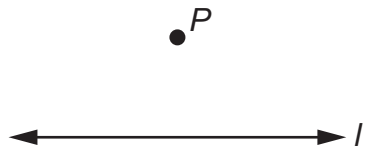
To ***construct a line perpendicular to a given line through a point not on the line***, follow the steps given:

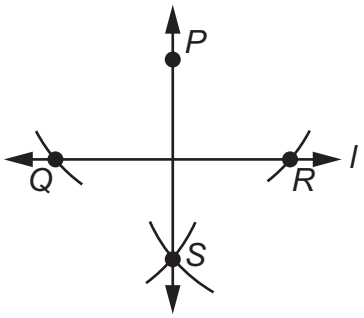


Given: Line *l* and point *P* that is not on line *l*

Construct: The line perpendicular to line *l* through point *P*

Procedure:

1. Place the point of a compass on point *P*.
2. Open the compass to a distance that is wide enough to draw two arcs across line *l*, one on each side of point *P*. Label these points *Q* and *R*.
3. From points *Q* and *R*, draw arcs on the opposite side of line *l* from point *P* so that the arcs intersect. Label the intersection point *S*.
4. Using a straightedge, draw $\overleftrightarrow{PS}$.
5. $\overline{PS}$ is perpendicular to line *l*.



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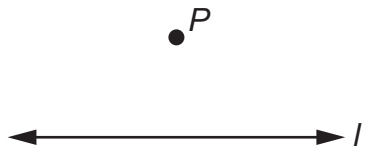
To ***construct a line parallel to a given line through a point not on the line***, follow the steps given:

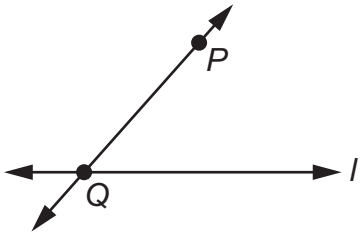


Given: Line *l* and point *P* that is not on line *l*

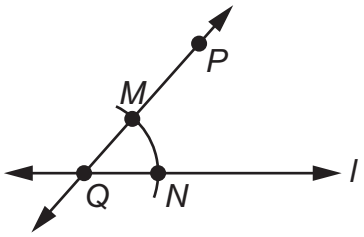
Construct: The line parallel to line *l* through point *P*

Procedure:

1. Draw a transversal line through point *P* crossing line *l* at a point. Label the point of intersection *Q*.



2. Open a compass to a width about half the distance from point *P* to point *Q*. Place the compass point on point *Q* and draw an arc that intersects both lines. Label the intersection of the arc and $\overline{PQ}$ as point *M* and the intersection of the arc and line *l* as point *N*.



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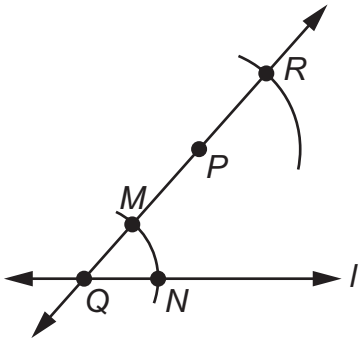
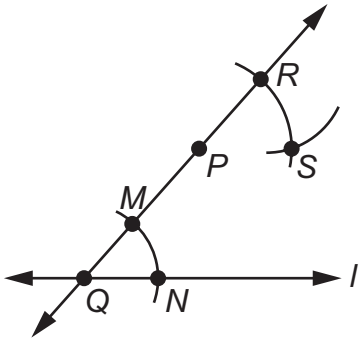
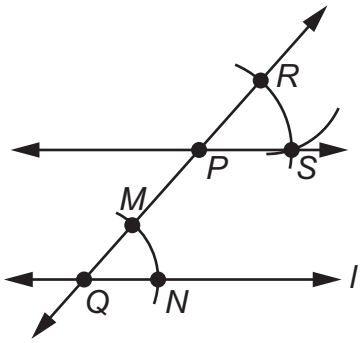
3. Without changing the compass width, place the compass point on point *P* and draw an arc that crosses $\overline{PQ}$ above point *P*. Note that this arc must have the same orientation as the arc drawn from point *M* to point *N*. Label the point of intersection *R*.



4. Set the compass width to the distance from point *M* to point *N*.
5. Place the compass point on point *R* and draw an arc that crosses the upper arc. Label the point of intersection *S*.



6. Using a straightedge, draw a line through points *P* and *S*.
7. $\overleftrightarrow{PS} \parallel l$



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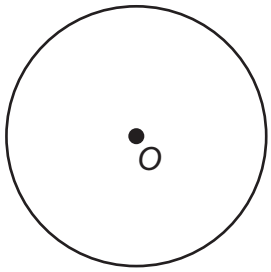
To ***construct an equilateral triangle inscribed in a circle***, follow the steps given:

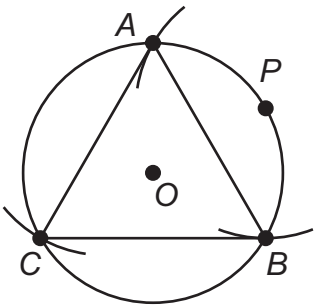


Given: Circle *O*

Construct: Equilateral $\triangle ABC$ inscribed in circle *O*

Procedure:

1. Mark a point anywhere on the circle and label it point *P*.
2. Open a compass to the radius of circle *O*.
3. Place the compass point on point *P* and draw an arc that intersects the circle at two points. Label the points *A* and *B*.
4. Using a straightedge, draw $\overline{AB}$.
5. Open the compass to the length of $\overline{AB}$.
6. Place the compass point on point *A*. Draw an arc from point *A* that intersects the circle. Label this point *C*.
7. Using a straightedge, draw $\overline{AC}$ and $\overline{BC}$.
8. Equilateral $\triangle ABC$ is inscribed in circle *O*.



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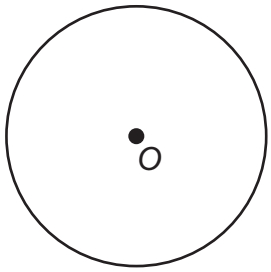
To ***construct a square inscribed in a circle***, follow the steps given:

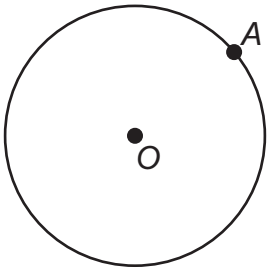


Given: Circle *O*

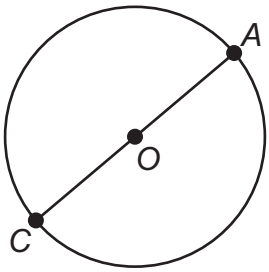
Construct: Square *ABCD* inscribed in circle *O*

Procedure:

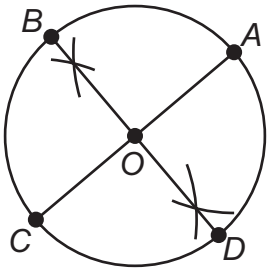
1. Mark a point anywhere on the circle and label it point *A*.



2. Using a straightedge, draw a diameter from point *A*. Label the other endpoint of the diameter as point *C*. This is diameter $\overline{AC}$.



3. Construct a perpendicular bisector of $\overline{AC}$ through the center of circle *O*. Label the points where it intersects the circle as point *B* and point *D*.



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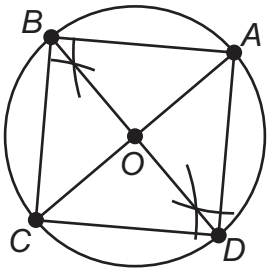
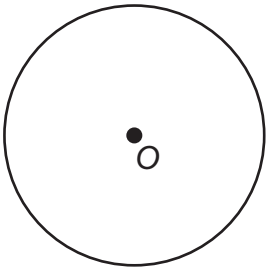
4. Using a straightedge, draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$.



5. Square *ABCD* is inscribed in circle *O*.

To ***construct a regular hexagon inscribed in a circle***, follow the steps given:

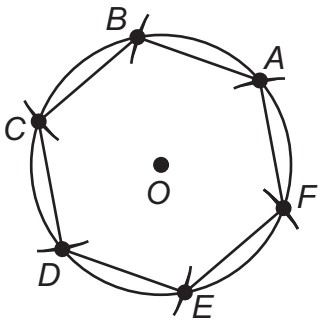


Given: Circle *O*

Construct: Regular hexagon *ABCDEF* inscribed in circle *O*

Procedure:

1. Mark a point anywhere on the circle and label it point *A*.
2. Open a compass to the radius of circle *O*.
3. Place the compass point on point *A* and draw an arc across the circle. Label this point *B*.
4. Without changing the width of the compass, place the compass point on point *B* and draw another arc across the circle. Label this point *C*.
5. Repeat this process from point *C* to a point *D*, from point *D* to a point *E*, and from point *E* to a point *F*.
6. Use a straightedge to draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, and $\overline{AF}$.
7. Regular hexagon *ABCDEF* is inscribed in circle *O*.



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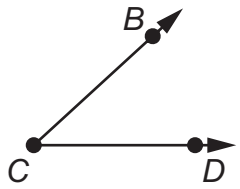
REVIEW EXAMPLES

- Allan drew angle *BCD*.

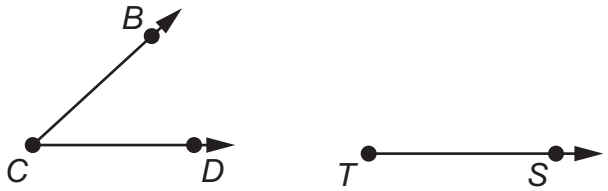


a. Copy angle *BCD*. List the steps you used to copy the angle. Label the copied angle *RTS*.

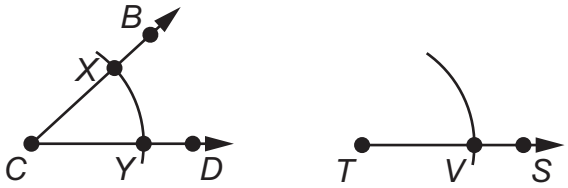
b. Without measuring the angles, how can you show they are congruent to one another?

Solution:

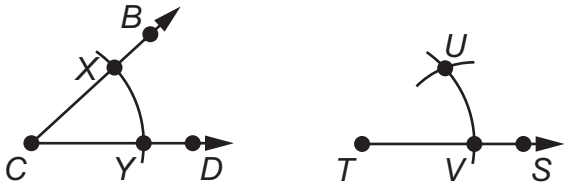
a. Draw point *T*. Draw $\overrightarrow{TS}$.



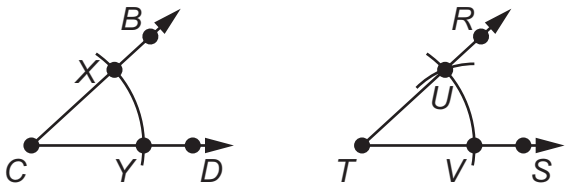
Place the point of a compass on point *C*. Draw an arc. Label the intersection points *X* and *Y*. Keep the compass width the same, and place the point of the compass on point *T*. Draw an arc and label the intersection point *V*.



Place the point of the compass on point *Y* and adjust the width to point *X*. Then place the point of the compass on point *V* and draw an arc that intersects the first arc. Label the intersection point *U*.



Draw $\overrightarrow{TU}$ and point *R* on $\overrightarrow{TU}$. Angle *BCD* has now been copied to form angle *RTS*.



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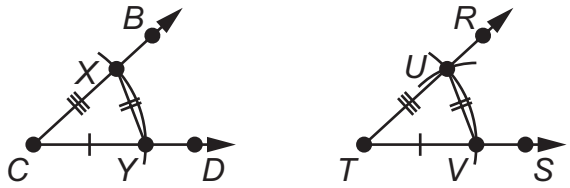
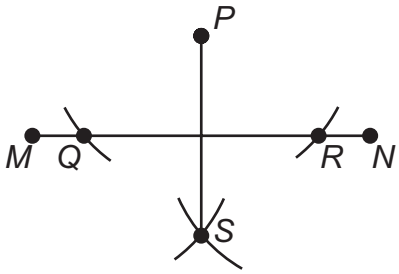
b. Connect points X and Y and points U and V to form $\triangle XCY$ and $\triangle UTV$. $\overline{CY}$ and $\overline{TV}$, $\overline{XY}$ and $\overline{UV}$, and $\overline{CX}$ and $\overline{TU}$ are congruent because they were drawn with the same compass width. So $\triangle XCY \cong \triangle UTV$ by SSS, and $\angle C \cong \angle T$ because congruent parts of congruent triangles are congruent.



♦ Construct a line segment perpendicular to $\overline{MN}$ from a point not on $\overline{MN}$. Explain the steps you used to make your construction.

Solution:

Draw a point P that is not on $\overline{MN}$. Place the point of a compass on point P. Draw an arc that intersects $\overline{MN}$ at two points. Label the intersection points Q and R. Without changing the width of the compass, place the compass point on point Q and draw an arc under $\overline{MN}$. Place the compass point on point R and draw another arc under $\overline{MN}$. Label the intersection point S. Draw $\overline{PS}$. Segment PS is perpendicular to and bisects $\overline{MN}$.



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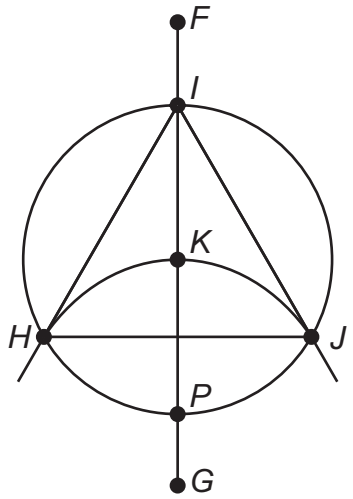
♦ Construct equilateral $\triangle HIJ$ inscribed in circle K. Explain the steps you used to make your construction.

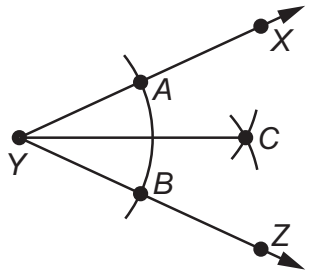
Solution:

(This is an alternate method from the method shown in the Key Ideas.) Use a compass to draw circle K. Draw segment $\overline{FG}$ through the center of circle K. Label the points where $\overline{FG}$ intersects circle K as points I and P. Using the compass setting you used when drawing the circle, place the compass on point P and draw an arc passing through point K. Label the points where the arc intersects circle K as points H and J. Draw $\overline{HJ}$, $\overline{IJ}$, and $\overline{HI}$. Triangle HIJ is an equilateral triangle inscribed in circle K.



SAMPLE ITEMS

1. Consider the construction of the angle bisector shown.



Which could have been the first step in creating this construction?

A. Place the compass point on point A and draw an arc inside $\angle Y$.
B. Place the compass point on point B and draw an arc inside $\angle Y$.
C. Place the compass point on vertex Y and draw an arc that intersects $\overline{YX}$ and $\overline{YZ}$.
D. Place the compass point on vertex Y and draw an arc that intersects point C.

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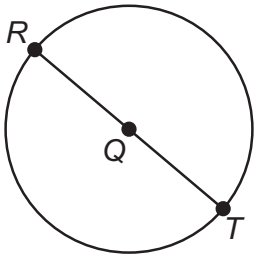
2. Consider the beginning of the construction of a square inscribed in circle *Q*.

Step 1: Label point *R* on circle *Q*.
Step 2: Draw a diameter through *R* and *Q*.
Step 3: Label the point where the diameter intersects the circle as point *T*.



What is the next step in this construction?

A. Draw radius $\overline{SQ}$.
B. Label point *S* on circle *Q*.
C. Construct a line segment parallel to $\overline{RT}$.
D. Construct the perpendicular bisector of $\overline{RT}$.

Answers to Unit 1.5 Sample Items

1. C 2. D

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1.6 Use Coordinates to Prove Simple Geometric Theorems Algebraically

MGSE9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

(Focus on quadrilaterals, right triangles, and circles.)

KEY IDEAS

To prove properties about special parallelograms on a coordinate plane, you can use the midpoint, distance, and slope formulas:

- The ***midpoint formula*** is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. This formula is used to find the coordinates of the midpoint of $\overline{AB}$, given $A(x_1, y_1)$ and $B(x_2, y_2)$.
- The ***distance formula*** is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. This formula is used to find the length of $\overline{AB}$, given $A(x_1, y_1)$ and $B(x_2, y_2)$.
- The ***slope formula*** is $m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$. This formula is used to find the slope of a line or line segment, given any two points on the line or line segment $A(x_1, y_1)$ and $B(x_2, y_2)$.

You can use properties of quadrilaterals to help prove theorems, such as the following:

- To prove a quadrilateral is a parallelogram, show that the opposite sides are parallel using the slope formula.
- To prove a quadrilateral is a rectangle, show that the opposite sides are parallel and the consecutive sides are perpendicular using the slope formula.
- To prove a quadrilateral is a rhombus, show that all four sides are congruent using the distance formula.
- To prove a quadrilateral is a square, show that all four sides are congruent and consecutive sides are perpendicular using the distance and slope formulas.

You can also use diagonals of a quadrilateral to help prove theorems, such as the following:

- To prove a quadrilateral is a parallelogram, show that its diagonals bisect each other using the midpoint formula.
- To prove a quadrilateral is a rectangle, show that its diagonals bisect each other and are congruent using the midpoint and distance formulas.
- To prove a quadrilateral is a rhombus, show that its diagonals bisect each other and are perpendicular using the midpoint and slope formulas.
- To prove a quadrilateral is a square, show that its diagonals bisect each other, are congruent, and are perpendicular using the midpoint, distance, and slope formulas.

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Important Tips

- When using the formulas for midpoint, distance, and slope, the order of the points does not matter. Either point can be (x_1, y_1), but be careful to always subtract in the same order.
- Parallel lines have the same slope. Perpendicular lines have slopes that are the negative reciprocal of each other.

REVIEW EXAMPLE

- Quadrilateral *ABCD* has vertices *A*(–1, 3), *B*(3, 5), *C*(4, 3), and *D*(0, 1). Is *ABCD* a rectangle? Explain how you know.

  Solution:

  First determine whether the figure is a parallelogram. If the figure is a parallelogram, then the diagonals bisect each other. If the diagonals bisect each other, then the midpoints of the diagonals are the same point. Use the midpoint formula to determine the midpoints for each diagonal.

  Midpoint $\overline{AC}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1 + 4}{2}, \frac{3 + 3}{2}\right) = \left(\frac{3}{2}, \frac{6}{2}\right) = (1.5, 3)$

  Midpoint $\overline{BD}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + 0}{2}, \frac{5 + 1}{2}\right) = \left(\frac{3}{2}, \frac{6}{2}\right) = (1.5, 3)$

  The diagonals have the same midpoint; therefore, the diagonals bisect each other and the figure is a parallelogram.

  A parallelogram with congruent diagonals is a rectangle. Determine whether the diagonals are congruent.

  Use the distance formula to find the length of the diagonals.

  $AC = \sqrt{(4 - (-1))^2 + (3 - 3)^2} = \sqrt{(5)^2 + (0)^2} = \sqrt{25 + 0} = \sqrt{25} = 5$

  $BD = \sqrt{(0 - 3)^2 + (1 - 5)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

  The diagonals are congruent because they have the same length.

  The figure is a parallelogram with congruent diagonals, so the figure is a rectangle.

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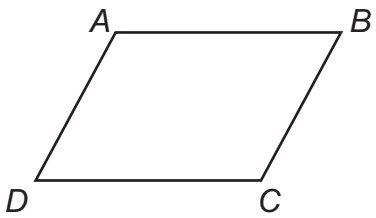
SAMPLE ITEMS

1. Which information is sufficient to show that a parallelogram is a rectangle?

A. The diagonals bisect each other.
B. The diagonals are congruent.
C. The diagonals are congruent and perpendicular.
D. The diagonals bisect each other and are perpendicular.

2. Look at quadrilateral *ABCD*.



Which information is needed to show that quadrilateral *ABCD* is a parallelogram?

A. Use the distance formula to show that diagonals *AC* and *BD* have the same length.
B. Use the slope formula to show that segments *AB* and *CD* are perpendicular and segments *AD* and *BC* are perpendicular.
C. Use the slope formula to show that segments *AB* and *CD* have the same slope and segments *AD* and *BC* have the same slope.
D. Use the distance formula to show that segments *AB* and *AD* have the same length and segments *CD* and *BC* have the same length.

Answers to Unit 1.6 Sample Items

1. B 2. C

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UNIT 2: RIGHT TRIANGLE TRIGONOMETRY

This unit investigates the properties of right triangles. The trigonometric ratios sine, cosine, and tangent along with the Pythagorean Theorem are used to solve right triangles in applied problems. The relationship between the sine and cosine of complementary angles is identified.

2.1 Right Triangle Relationships

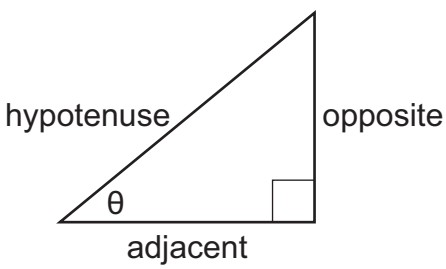
MGSE9-12.G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

MGSE9-12.G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

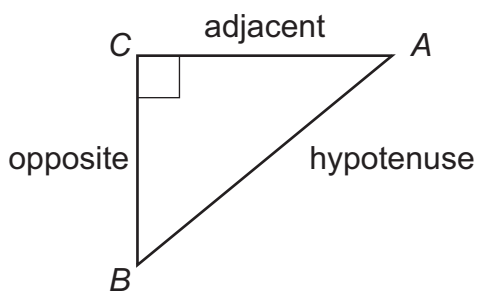
MGSE9-12.G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

KEY IDEAS

The trigonometric ratios ***sine***, ***cosine***, and ***tangent*** are defined as ratios of the lengths of the sides in a right triangle with a given acute angle measure. These terms are usually seen abbreviated as ***sin***, ***cos***, and ***tan***.



- $\sin\theta = \dfrac{\text{length of opposite side}}{\text{length of hypotenuse}}$
- $\cos\theta = \dfrac{\text{length of adjacent side}}{\text{length of hypotenuse}}$
- $\tan\theta = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$



- $\sin A = \dfrac{\text{length of opposite side}}{\text{length of hypotenuse}}$
- $\cos A = \dfrac{\text{length of adjacent side}}{\text{length of hypotenuse}}$
- $\tan A = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$

The two acute angles of any right triangle are complementary. As a result, if angles P and Q are complementary, $\sin P = \cos Q$ and $\sin Q = \cos P$.

When solving problems with right triangles, you can use both trigonometric ratios and the Pythagorean Theorem ($a^2 + b^2 = c^2$). There may be more than one way to solve the problem, so analyze the given information to help decide which method is the most efficient.

Important Tip

✍ The tangent of angle A is also equivalent to $\dfrac{\sin A}{\cos A}$.

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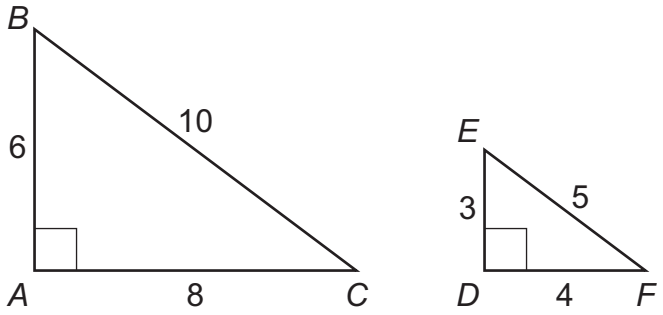
REVIEW EXAMPLES

♦ Triangles *ABC* and *DEF* are similar.



a. Find the ratio of the side opposite angle *B* to the hypotenuse in $\triangle ABC$.
b. What angle in $\triangle DEF$ corresponds to angle *B*?
c. Find the ratio of the side opposite angle *E* to the hypotenuse in $\triangle DEF$.
d. How does the ratio in part (a) compare to the ratio in part (c)?
e. Which trigonometric ratio does this represent?

Solution:

a. $\overline{AC}$ is opposite angle *B*. $\overline{BC}$ is the hypotenuse. The ratio of the side opposite angle *B* to the hypotenuse in $\triangle ABC$ is $\frac{8}{10} = \frac{4}{5}$.
b. Angle *E* in $\triangle DEF$ corresponds to angle *B* in $\triangle ABC$.
c. $\overline{DF}$ is opposite angle *E*. $\overline{EF}$ is the hypotenuse. The ratio of the side opposite angle *E* to the hypotenuse in $\triangle DEF$ is $\frac{4}{5}$.
d. The ratios are the same.
e. This represents sin *B* and sin *E*, because both are the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$.

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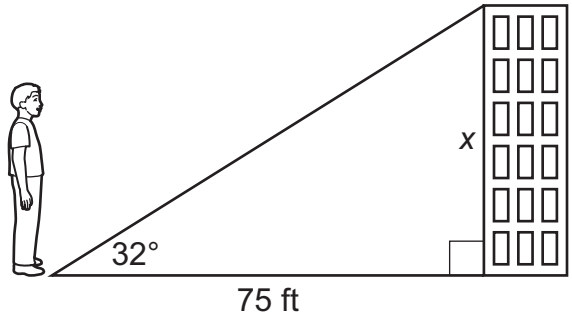
- Ricardo is standing 75 feet away from the base of a building. The ***angle of elevation*** from the ground where Ricardo is standing to the top of the building is 32°.



Note: figure not drawn to scale.

What is x, the height of the building, to the nearest tenth of a foot?

$$\begin{bmatrix} \sin 32° \approx 0.5299 \\ \cos 32° \approx 0.8480 \\ \tan 32° \approx 0.6249 \end{bmatrix}$$

Solution:

When finding the length of the side opposite the 32° angle and given the length of the side adjacent to the 32° angle, use the tangent ratio. Substitute x for the opposite side, 75 for the adjacent side, and 32° for the angle measure. Then solve.

$$\tan 32° = \frac{x}{75}$$

$$75\tan 32° = x$$

$$75 \bullet 0.6249 \approx x$$

$$46.9 \approx x$$

The building is about 46.9 feet tall.

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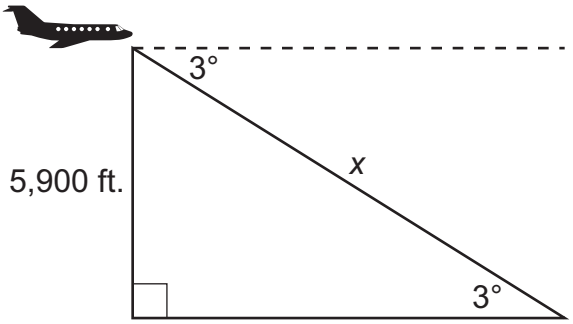
- An airplane is at an altitude of 5,900 feet. The airplane descends at an angle of 3°, called the ***angle of depression***.



Note: figure not drawn to scale.

About how far will the airplane travel in the air until it reaches the ground?

$$\begin{bmatrix} \sin 3° \approx 0.0523 \\ \cos 3° \approx 0.9986 \\ \tan 3° \approx 0.0524 \end{bmatrix}$$

Solution:

Use sin 3° to find the distance the airplane will travel until it reaches the ground, x. Substitute x for the hypotenuse, 5,900 for the opposite side, and 3° for the angle measure. Then solve.

$$\sin 3° = \frac{5{,}900}{x}$$

$$x \sin 3° = 5{,}900$$

$$x = \frac{5{,}900}{\sin 3°}$$

$$x \approx \frac{5{,}900}{0.0523}$$

$$x \approx 112{,}811$$

The airplane will travel about 113,000 feet until it reaches the ground.

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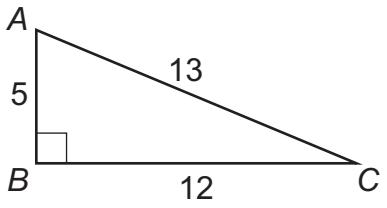
♦ Triangle *ABC* is a right triangle.



What is the best approximation for $m\angle C$?

$$\begin{bmatrix} \sin 67.4° \approx 0.923 \\ \cos 22.6° \approx 0.923 \\ \tan 42.7° \approx 0.923 \end{bmatrix}$$

Solution:

Find the trigonometric ratios for angle *C*.

$\sin C = \frac{5}{13} \approx 0.385$

$\cos C = \frac{12}{13} \approx 0.923$

$\tan C = \frac{5}{12} \approx 0.417$

Using the table, $\cos 22.6° \approx 0.923$, so $m\angle C \approx 22.6°$, or using trigonometric inverses, $\sin^{-1}\frac{5}{13} = 22.6$, $\cos^{-1}\frac{12}{13} = 22.6$, or $\tan^{-1}\frac{5}{12} = 22.6$.

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SAMPLE ITEMS

1. In right triangle *ABC*, angle *A* and angle *B* are complementary angles. The value of cos *A* is $\frac{5}{13}$. What is the value of sin *B*?

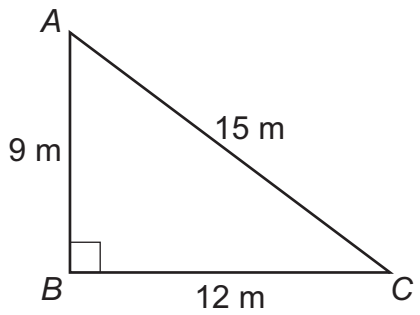
A. $\frac{5}{13}$

B. $\frac{12}{13}$

C. $\frac{13}{12}$

D. $\frac{13}{5}$

2. Triangle *ABC* is given below.



What is the value of cos *A*?

A. $\frac{3}{5}$

B. $\frac{3}{4}$

C. $\frac{4}{5}$

D. $\frac{5}{3}$

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3. In right triangle *HJK*, $\angle J$ is a right angle and tan $\angle H = 1$. Which statement about triangle *HJK* must be true?

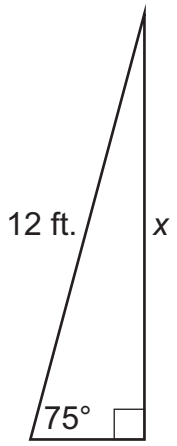
A. $\sin \angle H = \frac{1}{2}$

B. $\sin \angle H = 1$

C. $\sin \angle H = \cos \angle H$

D. $\sin \angle H = \frac{1}{\cos \angle H}$

4. A 12-foot ladder is leaning against a building at a 75° angle to the ground.



Which equation can be used to find how high the ladder reaches up the side of the building?

A. $\sin 75° = \frac{12}{x}$

B. $\tan 75° = \frac{12}{x}$

C. $\cos 75° = \frac{x}{12}$

D. $\sin 75° = \frac{x}{12}$

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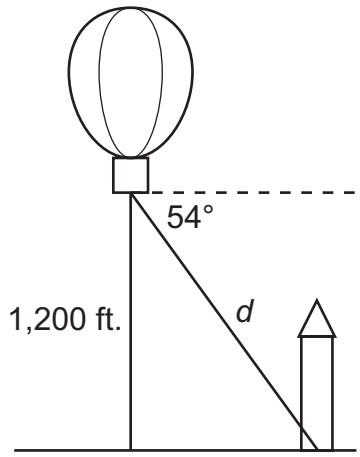
5. A hot-air balloon is 1,200 feet above the ground. The angle of depression from the basket of the hot-air balloon to the base of a monument is 54°.



Note: figure not drawn to scale.

Which equation can be used to find the distance, *d*, in feet, from the basket of the hot air balloon to the base of the monument?

A. $\sin 54° = \frac{d}{1200}$

B. $\sin 54° = \frac{1200}{d}$

C. $\cos 54° = \frac{d}{1200}$

D. $\cos 54° = \frac{1200}{d}$

Answers to Unit 2.1 Sample Items

1. A 2. A 3. C 4. D 5. B

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UNIT 3: CIRCLES AND VOLUME

This unit investigates the properties of circles and addresses finding the volume of solids. Properties of circles are used to solve problems involving arcs, angles, sectors, chords, tangents, and secants. Volume formulas are derived and used to calculate the volumes of cylinders, pyramids, cones, and spheres.

3.1 Understand and Apply Theorems about Circles

MGSE9-12.G.C.1 Understand that all circles are similar.

MGSE9-12.G.C.2 Identify and describe relationships among inscribed angles, radii, chords, tangents, and secants. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

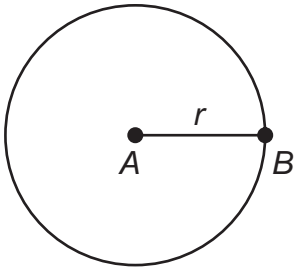
MGSE9-12.G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

MGSE9-12.G.C.4 Construct a tangent line from a point outside a given circle to the circle.

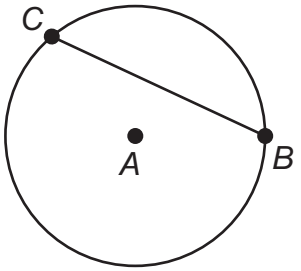
KEY IDEAS

A ***circle*** is the set of points in a plane equidistant from a given point, which is the center of the circle. All circles are similar.

A ***radius*** is a line segment from the center of a circle to any point on the circle. The word radius is also used to describe the length, *r*, of the segment. $\overline{AB}$ is a radius of circle *A*.



A ***chord*** is a line segment whose endpoints are on a circle. $\overline{BC}$ is a chord of circle *A*.



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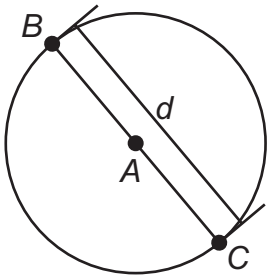
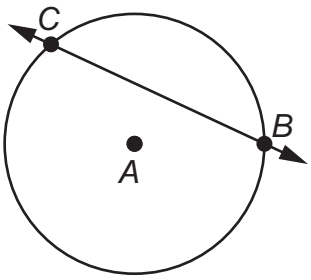
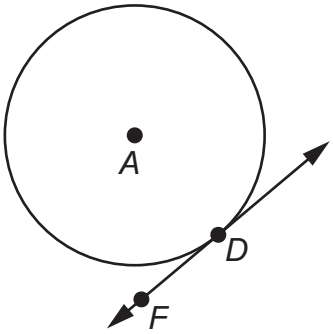
A ***diameter*** is a chord that passes through the center of a circle. The word diameter is also used to describe the length, *d,* of the segment. $\overline{BC}$ is a diameter of circle *A*.



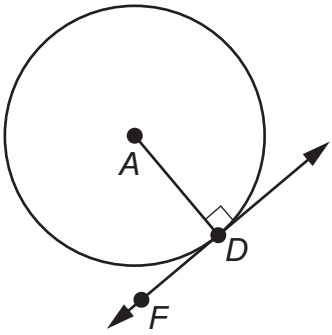
A ***secant line*** is a line that is in the plane of a circle and intersects the circle at exactly two points. Every chord lies on a secant line. $\overleftrightarrow{BC}$ is a secant line of circle *A*.



A ***tangent line*** is a line that is in the plane of a circle and intersects the circle at only one point, the ***point of tangency***. $\overleftrightarrow{DF}$ is tangent to circle *A* at the point of tangency, point *D.*



If a line is tangent to a circle, the line is perpendicular to the radius drawn to the point of tangency. $\overleftrightarrow{DF}$ is tangent to circle *A* at point *D*, so $\overline{AD} \perp \overline{DF}$.



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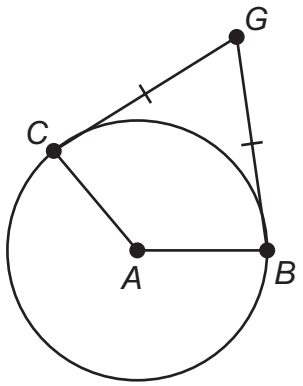
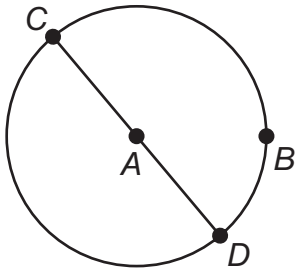
Tangent segments drawn from the same point are congruent. In circle A, $\overline{CG} \cong \overline{BG}$.

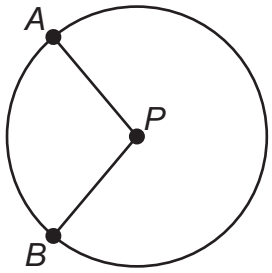


Circumference is the distance around a circle. The formula for the circumference, C, of a circle is $C = \pi d$, where d is the diameter of the circle. The formula is also written as $C = 2\pi r$, where r is the length of the radius of the circle. π is the ratio of circumference to diameter of any circle.

An ***arc*** is a part of the circumference of a circle. A ***minor arc*** has a measure less than 180°. Minor arcs are written using two points on a circle. A ***semicircle*** is an arc that measures exactly 180°. Semicircles are written using three points on a circle. This is done to show which half of the circle is being described. A ***major arc*** has a measure greater than 180°. Major arcs are written with three points to distinguish them from the corresponding minor arc. In circle A, $\overset{\frown}{CB}$ is a minor arc, $\overset{\frown}{CBD}$ is a semicircle, and $\overset{\frown}{CDB}$ is a major arc.



A ***central angle*** is an angle whose vertex is at the center of a circle and whose sides are radii of the circle. The measure of a central angle of a circle is equal to the measure of the intercepted arc. $\angle APB$ is a central angle for circle P, and $\overset{\frown}{AB}$ is the intercepted arc.



$$m\angle APB = m\overset{\frown}{AB}$$

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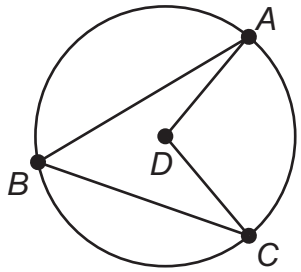
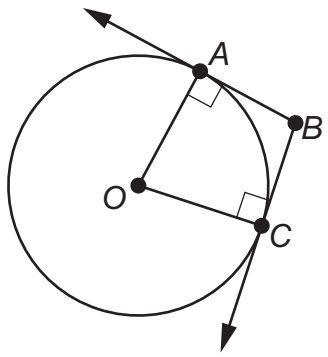
An ***inscribed angle*** is an angle whose vertex is on a circle and whose sides are chords of the circle. The measure of an angle inscribed in a circle is half the measure of the intercepted arc. For circle D, $\angle ABC$ is an inscribed angle, and $\overset{\frown}{AC}$ is the intercepted arc.



$$m\angle ABC = \frac{1}{2}\left(m\overset{\frown}{AC}\right) = \frac{1}{2}(m\angle ADC)$$

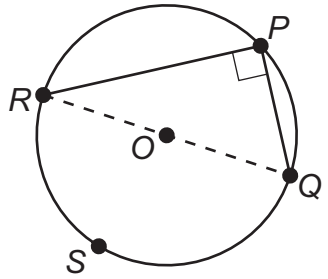
$$m\angle ADC = m\overset{\frown}{AC} = 2(m\angle ABC)$$

A ***circumscribed angle*** is an angle formed by two rays that are each tangent to a circle. These rays are perpendicular to radii of the circle. In circle O, the measure of the circumscribed angle is equal to 180° minus the measure of the central angle that forms the intercepted arc. The measure of the circumscribed angle can also be found by using the measures of two intercepted arcs.



$$m\angle ABC = 180° - m\angle AOC$$

When an inscribed angle intercepts a semicircle, the inscribed angle has a measure of 90°. For circle O, $\angle RPQ$ intercepts semicircle $\overset{\frown}{RSQ}$ as shown.



$$m\angle RPQ = \frac{1}{2}\left(m\overset{\frown}{RSQ}\right) = \frac{1}{2}(180°) = 90°$$

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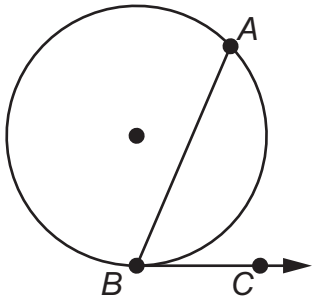
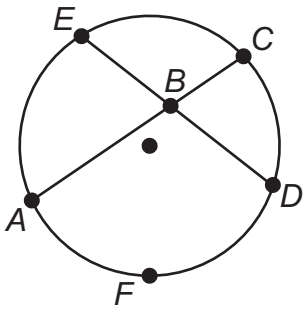
The measure of an angle formed by a tangent and a chord with its vertex on the circle is half the measure of the intercepted arc. $\overline{AB}$ is a chord for the circle, and $\overrightarrow{BC}$ is tangent to the circle at point B. So $\angle ABC$ is formed by a tangent and a chord.



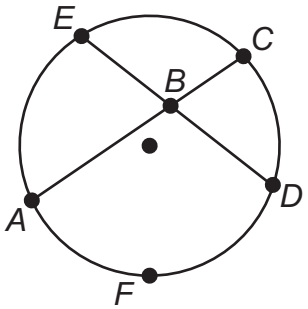
$$m\angle ABC = \frac{1}{2}\left(m\widehat{AB}\right)$$

When two chords intersect inside a circle, two pairs of vertical angles are formed. The measure of any one of the angles is half the sum of the measures of the arcs intercepted by the pair of vertical angles.



$$m\angle ABE = \frac{1}{2}\left(m\widehat{AE} + m\widehat{CD}\right)$$
$$m\angle ABD = \frac{1}{2}\left(m\widehat{AFD} + m\widehat{EC}\right)$$

When two chords intersect inside a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$$AB \bullet BC = EB \bullet BD$$

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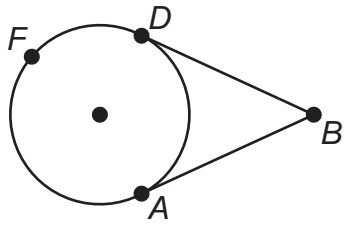
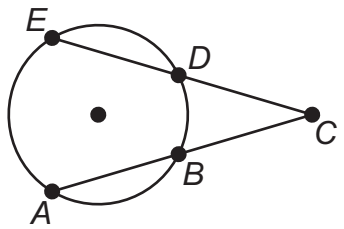
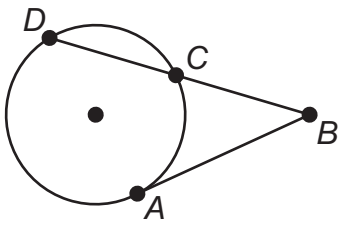
Angles outside a circle can be formed by the intersection of two tangents (circumscribed angle), two secants, or a secant and a tangent. For all three situations, the measure of the angle is half the difference of the measure of the larger intercepted arc and the measure of the smaller intercepted arc.



$$m\angle ABD = \frac{1}{2}\left(m\widehat{AFD} - m\widehat{AD}\right)$$

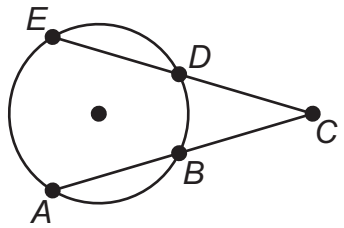


$$m\angle ACE = \frac{1}{2}\left(m\widehat{AE} - m\widehat{BD}\right)$$



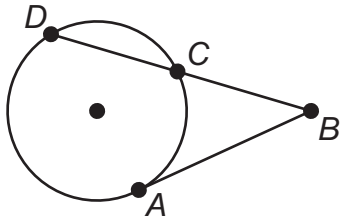
$$m\angle ABD = \frac{1}{2}\left(m\widehat{AD} - m\widehat{AC}\right)$$

When two ***secant segments*** intersect outside a circle, part of each secant segment is a segment formed outside the circle. The product of the length of one secant segment and the length of the segment formed outside the circle is equal to the product of the length of the other secant segment and the length of the segment formed outside the circle.



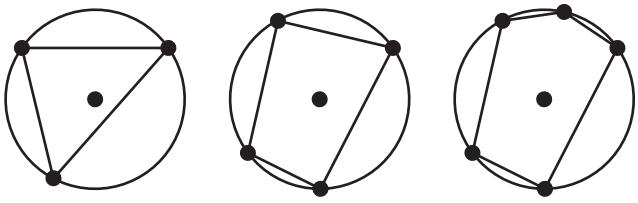
$$EC \bullet DC = AC \bullet BC$$

When a secant segment and a tangent segment intersect outside a circle, the product of the length of the secant segment and the length of the segment formed outside the circle is equal to the square of the length of the tangent segment.



$$DB \bullet CB = AB^2$$

An ***inscribed polygon*** is a polygon whose vertices all lie on a circle. This diagram shows a triangle, a quadrilateral, and a pentagon each inscribed in a circle.



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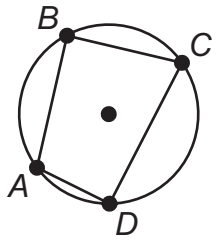
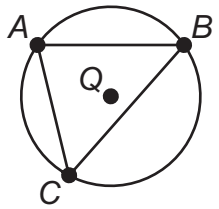
In a quadrilateral inscribed in a circle, the opposite angles are supplementary.



$$m\angle ABC + m\angle ADC = 180°$$

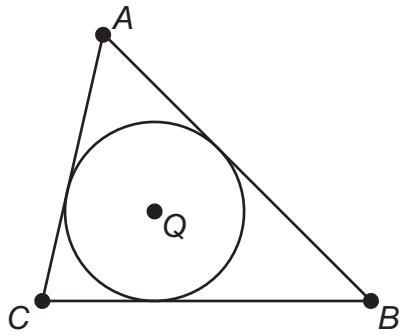
$$m\angle BCD + m\angle BAD = 180°$$

When a triangle is inscribed in a circle, the center of the circle is the ***circumcenter*** of the triangle. The circumcenter is equidistant from the vertices of the triangle. Triangle *ABC* is inscribed in circle *Q*, and point *Q* is the circumcenter of the triangle.



$$AQ = BQ = CQ$$

An ***inscribed circle*** is a circle enclosed in a polygon, where every side of the polygon is tangent to the circle. Specifically, when a circle is inscribed in a triangle, the center of the circle is the ***incenter*** of the triangle. The incenter is equidistant from the sides of the triangle. Circle *Q* is inscribed in triangle *ABC*, and point *Q* is the incenter of the triangle. Notice also that the sides of the triangle form circumscribed angles with the circle.



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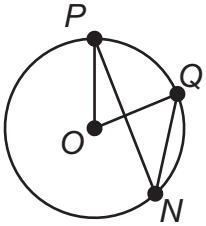
REVIEW EXAMPLES

♦ $\angle PNQ$ is inscribed in circle O, and $m\widehat{PQ} = 70°$.



a. What is the measure of $\angle POQ$?
b. What is the relationship between $\angle POQ$ and $\angle PNQ$?
c. What is the measure of $\angle PNQ$?

Solution:

a. The measure of a central angle is equal to the measure of the intercepted arc.
$m\angle POQ = m\widehat{PQ} = 70°$.

b. $\angle POQ$ is a central angle that intercepts $\widehat{PQ}$. $\angle PNQ$ is an inscribed angle that intercepts $\widehat{PQ}$. The measure of the central angle is equal to the measure of the intercepted arc. The measure of the inscribed angle is equal to one-half the measure of the intercepted arc. So $m\angle POQ = m\widehat{PQ}$, $m\angle PNQ = \frac{1}{2}\left(m\widehat{PQ}\right)$, and $m\angle POQ = 2(m\angle PNQ)$.

c. From part (b), $m\angle POQ = 2(m\angle PNQ)$
Substitute: $70° = 2(m\angle PNQ)$
Divide: $35° = m\angle PNQ$

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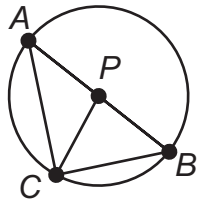
♦ In circle P below, $\overline{AB}$ is a diameter.



If $m\angle APC = 100°$, find the following:

a. $m\angle BPC$
b. $m\angle BAC$
c. $m\widehat{BC}$
d. $m\widehat{AC}$

Solution:

a. $\angle APC$ and $\angle BPC$ are supplementary, so $m\angle BPC = 180° - m\angle APC = 180° - 100° = 80°$.
b. $\angle BAC$ is an angle in $\triangle APC$. The sum of the measures of the angles of a triangle is 180°.

For $\triangle APC$: $m\angle APC + m\angle BAC + m\angle ACP = 180°$

You are given that $m\angle APC = 100°$.

Substitute: $100° + m\angle BAC + m\angle ACP = 180°$

Subtract 100° from both sides: $m\angle BAC + m\angle ACP = 80°$

Because two sides of $\triangle APC$ are radii of the circle, $\triangle APC$ is an isosceles triangle. This means that the two base angles are congruent, so $m\angle BAC = m\angle ACP$.

Substitute: $m\angle BAC$ for $m\angle ACP$: $m\angle BAC + m\angle BAC = 80°$

Add: $2(m\angle BAC) = 80°$

Divide: $m\angle BAC = 40°$

You could also use the answer from part (a) to solve for $m\angle BAC$. Part (a) shows $m\angle BPC = 80°$.

Because the central angle measure is equal to the measure of the intercepted arc, $m\angle BPC = m\widehat{BC} = 80°$.

Because an inscribed angle is equal to one-half the measure of the intercepted arc, $m\angle BAC = \frac{1}{2}(m\widehat{BC})$.

Substitute: $m\angle BAC = \frac{1}{2}(80°)$

Therefore, $m\angle BAC = 40°$.

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c. $\angle BAC$ is an inscribed angle intercepting $\overset{\frown}{BC}$. The intercepted arc is twice the measure of the inscribed angle.

$m\overset{\frown}{BC} = 2(m\angle BAC)$

From part (b), $m\angle BAC = 40°$.

Substitute: $m\overset{\frown}{BC} = 2 \bullet 40°$

$m\overset{\frown}{BC} = 80°$

You could also use the answer from part (a) to solve. Part (a) shows $m\angle BPC = 80°$. Because $\angle BPC$ is a central angle that intercepts $\overset{\frown}{BC}$, $m\angle BPC = m\overset{\frown}{BC} = 80°$.

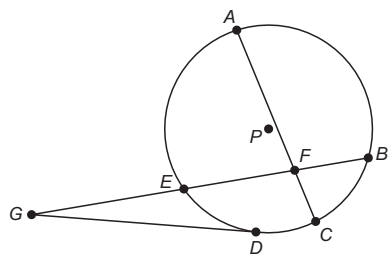
d. $\angle APC$ is a central angle intercepting $\overset{\frown}{AC}$. The measure of the intercepted arc is equal to the measure of the central angle.

$m\overset{\frown}{AC} = m\angle APC$

You are given $m\angle APC = 100°$.

Substitute: $m\overset{\frown}{AC} = 100°$

♦ In circle P below, $\overline{DG}$ is a tangent, $AF = 8$, $EF = 6$, $BF = 4$, and $EG = 8$.



Find CF and DG.

Solution:

First, find CF. Use the fact that $\overline{CF}$ is part of a pair of intersecting chords.

$$AF \bullet CF = EF \bullet BF$$
$$8 \bullet CF = 6 \bullet 4$$
$$8 \bullet CF = 24$$
$$CF = 3$$

Next, find DG. Use the fact that $\overline{DG}$ is tangent to the circle.

$$EG \bullet BG = DG^2$$
$$8 \bullet (8 + 6 + 4) = DG^2$$
$$8 \bullet 18 = DG^2$$
$$144 = DG^2$$
$$\pm 12 = DG$$
$$12 = DG \text{ (since length cannot be negative)}$$

$CF = 3$ and $DG = 12$

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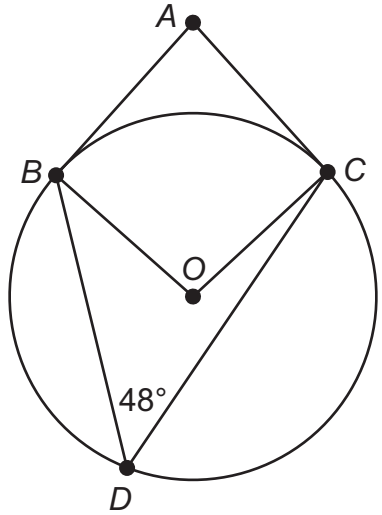
♦ In this circle, $\overline{AB}$ is tangent to the circle at point B, $\overline{AC}$ is tangent to the circle at point C, and point D lies on the circle. What is $m\angle BAC$?



Solution:

Method 1

First, find the measure of angle *BOC*. Angle *BDC* is an inscribed angle, and angle *BOC* is a central angle.

$$\begin{aligned} m\angle BOC &= 2(m\angle BDC) \\ &= 2 \bullet 48° \\ &= 96° \end{aligned}$$

Angle *BAC* is a circumscribed angle. Use the measure of angle *BOC* to find the measure of angle *BAC*.

$$\begin{aligned} m\angle BAC &= 180° - m\angle BOC \\ &= 180° - 96° \\ &= 84° \end{aligned}$$

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Method 2

Angle *BDC* is an inscribed angle. First, find the measures of $\overset{\frown}{BC}$ and $\overset{\frown}{BDC}$.

$$m\angle BDC = \frac{1}{2}\left(m\overset{\frown}{BC}\right)$$
$$48° = \frac{1}{2}\left(m\overset{\frown}{BC}\right)$$
$$2 \bullet 48° = m\overset{\frown}{BC}$$
$$96° = m\overset{\frown}{BC}$$

$$\begin{aligned} m\overset{\frown}{BDC} &= 360° - m\overset{\frown}{BC} \\ &= 360° - 96° \\ &= 264° \end{aligned}$$

Angle *BAC* is a circumscribed angle. Use the measures of $\overset{\frown}{BC}$ and $\overset{\frown}{BDC}$ to find the measure of angle *BAC*.

$$\begin{aligned} m\angle BAC &= \frac{1}{2}\left(m\overset{\frown}{BDC} - m\overset{\frown}{BC}\right) \\ &= \frac{1}{2}(264° - 96°) \\ &= \frac{1}{2}(168°) \\ &= 84° \end{aligned}$$

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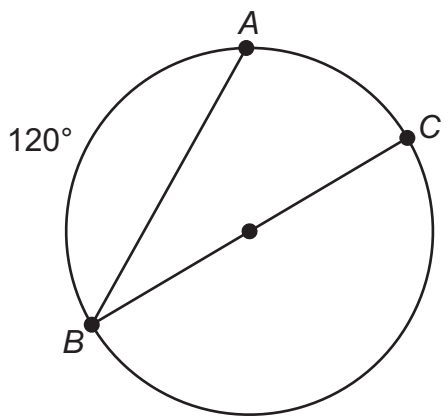
SAMPLE ITEMS

1. Circle *P* is dilated to form circle *P′*. Which statement is ALWAYS true?

A. The radius of circle *P* is equal to the radius of circle *P′*.
B. The length of any chord in circle *P* is greater than the length of any chord in circle *P′*.
C. The diameter of circle *P* is greater than the diameter of circle *P′*.
D. The ratio of the diameter to the circumference is the same for both circles.

2. In the circle shown, $\overline{BC}$ is a diameter and $m\widehat{AB} = 120°$.



What is the measure of $\angle ABC$?

A. 15°
B. 30°
C. 60°
D. 120°

Answers to Unit 3.1 Sample Items

1. D 2. B

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3.2 Find Arc Lengths and Areas of Sectors of Circles

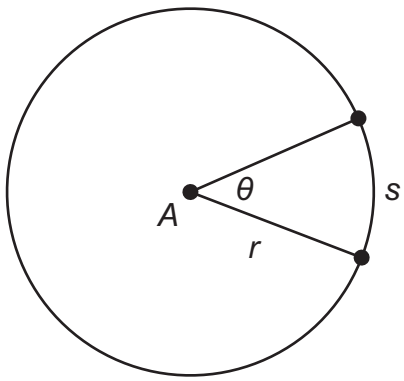
MGSE9-12.G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

KEY IDEAS

Circumference is the distance around a circle. The formula for the circumference, C, of a circle is $C = 2\pi r$, where r is the length of the radius of the circle.

Area is a measure of the amount of space a circle covers. The formula for the area, A, of a circle is $A = \pi r^2$, where r is the length of the radius of the circle.

Arc length is a portion of the circumference of a circle. To find the length of an arc, divide the number of degrees in the central angle of the arc by 360 and then multiply that amount by the circumference of the circle. The formula for the arc length, s, is $s = \frac{2\pi r\theta}{360}$, where θ is the degree measure of the central angle and r is the radius of the circle.



Important Tip

✍ Do not confuse arc length with the ***arc measure*** in degrees. Arc length depends on the size of the circle because it is part of the circumference of the circle. The measure of the arc is independent of (does not depend on) the size of the circle.

The following shows one way to remember the formula for arc length:

arc length = fraction of the circle × circumference = $s = \frac{2\pi r\theta}{360}$.

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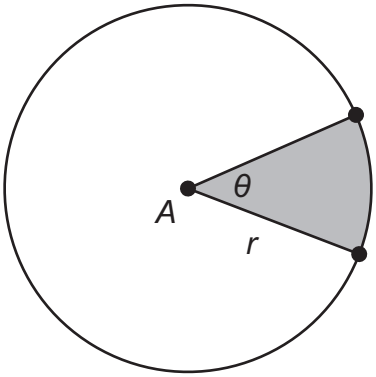
A ***sector*** of a circle is the region bounded by two radii of a circle and the resulting arc between them. To find the area of a sector, divide the number of degrees in the central angle of the arc by 360 and then multiply that amount by the area of the circle. The formula for the area of a sector is $\frac{\pi r^2 \theta}{360}$, where θ is the degree measure of the central angle and r is the radius of the circle.



Important Tip

✍ The following shows one way to remember the formula for the area of a sector:

area of a sector = fraction of the circle × area = $\frac{\theta}{360}(\pi r^2)$.

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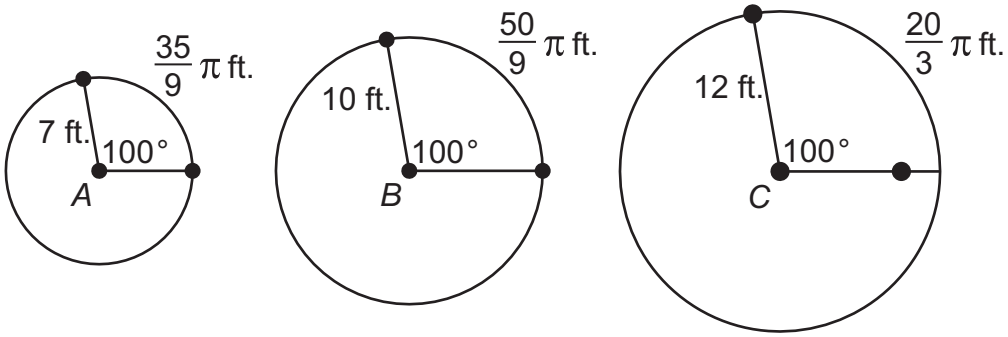
REVIEW EXAMPLES

♦ Circles *A*, *B*, and *C* have a central angle measuring 100°. The length of each radius and the length of each intercepted arc are shown.



a. What is the ratio of the radius of circle *B* to the radius of circle *A*?

b. What is the ratio of the length of the intercepted arc of circle *B* to the length of the intercepted arc of circle *A*?

c. Compare the ratios in parts (a) and (b).

d. What is the ratio of the radius of circle *C* to the radius of circle *B*?

e. What is the ratio of the length of the intercepted arc of circle *C* to the length of the intercepted arc of circle *B*?

f. Compare the ratios in parts (d) and (e).

g. Based on your observations of circles *A*, *B*, and *C*, what conjecture can you make about the length of the arc intercepted by a central angle and the radius?

h. What is the ratio of arc length to radius for each circle?

Solution:

a. Divide the radius of circle *B* by the radius of circle *A*: $\frac{\text{circle B}}{\text{circle A}} = \frac{10}{7}$

b. Divide the length of the intercepted arc of circle *B* by the length of the intercepted arc of circle *A*:

$$\frac{\frac{50}{9}\pi}{\frac{35}{9}\pi} = \frac{50\pi}{9} \bullet \frac{9}{35\pi} = \frac{10}{7}$$

c. The ratios are the same.

d. Divide the radius of circle *C* by the radius of circle *B*: $\frac{\text{circle C}}{\text{circle B}} = \frac{12}{10} = \frac{6}{5}$

e. Divide the length of the intercepted arc of circle *C* by the length of the intercepted arc of circle *B*:

$$\frac{\frac{20}{3}\pi}{\frac{50}{9}\pi} = \frac{20\pi}{3} \bullet \frac{9}{50\pi} = \frac{6}{5}$$

f. The ratios are the same.

g. When circles, such as circles *A*, *B*, and *C*, have the same central angle measure, the ratio of the lengths of the intercepted arcs is the same as the ratio of the radii.

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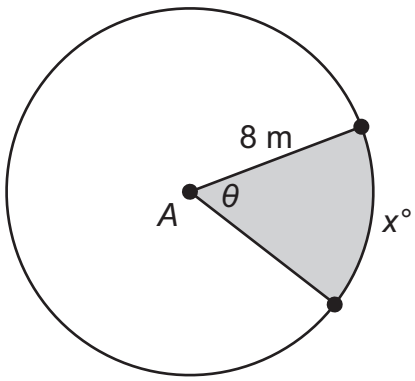
h. Circle A: $\dfrac{\frac{35}{9}\pi}{7} = \dfrac{35}{63}\pi = \dfrac{5}{9}\pi$

Circle B: $\dfrac{\frac{50}{9}\pi}{10} = \dfrac{50}{90}\pi = \dfrac{5}{9}\pi$

Circle C: $\dfrac{\frac{20}{3}\pi}{12} = \dfrac{20}{36}\pi = \dfrac{5}{9}\pi$

♦ Circle A is shown.



If $x = 50$, what is the area of the shaded sector of circle A?

Solution:

To find the area of the sector, divide the measure of the central angle of the arc in degrees by 360 and then multiply that amount by the area of the circle. The arc measure, x, is equal to the measure of the central angle, θ. The formula for the area of a circle is $A = \pi r^2$.

$A_{\text{sector}} = \dfrac{\pi r^2 \theta}{360}$ Area of sector of a circle with radius r and central angle θ in degrees

$A_{\text{sector}} = \dfrac{50\pi(8)^2}{360}$ Substitute 50 for θ and 8 for r.

$A_{\text{sector}} = \dfrac{5\pi(64)}{36}$ Rewrite the fraction and the power.

$A_{\text{sector}} = \dfrac{320\pi}{36}$ Multiply.

$A_{\text{sector}} = \dfrac{80\pi}{9}$ Rewrite.

The area of the sector is $\dfrac{80}{9}\pi$ square meters.

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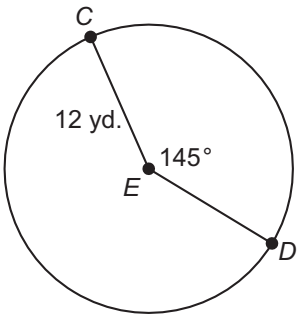
SAMPLE ITEMS

1. Circle *E* is shown.



What is the length of $\overset{\frown}{CD}$?

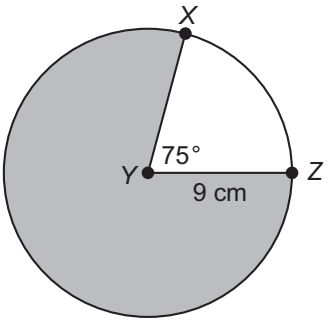
A. $\frac{29}{72}\pi$ yd.

B. $\frac{29}{6}\pi$ yd.

C. $\frac{29}{3}\pi$ yd.

D. $\frac{29}{2}\pi$ yd.

2. Circle *Y* is shown.



What is the area of the shaded part of the circle?

A. $\frac{57}{4}\pi$ cm^2

B. $\frac{135}{8}\pi$ cm^2

C. $\frac{405}{8}\pi$ cm^2

D. $\frac{513}{8}\pi$ cm^2

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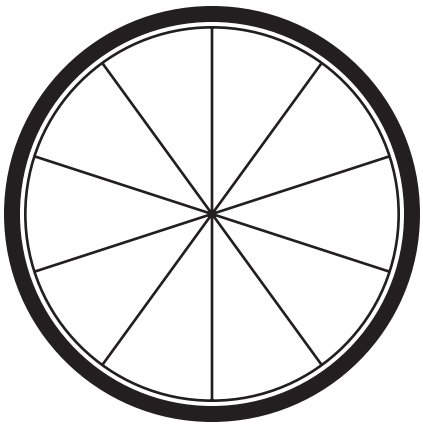
3. The spokes of a bicycle wheel form 10 congruent central angles. The diameter of the circle formed by the outer edge of the wheel is 18 inches.



What is the length, to the nearest tenth inch, of the outer edge of the wheel between two consecutive spokes?

A. 1.8 inches
B. 5.7 inches
C. 11.3 inches
D. 25.4 inches

Answers to Unit 3.2 Sample Items

1. C 2. D 3. B

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3.3 Explain Volume Formulas and Use Them to Solve Problems

MGSE9-12.G.GMD.1 Give informal arguments for geometric formulas.

a. Give informal arguments for the formulas of the circumference of a circle and area of a circle using dissection arguments and informal limit arguments.

b. Give informal arguments for the formula of the volume of a cylinder, pyramid, and cone using Cavalieri's principle.

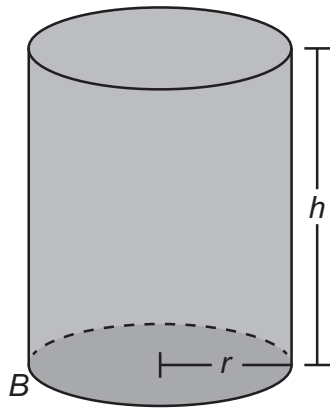
MGSE9-12.G.GMD.2 Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

MGSE9-12.G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

KEY IDEAS

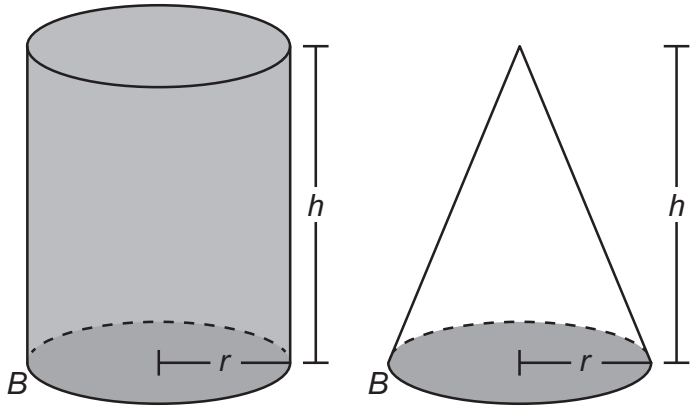
The ***volume*** of a figure is a measure of how much space it takes up. Volume is a measure of capacity.

The formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height. The volume formula can also be given as $V = Bh$, where B is the area of the base. In a cylinder, the base is a circle and the area of a circle is given by $A = \pi r^2$. Therefore, $V = Bh = \pi r^2 h$.



When a cylinder and a cone have congruent bases and equal heights, the volume of exactly three cones will fit into the cylinder. So, for a cone and cylinder that have the same radius r and height h, the volume of the cone is one-third of the volume of the cylinder.

The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height.



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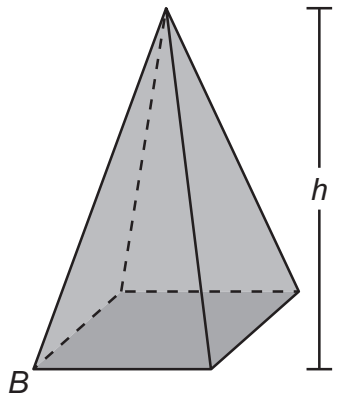
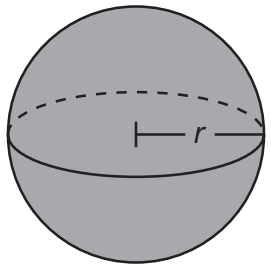
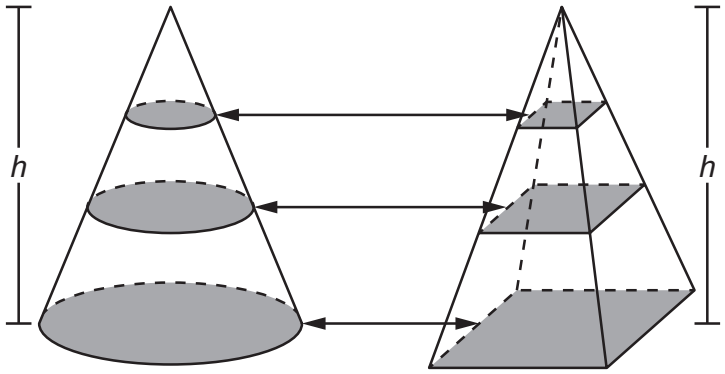
The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.



The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius.



Cavalieri's principle states that if two solids are between parallel planes and all cross sections at equal distances from their bases have equal areas, the solids have equal volumes. For example, this cone and this pyramid have the same height and the cross sections have the same area, so they have equal volumes.



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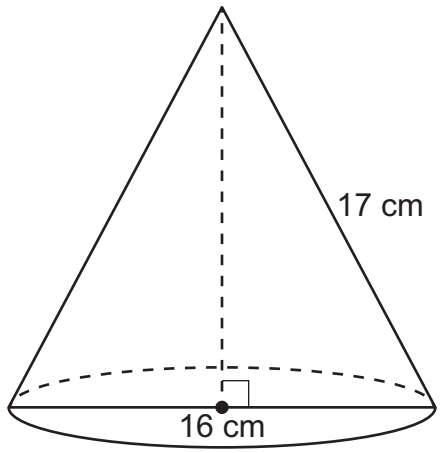
REVIEW EXAMPLES

- What is the volume of the cone shown below?



Solution:

The diameter of the cone is 16 cm. So the radius is 16 cm ÷ 2 = 8 cm. Use the Pythagorean Theorem, $a^2 + b^2 = c^2$, to find the height of the cone. Substitute 8 for b and 17 for c and solve for a:

$$a^2 + 8^2 = 17^2$$
$$a^2 + 64 = 289$$
$$a^2 = 225$$
$$a = 15$$

The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Substitute 8 for r and 15 for h:

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8)^2(15)$$

The volume is 320π cm^3.

- A sphere has a radius of 3 feet. What is the volume of the sphere?

Solution:

The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Substitute 3 for r and solve.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(3)^3$$
$$V = \frac{4}{3}\pi(27)$$
$$V = 36\pi \text{ ft}^3$$

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- A cylinder has a radius of 10 cm and a height of 9 cm. A cone has a radius of 10 cm and a height of 9 cm. Show that the volume of the cylinder is three times the volume of the cone.

  Solution:

  The formula for the volume of a cylinder is $V = \pi r^2 h$. Substitute 10 for r and 9 for h:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(10)^2(9) \\ &= \pi(100)(9) \\ &= 900\pi \text{ cm}^3 \end{aligned}$$

  The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Substitute 10 for r and 9 for h:

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(10)^2(9) \\ &= \frac{1}{3}\pi(100)(9) \\ &= 300\pi \text{ cm}^3 \end{aligned}$$

Divide: $900\pi \div 300\pi = 3$

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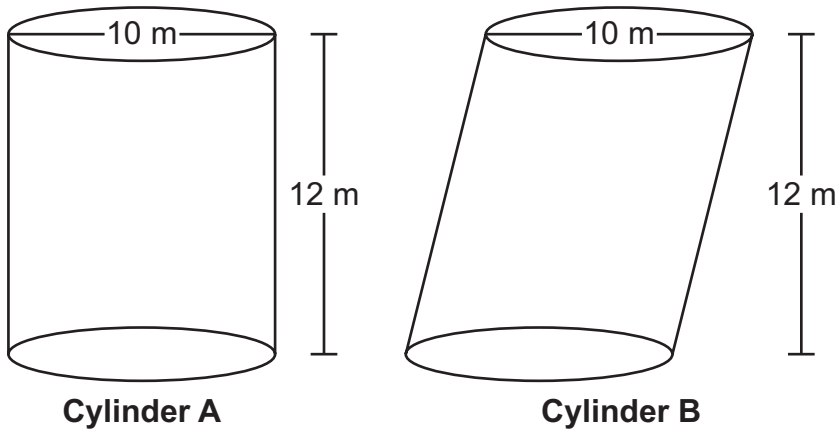
♦ Cylinder A and Cylinder B are shown below. What is the volume of each cylinder?

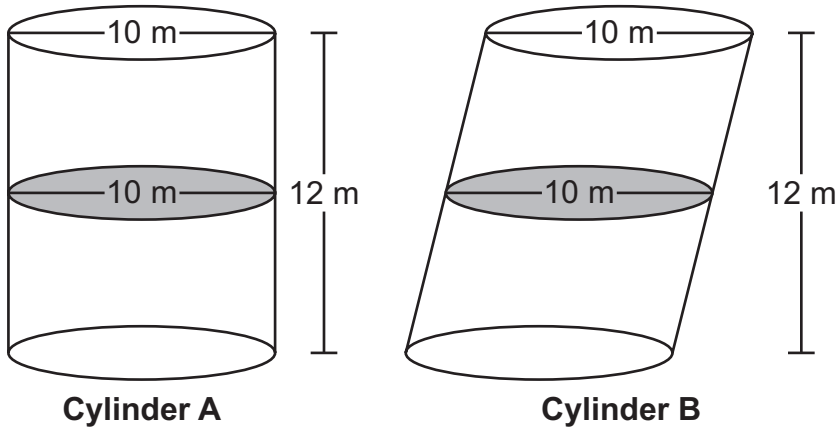


Solution**:**

To find the volume of Cylinder A, use the formula for the volume of a cylinder, which is $V = \pi r^2 h$. Divide the diameter by 2 to find the radius: $10 \div 2 = 5$. Substitute 5 for r and 12 for h:

$$\begin{aligned} V_{\text{Cylinder A}} &= \pi r^2 h \\ &= \pi(5)^2(12) \\ &= \pi(25)(12) \\ &= 300\pi \text{ m}^3 \\ &\approx 942 \text{ m}^3 \end{aligned}$$

Notice that Cylinder B has the same height and the same radius as Cylinder A. The only difference is that Cylinder B is slanted. For both cylinders, the cross section at every plane parallel to the bases is a circle with the same area. By Cavalieri's principle, the cylinders have the same volume; therefore, the volume of Cylinder B is 300π m^3, or about 942 m^3.



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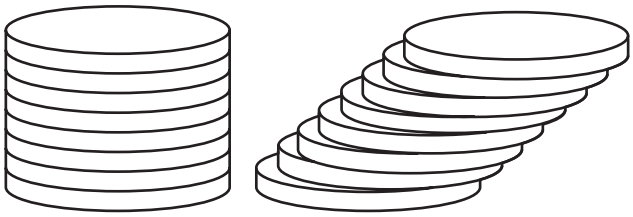
SAMPLE ITEMS

1. Jason constructed two cylinders using solid metal washers. The cylinders have the same height, but one of the cylinders is slanted as shown.



Which statement is true about Jason's cylinders?

A. The cylinders have different volumes because they have different radii.
B. The cylinders have different volumes because they have different surface areas.
C. The cylinders have the same volume because the washers are solid.
D. The cylinders have the same volume because they have the same cross-sectional area at every plane parallel to the bases.

2. What is the volume of a cylinder with a radius of 3 in. and a height of $\frac{9}{2}$ in.?

A. $\frac{81}{2}\pi$ in.^{3}

B. $\frac{27}{4}\pi$ in.^{3}

C. $\frac{27}{8}\pi$ in.^{3}

D. $\frac{9}{4}\pi$ in.^{3}

Answers to Unit 3.3 Sample Items

1. D 2. A

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UNIT 4: EXTENDING THE NUMBER SYSTEM

This unit investigates properties of square roots and rewriting expressions involving radicals. Sum and product of rational and irrational numbers are explored. Closure properties are explored in terms of number systems as well as polynomials.

4.1 Use Properties of Rational and Irrational Numbers

MGSE9-12.N.RN.2 Rewrite expressions involving radicals.

MGSE9-12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.

KEY IDEAS

The ***nth root*** of a number is the number that must be used as a factor *n* times to equal a given value. It can be notated with radicals and indices or with rational exponents. When a root does not have an index, the index is assumed to be 2.

$$\sqrt[\text{index}]{\text{radicand}}$$

Examples:

$$\sqrt{49} = \sqrt{7 \bullet 7} = 7$$

$$\sqrt{18} = \sqrt{9 \bullet 2} = 3\sqrt{2}$$

$$\sqrt{x^5} = \sqrt{x^4 \bullet x} = x^2\sqrt{x}$$

A rational number is a real number that can be represented as a ratio $\frac{p}{q}$ such that p and q are both integers and $q \neq 0$. All rational numbers can be expressed as a decimal that stops or repeats.

Examples: -0.5, 0, 7, $\frac{3}{2}$, $0.2\overline{6}$

An irrational number is a real number that cannot be expressed as a ratio $\frac{p}{q}$ such that p and q are both integers and $q \neq 0$. Irrational numbers cannot be represented by decimals that stop or repeat.

Examples: $\sqrt{3}$, π, $\frac{\sqrt{5}}{2}$

The sum, product, or difference of two rational numbers is always a rational number. The quotient of two rational numbers is always rational when the divisor is not zero. The sum of an irrational number and a rational number is always irrational. The product of a nonzero rational number and an irrational number is always irrational. The sum or product of rational numbers is rational.

Example: The sum is irrational since it cannot be written as a fraction and the sum of a rational number and an irrational number is irrational.

> Let a be an irrational number, and let b be a rational number. Suppose that the sum of a and b is a rational number, c. If you can show that this is not true, it is the same as proving the original statement.

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Let $b = \frac{p}{q}$, where p and q are integers and $q \neq 0$. Let $c = \frac{m}{n}$, where m and n are integers and $n \neq 0$. Substitute $\frac{p}{q}$ and $\frac{m}{n}$ for b and c. Then subtract to find a.

$$a + b = c$$
$$a + \frac{p}{q} = \frac{m}{n}$$
$$a = \frac{m}{n} - \frac{p}{q}$$
$$a = \frac{mq}{nq} - \frac{pn}{nq}$$
$$a = \frac{mq - pn}{nq}$$

The set of integers is closed under multiplication and subtraction, so $\frac{mq - pn}{nq}$ is an integer divided by an integer. This means that a is rational. However, a was assumed to be irrational, so this is a contradiction. This means that c must be irrational. So, the sum of an irrational number and a rational number is irrational.

Example: Is the sum of 0.75 and –2.25 a rational or an irrational number?

The sum is a rational number. The sum is –1.50, which can be rewritten as the fraction $\frac{-150}{100}$.

Example: Is the sum of $\frac{1}{2}$ and $\sqrt{2}$ a rational or an irrational number?

The sum is an irrational number. The square root of 2 is a decimal that does not terminate or repeat. Therefore, the actual sum can be written only as $\frac{1}{2} + \sqrt{2}$.

Example: Is the product of –0.5 and $\sqrt{3}$ a rational or an irrational number? Explain your reasoning.

The product is an irrational number. The square root of 3 is a decimal that does not terminate or repeat. Therefore, the product can be written only as $-0.5\sqrt{3}$.

To rewrite square root expressions, you can use properties of square roots where a and b are real numbers with $a > 0$ and $b > 0$.

- Product Property: $\sqrt{ab} = \sqrt{a}\sqrt{b}$
- Quotient Property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples:

$$\sqrt{32} = \sqrt{16 \bullet 2} = \sqrt{16} \bullet \sqrt{2} = 4\sqrt{2}$$

$$3\sqrt{700} = 3\sqrt{7 \bullet 100} = 3 \bullet 10\sqrt{7} = 30\sqrt{7}$$

$$\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$$

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Two radical expressions that have the same index and the same radicand are called ***like radicals***. To add or subtract like radicals, you can use the Distributive Property.

Example:

$\sqrt{8} + \sqrt{2}$	
$\sqrt{4 \cdot 2} + \sqrt{2}$	Factor out the perfect square.
$\sqrt{4} \cdot \sqrt{2} + \sqrt{2}$	Use the product property of square roots.
$2\sqrt{2} + \sqrt{2}$	Compute the square root.
$(2 + 1)\sqrt{2}$	Use the Distributive Property.
$3\sqrt{2}$	Add.

REVIEW EXAMPLES

- Rewrite $\sqrt{2}(\sqrt{12} - \sqrt{3})$.

  Solution:

$\sqrt{2}(\sqrt{12} - \sqrt{3})$	Original expression
$\sqrt{2} \cdot \sqrt{12} - \sqrt{2} \cdot \sqrt{3}$	Distributive Property
$\sqrt{2} \cdot \sqrt{4 \cdot 3} - \sqrt{2} \cdot \sqrt{3}$	Factor out the perfect square.
$\sqrt{2} \cdot \sqrt{4} \cdot \sqrt{3} - \sqrt{2} \cdot \sqrt{3}$	Product Property
$2 \cdot \sqrt{2} \cdot \sqrt{3} - \sqrt{2} \cdot \sqrt{3}$	Evaluate the square root.
$2\sqrt{6} - \sqrt{6}$	Product Property
$(2 - 1)\sqrt{6}$	Distributive Property
$\sqrt{6}$	Subtract.

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- Write $\sqrt{\frac{18}{25}}$ in an equivalent form where no radical has a perfect square factor and there is no radical in the denominator.

  Solution:

$\sqrt{\frac{18}{25}}$	Original expression
$\frac{\sqrt{18}}{\sqrt{25}}$	Quotient Property
$\frac{\sqrt{18}}{5}$	Evaluate the square root.
$\frac{\sqrt{9 \bullet 2}}{5}$	Factor out the perfect square.
$\frac{\sqrt{9} \bullet \sqrt{2}}{5}$	Product Property
$\frac{3\sqrt{2}}{5}$	Evaluate the square root.

- Write $\sqrt{(4p^2)^3}$ in an equivalent form without a square root. Assume that p is nonnegative.

  Solution:

$\sqrt{(4p^2)^3}$	Original expression
$\sqrt{64p^6}$	Power of Powers Rule
$\sqrt{64}\sqrt{p^6}$	Product Property
$8p^3$	Evaluate the square roots.

- Look at the expression below.

$$(5 + \sqrt{2}) + 2\sqrt{2}$$

  Is the value of the expression rational or irrational? Explain.

  Solution:

  Use the associative property to add like terms.

$$\begin{aligned}(5 + \sqrt{2}) + 2\sqrt{2} &= 5 + (\sqrt{2} + 2\sqrt{2}) \\ &= 5 + 3\sqrt{2}\end{aligned}$$

  Since $3\sqrt{2}$ is the product of a rational number, 3, and an irrational number, $\sqrt{2}$, and the product of an irrational number and a rational number is always irrational, $3\sqrt{2}$ is irrational. Since 5 is a rational number and $3\sqrt{2}$ is an irrational number and the sum of a rational number and an irrational number is always irrational, the value of the expression $5 + 3\sqrt{2}$ is irrational.

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- Explain why the product of $\pi \bullet 5$ is irrational.

  Solution:

  The product of $\pi \bullet 5$ is 5π. Assume 5π is a rational number. The quotient of two rational numbers is rational. Therefore, $\frac{5\pi}{5}$ would be a rational number, and $\frac{5\pi}{5} = \pi$. This would mean that π is a rational number. This is a contradiction because π is an irrational number. So 5π is irrational.

- Is the value of the expression $\sqrt{8}(5\sqrt{8} + \sqrt{2})$ rational or irrational? Explain how you found your answer.

  Solution:

$$\begin{aligned}\sqrt{8}(5\sqrt{8} + \sqrt{2}) &= 5\sqrt{8} \bullet \sqrt{8} + \sqrt{2} \bullet \sqrt{8} \\ &= 5\sqrt{64} + \sqrt{16} \\ &= 5 \bullet 8 + 4 \\ &= 40 + 4 \\ &= 44\end{aligned}$$

  The value of the expression is 44, which is rational.

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SAMPLE ITEMS

1. Which expression is equivalent to $\sqrt{32} - \sqrt{8}$?

A. $2\sqrt{2}$
B. $6\sqrt{2}$
C. $2\sqrt{6}$
D. $2\sqrt{10}$

2. Which expression is equivalent to $\sqrt{\frac{27}{16}}$?

A. $\frac{4\sqrt{3}}{3}$

B. $\frac{2\sqrt{3}}{3}$

C. $\frac{3\sqrt{3}}{4}$

D. $\frac{4\sqrt{3}}{9}$

3. Which expression has a value that is a rational number?

A. $\sqrt{10} + 16$
B. $2(\sqrt{5} + \sqrt{7})$
C. $\sqrt{9} + \sqrt{4}$
D. $\sqrt{3} + 0$

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4. Which statement is true about the value of $(\sqrt{8} + 4) \cdot 4$?

A. It is rational because the product of two rational numbers is rational.
B. It is rational because the product of a rational number and an irrational number is rational.
C. It is irrational because the product of two irrational numbers is irrational.
D. It is irrational because the product of an irrational number and a rational number is irrational.

5. Let *a* be a nonzero rational number and *b* be an irrational number. Which of these MUST be a rational number?

A. $b + 0$
B. $a + a$
C. $a + b$
D. $b + b$

Answers to Unit 4.1 Sample Items

1. A 2. C 3. C 4. D 5. B

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4.2 Perform Arithmetic Operations on Polynomials

MGSE9-12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.

KEY IDEAS

A ***polynomial*** is an expression made from one or more terms that involve constants, variables, and exponents. Examples: $3x$, $x^3 + 5x^2 + 4$, $a^2b - 2ab + b^2$

To add and subtract polynomials, combine like terms. In a polynomial, like terms have the same variables and are raised to the same powers.

Examples:

$$\begin{aligned} 7x + 6 + 5x - 3 &= 7x + 5x + 6 - 3 \\ &= 12x + 3 \end{aligned}$$

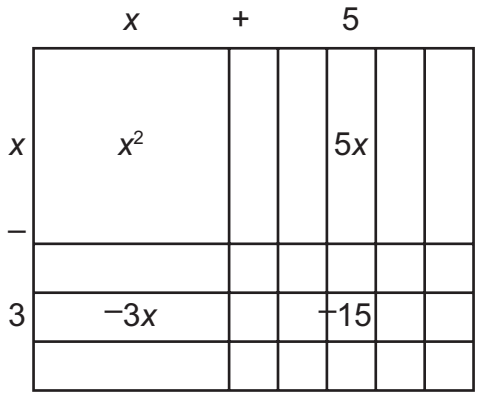
$$\begin{aligned} 13a + 1 - (5a - 4) &= 13a + 1 - 5a + 4 \\ &= 13a - 5a + 1 + 4 \\ &= 8a + 5 \end{aligned}$$

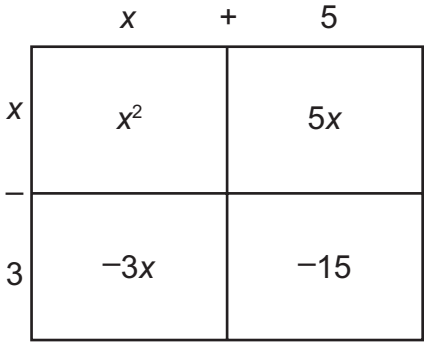
To multiply polynomials, use the Distributive Property. Multiply every term in the first polynomial by every term in the second polynomial. To completely simplify, add like terms after multiplying.

Example:

$$\begin{aligned} (x + 5)(x - 3) &= (x)(x) + (-3)(x) + (5)(x) + (5)(-3) \\ &= x^2 - 3x + 5x - 15 \\ &= x^2 + 2x - 15 \end{aligned}$$

The multiplication can also be represented with tiles and area models.





Polynomials are closed under addition, subtraction, and multiplication, similar to the set of integers. This means that the sum, difference, or product of two polynomials is always a polynomial.

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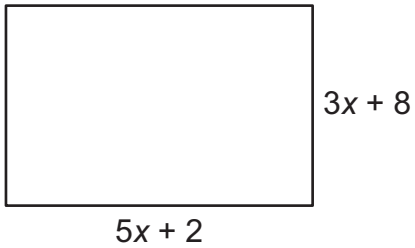
REVIEW EXAMPLES

- The dimensions of a rectangle are shown.



What is the perimeter, in units, of the rectangle?

Solution:

Substitute $5x + 2$ for l and $3x + 8$ for w in the formula for the perimeter of a rectangle:

$$P = l + l + w + w$$
$$P = 2l + 2w$$
$$P = 2(5x + 2) + 2(3x + 8)$$
$$P = 10x + 4 + 6x + 16$$
$$P = 10x + 6x + 4 + 16$$
$$P = 16x + 20 \text{ units}$$

- Rewrite the expression $(x^3 + 2x^2 - x) - (-x^3 + 2x^2 + 6)$.

Solution:

Distribute the negative and then combine like terms:

$$\begin{aligned}(x^3 + 2x^2 - x) - (-x^3 + 2x^2 + 6) &= x^3 + 2x^2 - x + x^3 - 2x^2 - 6\\ &= x^3 + x^3 + 2x^2 - 2x^2 - x - 6\\ &= 2x^3 - x - 6\end{aligned}$$

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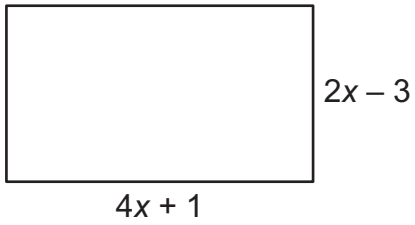
♦ The dimensions of a patio, in feet, are shown below.



What is the area of the patio, in square feet?

Solution:

Substitute $4x + 1$ for b and $2x - 3$ for h in the formula for the area of a rectangle:

$$A = bh$$

$$A = (4x + 1)(2x - 3)$$

$$A = 8x^2 - 12x + 2x - 3$$

$$A = 8x^2 - 10x - 3 \text{ square feet}$$

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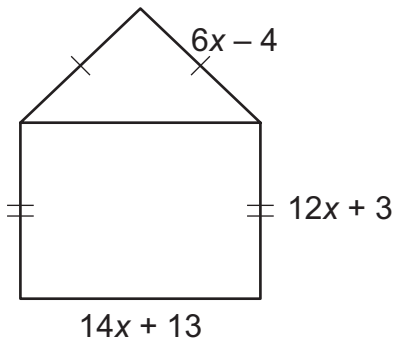
SAMPLE ITEMS

1. What is the product of $7x - 4$ and $8x + 5$?

A. $15x + 1$
B. $30x + 2$
C. $56x^2 + 3x - 20$
D. $56x^2 - 3x + 20$

2. A model of a house is shown.



What is the perimeter, in units, of the model?

A. $32x + 12$
B. $46x + 25$
C. $50x + 11$
D. $64x + 24$

3. Which expression has the same value as the expression $(8x^2 + 2x - 6) - (5x^2 - 3x + 2)$?

A. $3x^2 - x - 4$
B. $3x^2 + 5x - 8$
C. $13x^2 - x - 8$
D. $13x^2 - 5x - 4$

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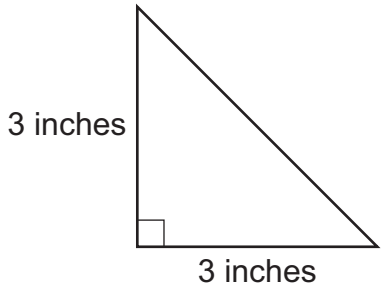
4. Kelly makes two different-sized ceramic tiles in the shape of a right isosceles triangle. This diagram shows the leg lengths of the smaller tile.



Kelly makes the larger tile by increasing the length of each leg of the smaller tile by *x* inches. Which expression represents the length, in inches, of the hypotenuse of the larger tile?

A. $18 + x$

B. $(x + 3)^2$

C. $(x + 3)\sqrt{2}$

D. $3\sqrt{2} + x$

Answers to Unit 4.2 Sample Items

1. C 2. C 3. B 4. C

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UNIT 5: QUADRATIC FUNCTIONS

This unit investigates quadratic functions. Students study the structure of expressions and write expressions in equivalent forms. They solve quadratic equations by inspection, by completing the square, by factoring, and by using the quadratic formula. Students also graph quadratic functions and analyze characteristics of those functions, including end behavior. They write functions for various situations and build functions from other functions, using operations as needed. Given two-variable data, students fit a function to the data and use it to make predictions.

5.1 Interpret the Structure of Expressions

MGSE9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

> **MGSE9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients, in context.
>
> **MGSE9-12.A.SSE.1b** Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

MGSE9-12.A.SSE.2 Use the structure of an expression to rewrite it in different equivalent forms. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

KEY IDEAS

An ***algebraic expression*** contains variables, numbers, and operation symbols. For example, two expressions are $5x^2 - 10x + 15$ and $30x^2 + 6x$.

A ***term*** in an algebraic expression can be a constant, a variable, or a constant multiplied by a variable or variables. Every term is separated by a plus sign. For example, in the expression $5x^2 - 3x + 8$, the terms are $5x^2$, $-3x$, and 8.

A ***coefficient*** is the constant number that is multiplied by a variable in a term. For example, in the expression $5x^2 - 3x + 8$, the coefficient of the x^2 term is 5 and the coefficient of the x term is –3.

A ***common factor*** is a variable or number that terms can be divided by without a remainder. ***Factors*** are numbers multiplied together to get another number. Example: For the terms $30x^2$ and $6x$, the common factors are 1, 2, 3, 6, and x.

A ***common factor of an expression*** is a number or term that the entire expression can be divided by without a remainder. Example: For the expression $30x^2 + 6x$, a common factor of the expression is $6x$ because $30x^2 + 6x = 6x(5x + 1)$. Notice that any of the common factors discussed in the previous example would be common factors of the expression.

If parts of an expression are independent of each other, the expression can be interpreted in different ways. Example: In the expression $\frac{1}{2}h(b_1 + b_2)$, the factors h and $(b_1 + b_2)$ are independent of each other. It can be interpreted as the product of h and a term that does not depend on h.

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There are many methods for factoring quadratic expressions to help find equivalent forms. We will look at just two possible ways. Remember that the standard form of a quadratic function is $ax^2 + bx + c$.

Suppose we have the expression $2x^2 + 4x - 30$. Each term has a common factor of 2. We can factor out that 2 to have a quadratic function that is easier to factor.

$$2x^2 + 4x - 30 = 2(x^2 + 2x - 15)$$

Now, looking at the quadratic $(x^2 + 2x - 15)$, there is no coefficient on the x^2. We consider the factors of the constant term, $c = -15$, that would add up to the b coefficient, 2. The factors of –15 are 1 and –15, or –1 and 15, or 3 and –5, or –3 and 5. Of these factors, only one pair adds to 2: –3 and 5. Therefore, the quadratic function can be factored to $(x^2 + 2x - 15) = (x + 5)(x - 3)$. So going back to what we began with: $2x^2 + 4x - 30 = 2(x + 5)(x - 3)$.

The second method we will explore is called the *ac* method. We will use the expression $8x^2 + 10x - 3$ to explain how this method works. The terms have no common factor and there is a coefficient on the x^2 term. Using the standard form of a quadratic function, $ax^2 + bx + c$, we take the coefficients a and c and multiply them.

$$8 \bullet (-3) = -24$$

Now we consider the factors of –24 that would add to 10 (the b coefficient), which are 12 and –2. Now we rewrite our bx term using these factors as shown. Then we can use grouping and common factors to factor further.

$$\begin{aligned} 8x^2 + 10x - 3 &= 8x^2 + (12x - 2x) - 3 \\ &= (8x^2 + 12x) + (-2x - 3) \\ &= 4x(2x + 3) + (-1)(2x + 3) \\ &= (4x - 1)\,(2x + 3) \end{aligned}$$

The structure of some expressions can be used to help rewrite them. For example, some fourth-degree expressions are in quadratic form.

Example: $x^4 + 5x^2 + 4 = (x^2 + 4)(x^2 + 1)$

Example: $x^4 - y^4 = (x^2 - y^2)(x^2 + y^2) = (x - y)(x + y)(x^2 + y^2)$

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REVIEW EXAMPLES

- Consider the expression $3n^2 + n + 2$.

  a. What is the coefficient of n?

  b. What terms are being added in the expression?

  Solution:

  a. 1

  b. $3n^2$, n, and 2

- Factor the expression $16a^2 - 81$.

  Solution:

  The expression $16a^2 - 81$ is quadratic in form because it is the difference of two squares ($16a^2 = (4a)^2$ and $81 = 9^2$) and both terms of the binomial are perfect squares. The difference of squares can be factored as

  $x^2 - y^2 = (x + y)(x - y)$

$16a^2 - 81$	Original expression
$(4a + 9)(4a - 9)$	Factor the binomial (difference of two squares).

- Factor the expression $12x^2 + 14x - 6$.

  Solution:

$12x^2 + 14x - 6$	Original expression
$2(6x^2 + 7x - 3)$	Factor the trinomial (common factor).
$2(3x - 1)(2x + 3)$	Factor.

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SAMPLE ITEMS

1. In which expression is the coefficient of the *n* term –1?

A. $3n^2 + 4n - 1$
B. $-n^2 + 5n + 4$
C. $-2n^2 - n + 5$
D. $4n^2 + n - 5$

2. Which expression is equivalent to $121x^2 - 64y^2$?

A. $(11x - 16y)(11x + 16y)$
B. $(11x - 16y)(11x - 16y)$
C. $(11x + 8y)(11x + 8y)$
D. $(11x + 8y)(11x - 8y)$

3. The expression s^2 is used to calculate the area of a square, where *s* is the side length of the square. What does the expression $(8x)^2$ represent?

A. the area of a square with a side length of 8
B. the area of a square with a side length of 16
C. the area of a square with a side length of $4x$
D. the area of a square with a side length of $8x$

Answers to Unit 5.1 Sample Items

1. C 2. D 3. D

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5.2 Write Expressions in Equivalent Forms to Solve Problems

MGSE9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

MGSE9-12.A.SSE.3a Factor any quadratic expression to reveal the zeros of the function defined by the expression.

MGSE9-12.A.SSE.3b Complete the square in a quadratic expression to reveal the maximum and minimum value of the function defined by the expression.

KEY IDEAS

The ***zeros***, ***roots***, or ***x-intercepts*** of a function are the values of the variable that make the function equal to zero. When the function is written in factored form, the Zero Product Property can be used to find the zeros of the function. The Zero Product Property states that if the product of two factors is zero, then one or both of the factors must be zero. So, the zeros of the function are the values that make either factor equal to zero.

Example:

$x^2 - 7x + 12 = 0$	Original equation
$(x - 3)(x - 4) = 0$	Factor.

Set each factor equal to zero and solve.

$x - 3 = 0$	$x - 4 = 0$
$x = 3$	$x = 4$

The zeros of the function $y = x^2 - 7x + 12$ are $x = 3$ and $x = 4$.

To ***complete the square*** of a quadratic function means to write a function as the square of a sum. The standard form for a quadratic expression is $\boldsymbol{ax^2 + bx + c}$, where $a \neq 0$. When $a = 1$, completing the square of the function $x^2 + bx = d$ gives $\left(x + \frac{b}{2}\right)^2 = d + \left(\frac{b}{2}\right)^2$. To complete the square when the value $a \neq 1$, factor the value of a from the expression.

Example:

To complete the square, take half of the coefficient of the x-term, square it, and add it to both sides of the equation.

$x^2 + bx = d$	Original expression
$x^2 + bx + \left(\frac{b}{2}\right)^2 = d + \left(\frac{b}{2}\right)^2$	The coefficient of x is b. Half of b is $\frac{b}{2}$. Add the square of $\frac{b}{2}$ to both sides of the equation.
$\left(x + \frac{b}{2}\right)^2 = d + \left(\frac{b}{2}\right)^2$	The expression on the left side of the equation is a perfect square trinomial. Factor to write it as a binomial squared.

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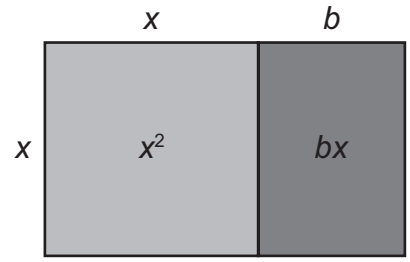
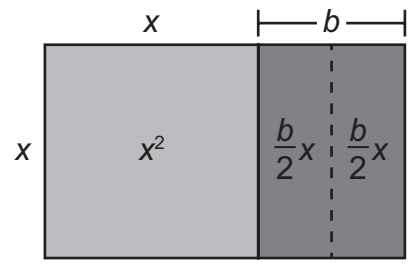
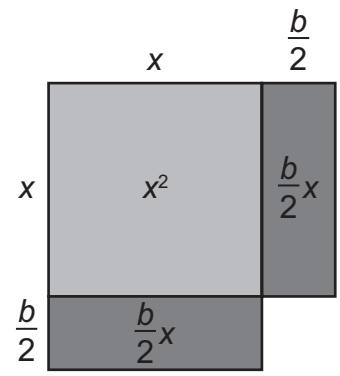
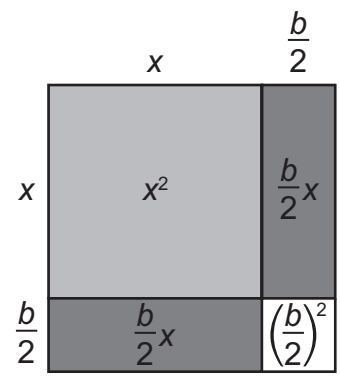
This figure shows how a model can represent completing the square of the expression $x^2 + bx$, where b is positive.

This model represents the expression $x^2 + bx$. To complete the square, create a model that is a square.	
Split the rectangle for bx into two rectangles that represent $\frac{b}{2}x$.	
Rearrange the two rectangles that represent $\frac{b}{2}x$.	
The missing piece of the square measures $\frac{b}{2}$ by $\frac{b}{2}$. Add a square with these dimensions to complete the model of the square. The large square has a side length of $x + \frac{b}{2}$, so this model represents $\left(x + \frac{b}{2}\right)^2$ minus $\left(\frac{b}{2}\right)^2$. $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$.	

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Examples:

Complete the square:

$x^2 + 3x + 7$

$\left(x^2 + 3x + \left(\frac{3}{2}\right)^2\right) + 7 - \left(\frac{3}{2}\right)^2$

$\left(x + \frac{3}{2}\right)^2 + \frac{19}{4}$

Complete the square:

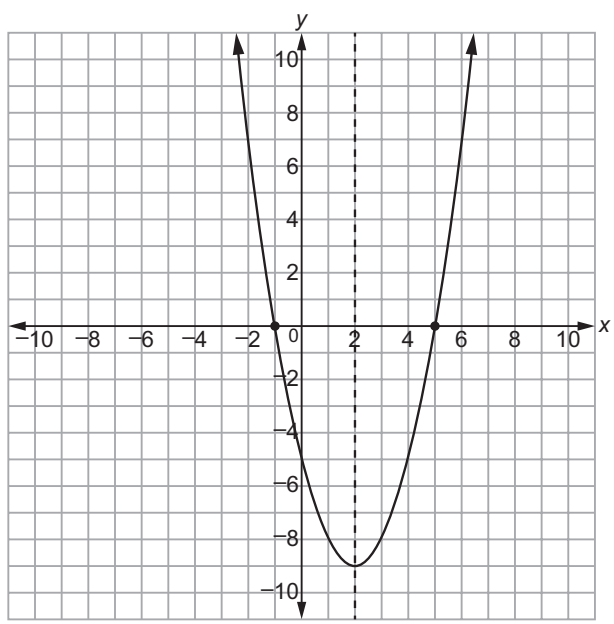
$x^2 + 3x + 7 = 0$

$x^2 + 3x + \left(\frac{3}{2}\right)^2 = -7 + \left(\frac{3}{2}\right)^2$

$\left(x + \frac{3}{2}\right)^2 = -\frac{19}{4}$

Every quadratic function has a ***minimum*** or a ***maximum***. This minimum or maximum is located at the ***vertex*** (h, k). The vertex (h, k) also identifies the ***axis of symmetry*** and the minimum or maximum value of the function. The axis of symmetry is $x = h$.

Example: The quadratic equation $f(x) = x^2 - 4x - 5$ is shown in this graph. The minimum of the function occurs at the vertex (2, –9). The zeros or x-intercepts of the function are (–1, 0) and (5, 0). The axis of symmetry is $x = 2$.



The ***vertex form*** of a quadratic function is $f(x) = a(x - h)^2 + k$ where (h, k) is the vertex. One way to convert an equation from standard form to vertex form is to complete the square.

The vertex of a quadratic function can also be found by using the ***standard form*** and determining the value $\frac{-b}{2a}$. The vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

Important Tips

- When you complete the square, make sure you are only changing the form of the expression and not changing the value.
- When completing the square in an **expression**, add **and subtract** half of the coefficient of the x-term squared.
- When completing the square in an **equation**, add half of the coefficient of the x-term squared **to both sides of the equation.**

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REVIEW EXAMPLES

- Write $f(x) = 2x^2 + 12x + 1$ in vertex form.

  Solution Method 1:

  The function is in standard form, where $a = 2$, $b = 12$, and $c = 1$.

$2x^2 + 12x + 1$	Original expression
$2(x^2 + 6x) + 1$	Factor out 2 from the quadratic and linear terms.
$2(x^2 + 6x + (3)^2 - (3)^2) + 1$	Add and subtract the square of half of the coefficient of the linear term.
$2(x^2 + 6x + (3)^2) - 2(9) + 1$	Remove the subtracted term from the parentheses. Remember to multiply by a.
$2(x^2 + 6x + (3)^2) - 17$	Combine the constant terms.
$2(x + 3)^2 - 17$	Write the perfect square trinomial as a binomial squared.

  The vertex of the function is (–3, –17).

  Solution Method 2:

  The vertex of a quadratic function can also be found by writing the polynomial in standard form and determining the value of $\frac{-b}{2a}$. The vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

  For $f(x) = 2x^2 + 12x + 1$, $a = 2$, $b = 12$, and $c = 1$.

  $$\frac{-b}{2a} = \frac{-12}{2(2)} = \frac{-12}{4} = -3$$

  $$\begin{aligned} f(-3) &= 2(-3)^2 + 12(-3) + 1 \\ &= 2(-3)^2 - 36 + 1 \\ &= 18 - 36 + 1 \\ &= -17 \end{aligned}$$

  The vertex of the function is (–3, –17).

- The function $h(t) = -t^2 + 8t + 2$ represents the height, in feet, of a stream of water being squirted out of a fountain after t seconds. What is the maximum height of the water?

  Solution:

  The function is in standard form, where $a = -1$, $b = 8$, and $c = 2$.

  The x-coordinate of the vertex is $\frac{-b}{2a} = \frac{-8}{2(-1)} = 4$.

  The y-coordinate of the vertex is $h(4) = -(4)^2 + 8(4) + 2 = 18$.

  The vertex of the function is (4, 18). So, the maximum height of the water occurs at 4 seconds and is 18 feet.

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- What are the zeros of the function represented by the quadratic expression $x^2 + 6x - 27$?

  Solution:

  Factor the expression: $x^2 + 6x - 27 = (x + 9)(x - 3)$.

	x	$+\ 9$
x	x^2	$9x$
$-\ 3$	$-3x$	-27

  Set each factor equal to 0 and solve for x.

  $x + 9 = 0$ $\qquad$ $x - 3 = 0$

  $x = -9$ $\qquad$ $x = 3$

  The zeros are $x = -9$ and $x = 3$. This means that $f(-9) = 0$ and $f(3) = 0$.

- What are the zeros of the function represented by the quadratic expression $2x^2 - 5x - 3$?

  Solution:

  Factor the expression: $2x^2 - 5x - 3 = (2x + 1)(x - 3)$.

  Set each factor equal to 0 and solve for x.

  $2x + 1 = 0$ $\qquad$ $x - 3 = 0$

  $x = -\frac{1}{2}$ $\qquad$ $x = 3$

  The zeros are $x = -\frac{1}{2}$ and $x = 3$.

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SAMPLE ITEMS

1. What are the zeros of the function represented by the quadratic expression $2x^2 + x - 3$?

A. $x = -\frac{3}{2}$ and $x = 1$

B. $x = -\frac{2}{3}$ and $x = 1$

C. $x = -1$ and $x = \frac{2}{3}$

D. $x = -1$ and $x = -\frac{3}{2}$

2. What is the vertex of the graph of $f(x) = x^2 + 10x - 9$?

A. (5, 66)
B. (5, –9)
C. (–5, –9)
D. (–5, –34)

3. Which of these is the result of completing the square for the expression $x^2 + 8x - 30$?

A. $(x + 4)^2 - 30$
B. $(x + 4)^2 - 46$
C. $(x + 8)^2 - 30$
D. $(x + 8)^2 - 94$

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4. The expression $-x^2 + 70x - 600$ represents a company's profit for selling x items. For which number(s) of items sold is the company's profit equal to $0?

A. 0 items
B. 35 items
C. 10 items and 60 items
D. 20 items and 30 items

Answers to Unit 5.2 Sample Items

1. A 2. D 3. B 4. C

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5.3 Create Equations That Describe Numbers or Relationships

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions (integer inputs only).

MGSE9-12.A.CED.2 Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P\left(1 + \frac{r}{n}\right)^{nt}$ has multiple variables.)

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm's law $V = IR$ to highlight resistance R; Rearrange area of a circle formula $A = \pi r^2$ to highlight the radius r.*

KEY IDEAS

Quadratic equations can be written to model real-world situations.

Here are some examples of real-world situations that can be modeled by quadratic functions:

- Finding the area of a shape: Given that the length of a rectangle is 5 units more than the width, the area of the rectangle in square units can be represented by $A = x(x + 5)$, where x is the width and $x + 5$ is the length.
- Finding the product of consecutive integers: Given a number, n, the next consecutive number is $n + 1$ and the next consecutive even (or odd) number is $n + 2$. The product, P, of two consecutive numbers is $P = n(n + 1)$.
- Finding the height of a projectile that is dropped: When heights are given in metric units, the equation used is $h(t) = -4.9t^2 + v_o t + h_o$, where v_o is the initial velocity, in meters per second, and h_o is the initial height, in meters. The coefficient –4.9 represents half the force of gravity. When heights are given in customary units, the equation used is $h(t) = -16t^2 + v_o t + h_o$, where v_o is the initial velocity, in feet per second, and h_o is the initial height, in feet. The coefficient –16 represents half the force of gravity. For example, the height, in feet, of a ball thrown with an initial velocity of 60 feet per second and an initial height of 4 feet can be represented by $h(t) = -16t^2 + 60t + 4$, where t is seconds.

You can use the properties of equality to solve for a variable in an equation. Use inverse operations on both sides of the equation until you have isolated the variable.

Example: What is the value of r when $S = 0$ for the equation $S = 2\pi r^2 + 2\pi rh$ for r?

First, factor the expression $2\pi r^2 + 2\pi rh$.

$$2\pi r(r + h)$$

Next, set each factor equal to 0.

$$2\pi r = 0, \qquad r + h = 0$$
$$r = 0, \qquad r = -h$$

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To graph a quadratic equation, find the vertex of the graph and the zeros of the equation. The axis of symmetry goes through the vertex and divides the graph into two sides that are mirror images of each other. To draw the graph, you can plot points on one side of the parabola and use symmetry to find the corresponding points on the other side of the parabola.

Example: Graph the quadratic equation $y = x^2 + 5x + 6$.

First, we can find the zeros by solving for x when $y = 0$. This is where the graph crosses the x-axis.

$$0 = x^2 + 5x + 6$$

$$0 = (x + 2)(x + 3)$$

$$x + 2 = 0,\ x + 3 = 0$$

$x = -2$, $x = -3$; this gives us the points (–2, 0) and (–3, 0).

Next, we can find the axis of symmetry by finding the vertex. The axis of symmetry is the equation $x = -\frac{b}{2a}$. To find the vertex, we first find the axis of symmetry.

$$x = -\frac{5}{2(1)} = -\frac{5}{2}$$

Now we can find the value of the y-coordinate of the vertex.

$$\begin{aligned} y &= \left(-\frac{5}{2}\right)^2 + 5\left(-\frac{5}{2}\right) + 6 \\ &= \frac{25}{4} + \left(-\frac{25}{2}\right) + 6 \\ &= \frac{25}{4} + -\frac{50}{4} + \frac{24}{4} \\ &= -\frac{25}{4} + \frac{24}{4} \\ &= -\frac{1}{4} \end{aligned}$$

So, the vertex is located at $\left(-\frac{5}{2}, -\frac{1}{4}\right)$.

Next, we can find two more points to continue the curve. We can use the y-intercept to find the first of the two points.

$y = (0)^2 + 5(0) + 6 = 6$. The y-intercept is at (0, 6).

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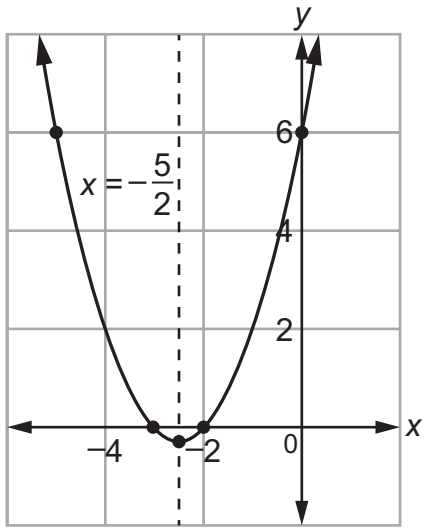
This point is 2.5 more than the axis of symmetry, so the last point will be 2.5 less than the axis of symmetry. The point 2.5 less than the axis of symmetry with a y-value of 6 is (–5, 6).



The axis of symmetry is the midpoint for each corresponding pair of x-coordinates with the same y-value. If (x_1, y) is a point on the graph of a parabola and $x = h$ is the axis of symmetry, then (x_2, y) is also a point on the graph, and x_2 can be found using this equation: $\frac{x_1 + x_2}{2} = h$. In the example shown, we can use the zeros (–3, 0) and (–2, 0) to find the axis of symmetry.

$$\frac{-3 + -2}{2} = -\frac{5}{2} = -2.5, \text{ so } x = -2.5$$

REVIEW EXAMPLES

♦ The product of two consecutive positive integers is 132.

a. Write an equation to model the situation.
b. What are the two consecutive integers?

Solution:

a. Let n represent the lesser of the two integers. Then $n + 1$ represents the greater of the two integers. So, the equation is $n(n + 1) = 132$.
b. Solve the equation for n.

$n(n + 1) = 132$	Original equation
$n^2 + n = 132$	Distributive Property
$n^2 + n - 132 = 0$	Subtraction Property of Equality
$(n + 12)(n - 11) = 0$	Factor.

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Set each factor equal to 0 and solve for n.

$n + 12 = 0$ $\qquad$ $n - 11 = 0$

$n = -12$ $\qquad$ $n = 11$

Because the two consecutive integers are both positive, $n = -12$ cannot be the solution. So, $n = 11$ is the solution, which means that the two consecutive integers are 11 and 12.

♦ The formula for the volume of a cylinder is $V = \pi r^2 h$.

a. Solve the formula for r.

b. If the volume of a cylinder is 200π cubic inches and the height of the cylinder is 8 inches, what is the radius of the cylinder?

Solution:

a. Solve the formula for r.

$V = \pi r^2 h$	Original formula
$\frac{V}{\pi h} = r^2$	Division Property of Equality
$\pm\sqrt{\frac{V}{\pi h}} = r$	Take the square root of both sides.
$\sqrt{\frac{V}{\pi h}} = r$	Choose the positive value because the radius cannot be negative.

b. Substitute 200π for V and 8 for h and evaluate.

$$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{200\pi}{\pi(8)}} = \sqrt{\frac{200}{8}} = \sqrt{25} = 5$$

The radius of the cylinder is 5 inches.

♦ Graph the function represented by the equation $y = 3x^2 - 6x - 9$.

Solution:

Find the zeros of the equation.

$0 = 3x^2 - 6x - 9$	Set the equation equal to 0.
$0 = 3(x^2 - 2x - 3)$	Factor out 3.
$0 = 3(x - 3)(x + 1)$	Factor.
$0 = (x - 3)(x + 1)$	Division Property of Equality

Set each factor equal to 0 and solve for x.

$x - 3 = 0$ $\qquad$ $x + 1 = 0$

$x = 3$ $\qquad$ $x = -1$

The zeros are at $x = -1$ and $x = 3$.

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Find the vertex of the graph.

$$\frac{-b}{2a} = \frac{-(-6)}{2(3)} = \frac{6}{6} = 1$$

Substitute 1 for x in the original equation to find the y-value of the vertex:

$$\begin{aligned} 3(1)^2 - 6(1) - 9 &= 3 - 6 - 9 \\ &= -12 \end{aligned}$$

Graph the two x-intercepts (3, 0) and (–1, 0) and the vertex (1, –12).

Another descriptive point is the y-intercept. You can find the y-intercept by substituting 0 for x.

$$\begin{aligned} y &= 3x^2 - 6x - 9 \\ y &= 3(0)^2 - 6(0) - 9 \\ y &= -9 \end{aligned}$$

You can find more points for your graph by substituting x-values into the function. Find the y-value when $x = -2$.

$$\begin{aligned} y &= 3x^2 - 6x - 9 \\ y &= 3(-2)^2 - 6(-2) - 9 \\ y &= 3(4) + 12 - 9 \\ y &= 15 \end{aligned}$$

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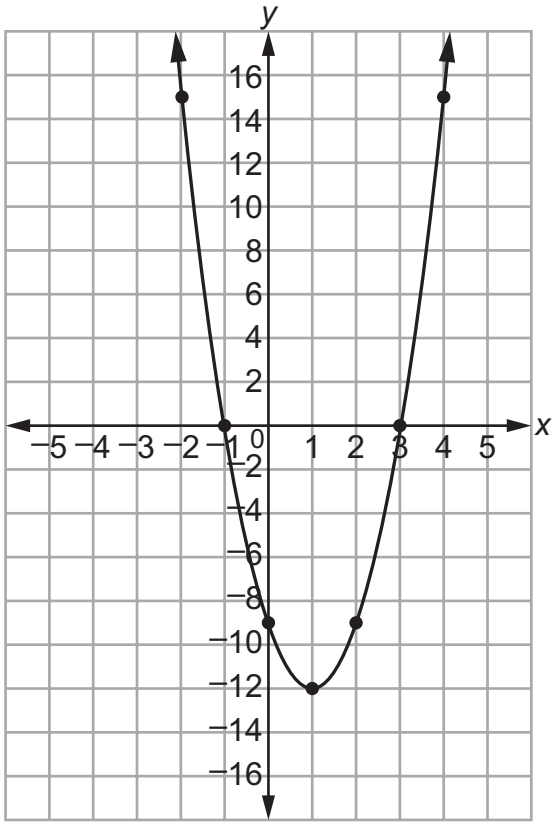
Graph the points (0, –9) and (–2, 15). Then use the concept of symmetry to draw the rest of the function. The axis of symmetry is $x = 1$. So, the mirror image of (0, –9) is (2, –9) and the mirror image of (–2, 15) is (4, 15).



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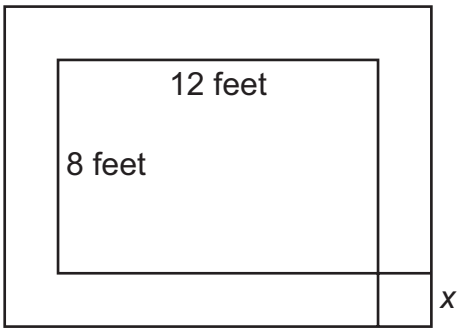
- A garden measuring 8 feet by 12 feet will have a walkway around it. The walkway has a uniform width, and the area covered by the garden and the walkway is 192 square feet. What is the width of the walkway?



The length of the garden and walkway is $x + 12 + x = 12 + 2x$. The width of the garden and walkway is $x + 8 + x = 8 + 2x$. The area covered by the garden and the walkway is shown.

$$192 = (12 + 2x)(8 + 2x)$$
$$192 - 192 = 4x^2 + 40x + 96 - 192$$
$$0 = 4x^2 + 40x - 96$$
$$0 = 4(x^2 + 10x - 24)$$
$$0 = 4(x + 12)(x - 2)$$

This means x could be -12 or 2. The walkway cannot be a negative length so the width of the walkway must be 2 feet.

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SAMPLE ITEM

1. The formula for the area of a circle is $A = \pi r^2$. Which equation shows the formula in terms of r?

A. $r = \frac{2A}{\pi}$

B. $r = \frac{\sqrt{A}}{\pi}$

C. $r = \sqrt{\frac{A}{\pi}}$

D. $r = \frac{A}{2\pi}$

Answer to Unit 5.3 Sample Item

1. C

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5.4 Solve Equations and Inequalities in One Variable

MGSE9-12.A.REI.4 Solve quadratic equations in one variable.

MGSE9-12.A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from $ax^2 + bx + c = 0$.

MGSE9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation (limit to real number solutions).

KEY IDEAS

When quadratic equations do not have a linear term, you can solve the equation by taking the ***square root*** of each side of the equation. Remember, every square root has a positive value and a negative value. Earlier in the guide, we eliminated the negative answers when they represented length or distance.

Example:

$3x^2 - 147 = 0$	
$3x^2 = 147$	Addition Property of Equality
$x^2 = 49$	Multiplicative Inverse Property
$x = \pm 7$	Take the square root of both sides.

Check your answers:

$$\begin{aligned} 3(7)^2 - 147 &= 3(49) - 147 \\ &= 147 - 147 \\ &= 0 \end{aligned}$$

$$\begin{aligned} 3(-7)^2 - 147 &= 3(49) - 147 \\ &= 147 - 147 \\ &= 0 \end{aligned}$$

You can ***factor*** some quadratic equations to find the solutions. To do this, rewrite the equation in standard form set equal to zero ($ax^2 + bx + c = 0$). Factor the expression, set each factor to 0 (by the Zero Product Property), and then solve for x in each resulting equation. This will provide two rational values for x.

Example:

$x^2 - x = 12$	
$x^2 - x - 12 = 0$	Addition Property of Equality
$(x - 4)(x + 3) = 0$	Factor.

Set each factor equal to 0 and solve.

$x - 4 = 0$	$x + 3 = 0$
$x = 4$	$x = -3$

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Check your answers:

$4^2 - 4 = 16 - 4$ $\qquad$ $(-3)^2 - (-3) = 9 + 3$

$= 12$ $\qquad$ $= 12$

You can complete the square to solve a quadratic equation. First, write the equation that represents the function in standard form, $ax^2 + bx + c = 0$. Subtract the constant from both sides of the equation: $ax^2 + bx = -c$. Divide both sides of the equation by a: $x^2 + \frac{b}{a}x = \frac{-c}{a}$. Add the square of half the coefficient of the x-term to both sides: $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$. Write the perfect square trinomial as a binomial squared: $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$. Take the square root of both sides of the equation and solve for x. This method works best when a is 1 and b is even.

Example:

$x^2 - 6x - 7 = 0$	
$x^2 - 6x = 7$	Addition Property of Equality
$x^2 - 6x + \left(\frac{-6}{2}\right)^2 = 7 + \left(\frac{-6}{2}\right)^2$	Addition Property of Equality
$x^2 - 6x + (-3)^2 = 7 + (-3)^2$	
$(x - 3)^2 = 7 + 9$	Distribution Property
$(x - 3)^2 = 16$	
$x - 3 = \pm 4$	Take the square root of both sides.
$x = 3 \pm 4$	Addition Property of Equality
$x = 3 + 4 = 7$; $x = 3 - 4 = -1$	Solve for x for both operations.

All quadratic equations can be solved using the quadratic formula. The ***quadratic formula*** is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $ax^2 + bx + c = 0$. The quadratic formula will yield real solutions. We can solve the previous equation using the quadratic formula.

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Example: $5x^2 - 6x - 8 = 0$, where $a = 5$, $b = -6$, and $c = -8$.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-8)}}{2(5)}$$

$$x = \frac{6 \pm \sqrt{36 - 4(-40)}}{10}$$

$$x = \frac{6 \pm \sqrt{36 - (-160)}}{10}$$

$$x = \frac{6 \pm \sqrt{36 + (160)}}{10}$$

$$x = \frac{6 \pm \sqrt{196}}{10}$$

$$x = \frac{6 \pm 14}{10}$$

$$x = \frac{6 + 14}{10} = \frac{20}{10} = 2; \quad x = \frac{6 - 14}{10} = \frac{-8}{10} = \frac{-4}{5}$$

Important Tip

✍ While there may be several methods that can be used to solve a quadratic equation, some methods may be easier than others for certain equations.

REVIEW EXAMPLES

♦ Solve the equation $x^2 - 10x + 25 = 0$ by factoring.

Solution:

Factor: $x^2 - 10x + 25 = (x - 5)(x - 5)$.

Both factors are the same, so solve the equation:

$x - 5 = 0$

$x = 5$

♦ Solve the equation $x^2 - 100 = 0$ by using square roots.

Solution:

Solve the equation using square roots.

$x^2 = 100$ Add 100 to both sides of the equation.

$x = \pm\sqrt{100}$ Take the square root of both sides of the equation.

$x = \pm 10$ Evaluate.

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- Solve the equation $4x^2 - 7x + 3 = 0$ using the quadratic formula.

  Solution:

  Solve the equation using the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $ax^2 + bx + c = 0$.

  Given the equation in standard form, the following values will be used in the formula:

  $a = 4$, $b = -7$, and $c = 3$

  $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(3)}}{2(4)}$ Substitute each value into the quadratic formula.

  $x = \dfrac{7 \pm \sqrt{1}}{8}$ Simplify the expression.

  $x = \dfrac{7 + 1}{8} = 1$ and $x = \dfrac{7 - 1}{8} = \dfrac{6}{8} = \dfrac{3}{4}$ Evaluate.

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SAMPLE ITEMS

1. What are the solutions to the equation $2x^2 - 2x - 12 = 0$?

A. $x = -4, x = 3$
B. $x = -3, x = 4$
C. $x = -2, x = 3$
D. $x = -6, x = 2$

2. What are the solutions to the equation $6x^2 - x - 40 = 0$?

A. $x = -\frac{8}{3}, x = -\frac{5}{2}$

B. $x = -\frac{8}{3}, x = \frac{5}{2}$

C. $x = \frac{5}{2}, x = \frac{8}{3}$

D. $x = -\frac{5}{2}, x = \frac{8}{3}$

3. What are the solutions to the equation $x^2 - 5x = 14$?

A. $x = -7, x = -2$
B. $x = -14, x = -1$
C. $x = -2, x = 7$
D. $x = -1, x = 14$

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4. An object is thrown into the air with an initial velocity of 5 m/s from a height of 9 m. The equation $h(t) = -4.9t^2 + 5t + 9$ models the height of the object in meters after t seconds.

About how many seconds does it take for the object to hit the ground? Round your answer to the nearest tenth of a second.

A. 0.9
B. 1.5
C. 2.0
D. 9.0

Answers to Unit 5.4 Sample Items

1. C 2. D 3. C 4. C

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5.5 Interpret Functions That Arise in Applications in Terms of the Context

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior.

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

KEY IDEAS

An ***x-intercept***, ***root***, or ***zero*** of a function is the *x*-coordinate of a point where the function crosses the *x*-axis. A function may have multiple *x*-intercepts. To find the *x*-intercepts of a quadratic function, set the function equal to 0 and solve for *x*. This can be done by factoring, completing the square, or using the quadratic formula.

The ***y-intercept*** of a function is the *y*-coordinate of the point where the function crosses the *y*-axis. A function may have zero *y*-intercepts or one *y*-intercept. To find the *y*-intercept of a quadratic function, find the value of the function when *x* equals 0.

A function is ***increasing*** over an interval when the values of *y* increase as the values of *x* increase over that interval. The interval is represented in terms of *x*.

A function is ***decreasing*** over an interval when the values of *y* decrease as the values of *x* increase over that interval. The interval is represented in terms of *x*.

Every quadratic function has a ***minimum*** or ***maximum***, which is located at the vertex. When the function is written in standard form, the *x*-coordinate of the vertex is $\frac{-b}{2a}$. To find the *y*-coordinate of the vertex, substitute the value of $\frac{-b}{2a}$ into the function and evaluate. When the value of *a* is positive, the graph opens up, and the vertex is the minimum point. When the value of *a* is negative, the graph opens down, and the vertex is the maximum point.

The ***end behavior*** of a function describes how the values of the function change as the *x*-values approach negative infinity and positive infinity.

The ***domain*** of a function is the set of values for which it is possible to evaluate the function. The domain of a quadratic function is typically all real numbers, although in real-world applications it may only make sense to look at the domain values on a particular interval. For example, time must be a nonnegative number.

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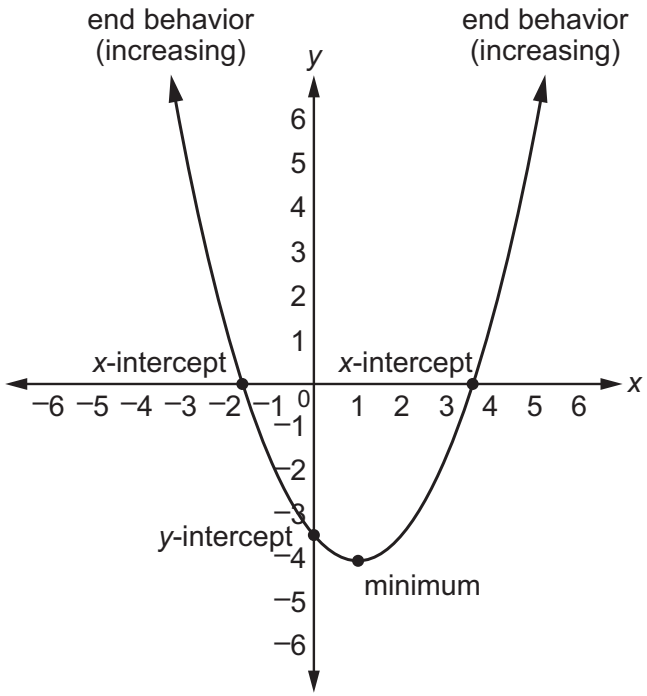
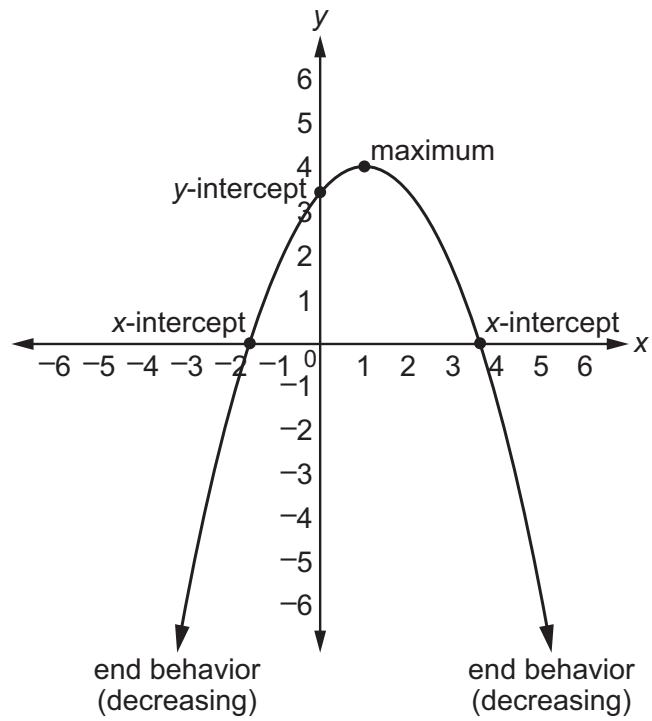
The ***average rate of change*** of a function over a specified interval is the change in the *y*-value divided by the change in the *x*-value for two distinct points on a graph. To calculate the average rate of change of a function over the interval from *a* to *b*, evaluate the expression $m = \frac{y_2 - y_1}{x_2 - x_1}$.



Parabolas have this shape when $a > 0$.



Parabolas have this shape when $a < 0$.

REVIEW EXAMPLES

♦ A ball is thrown into the air from a height of 4 feet at time $t = 0$. The function that models this situation is $h(t) = -16t^2 + 63t + 4$, where t is measured in seconds and h is the height in feet.

a. What is the height of the ball after 2 seconds?
b. When will the ball reach a height of 50 feet?
c. What is the maximum height of the ball?
d. When will the ball hit the ground?
e. What domain makes sense for the function?

Solution:

a. To find the height of the ball after 2 seconds, substitute 2 for t in the function.
$h(2) = -16(2)^2 + 63(2) + 4 = -16(4) + 126 + 4 = -64 + 126 + 4 = 66$

The height of the ball after 2 seconds is 66 feet.

b. To find when the ball will reach a height of 50 feet, find the value of t that makes $h(t) = 50$.
$50 = -16t^2 + 63t + 4$

$0 = -16t^2 + 63t - 46$

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Use the quadratic formula with $a = -16$, $b = 63$, and $c = -46$.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-63 \pm \sqrt{(63)^2 - 4(-16)(-46)}}{2(-16)}$$

$$t = \frac{-63 \pm \sqrt{3969 - 2944}}{-32}$$

$$t = \frac{-63 \pm \sqrt{1025}}{-32}$$

$t \approx 0.97$ or $t \approx 2.97$. So, the ball is at a height of 50 feet after approximately 0.97 second and 2.97 seconds.

c. To find the maximum height, find the vertex of $h(t)$.

The x-coordinate of the vertex is equal to $\frac{-b}{2a}$: $\frac{-63}{2(-16)} \approx 1.97$. To find the y-coordinate, find $h(1.97)$:

$$h(1.97) = -16(1.97)^2 + 63(1.97) + 4 \approx 66$$

The maximum height of the ball is about 66 feet.

d. To find when the ball will hit the ground, find the value of t that makes $h(t) = 0$ (because 0 represents 0 feet from the ground).

$0 = -16t^2 + 63t + 4$

Using the quadratic formula (or by factoring), $t = -0.0625$ or $t = 4$.

Time cannot be negative, so $t = -0.0625$ is not a solution. The ball will hit the ground after 4 seconds.

e. Time must always be nonnegative and can be expressed by any fraction or decimal. The ball is thrown at 0 seconds and reaches the ground after 4 seconds. So, the domain $0 \leq t \leq 4$ makes sense for function $h(t)$.

♦ This table shows a company's profit, p, in thousands of dollars, over time, t, in months.

Time, *t* (months)	**Profit, *p* (thousands of dollars)**
3	18
7	66
10	123
15	258
24	627

a. Describe the average rate of change in terms of the given context.

b. What is the average rate of change of the profit between 3 and 7 months?

c. What is the average rate of change of the profit between 3 and 24 months?

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Solution:

a. The average rate of change represents the rate at which the company earns a profit.

b. Use the expression for average rate of change. Let $x_1 = 3$, $x_2 = 7$, $y_1 = 18$, and $y_2 = 66$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{66 - 18}{7 - 3} = \frac{48}{4} = 12$$

The average rate of change between 3 and 7 months is 12 thousand dollars ($12,000) per month.

c. Use the expression for average rate of change. Let $x_1 = 3$, $x_2 = 24$, $y_1 = 18$, and $y_2 = 627$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{627 - 18}{24 - 3} = \frac{609}{21} = 29$$

The average rate of change between 3 and 24 months is 29 thousand dollars ($29,000) per month.

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SAMPLE ITEMS

1. A flying disk is thrown into the air from a height of 25 feet at time $t = 0$. The function that models this situation is $h(t) = -16t^2 + 75t + 25$, where t is measured in seconds and h is the height in feet. What values of t best describe the time when the disk is flying in the air?

A. $0 < t < 5$
B. $0 < t < 25$
C. all real numbers
D. all positive integers

2. Use this table to answer the question.

x	$f(x)$
–2	15
–1	9
0	5
1	3
2	3

What is the average rate of change of x over the interval $-2 \leq x \leq 0$?

A. –10
B. –5
C. 5
D. 10

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3. What is the end behavior of the graph of $f(x) = -0.25x^2 - 2x + 1$?

A. As x increases, $f(x)$ increases. As x decreases, $f(x)$ decreases.
B. As x increases, $f(x)$ decreases. As x decreases, $f(x)$ decreases.
C. As x increases, $f(x)$ increases. As x decreases, $f(x)$ increases.
D. As x increases, $f(x)$ decreases. As x decreases, $f(x)$ increases.

Answers to Unit 5.5 Sample Items

1. A 2. B 3. B

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5.6 Analyze Functions Using Different Representations

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima (as determined by the function or by context).

MGSE9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MGSE9-12.F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. *For example, compare and contrast quadratic functions in standard, vertex, and intercept forms.*

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.

KEY IDEAS

Functions can be represented algebraically, graphically, numerically (in tables), or verbally (by description).

Examples:

Algebraically: $f(x) = x^2 + 2x$

Verbally (by description): a function that represents the sum of the square of a number and twice the number

Numerically (in a table):

x	$f(x)$
–1	–1
0	0
1	3
2	8

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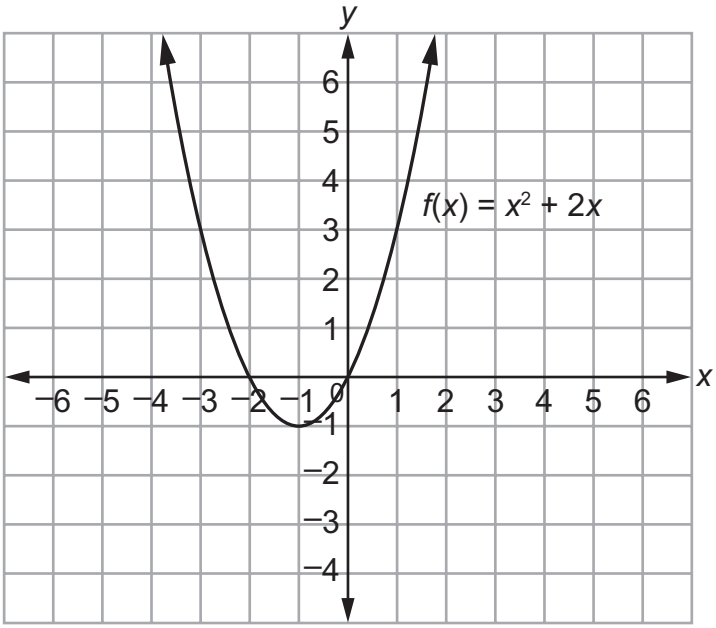
Graphically:



You can compare key features of two functions represented in different ways. For example, if you are given an equation of a quadratic function and a graph of another quadratic function, you can calculate the vertex of the first function and compare it to the vertex of the graphed function.

REVIEW EXAMPLES

- Graph the function $f(x) = x^2 - 5x - 24$.

  Solution:

  Use the algebraic representation of the function to find the key features of the graph of the function.

  Find the zeros of the function.

  $0 = x^2 - 5x - 24$ Set the function equal to 0.

  $0 = (x - 8)(x + 3)$ Factor.

  Set each factor equal to 0 and solve for x.

  $$x - 8 = 0 \qquad x + 3 = 0$$
  $$x = 8 \qquad x = -3$$

  The zeros are at $x = -3$ and $x = 8$.

  Find the vertex of the function.

  $$x = \frac{-b}{2a} = \frac{-(-5)}{2(1)} = \frac{5}{2} = 2.5$$

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Substitute 2.5 for x in the original function to find $f(2.5)$:

$$f(x) = x^2 - 5x - 24$$
$$f(2.5) = (2.5)^2 - 5(2.5) - 24 = 6.25 - 12.5 - 24 = -30.25$$

The vertex is (2.5, –30.25).

Find the y-intercept by finding $f(0)$.

$$f(x) = x^2 - 5x - 24$$
$$f(0) = (0)^2 - 5(0) - 24 = -24$$

The y-intercept is (0, –24). Use symmetry to find another point. The line of symmetry is $x = 2.5$.

$$\frac{0 + x}{2} = 2.5$$

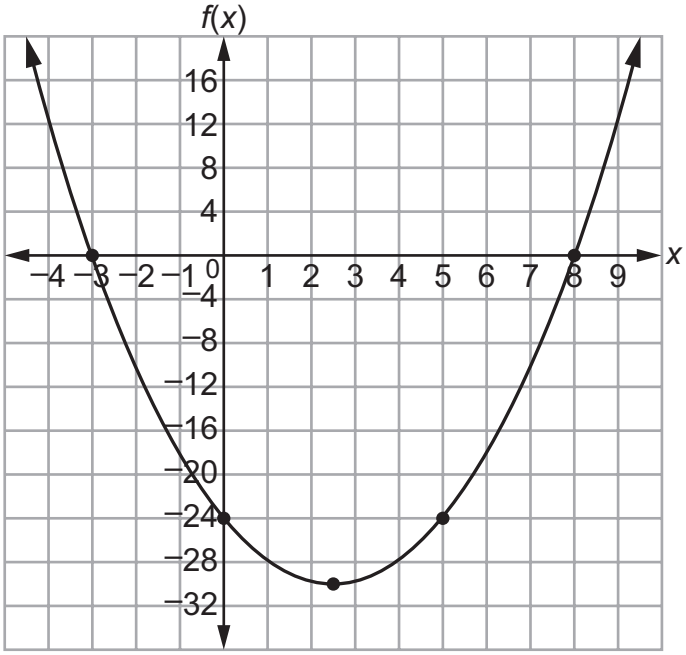
$$0 + x = 5$$

$$x = 5$$

So, point (5, –24) is also on the graph.

Plot the points (–3, 0), (8, 0), (2.5, –30.25), (0, –24), and (5, –24). Draw a smooth curve through the points.

We can also use the value of a in the function to determine if the graph opens up or down.
In $f(x) = x^2 - 5x - 24$, $a = 1$. Since $a > 0$, the graph opens up.



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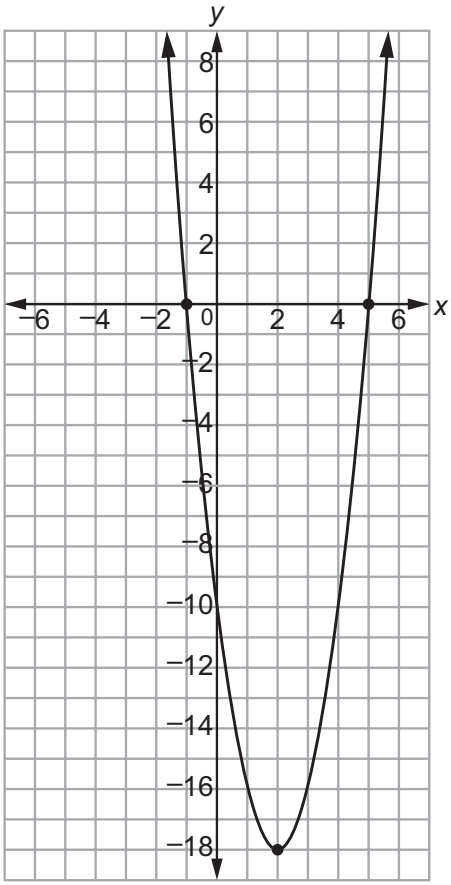
- This graph shows a function $f(x)$.



Compare the graph of $f(x)$ to the graph of the function given by the equation $g(x) = 4x^2 + 6x - 18$. Which function has the lesser minimum value? How do you know?

Solution:

The minimum value of a quadratic function that opens up is the y-value of the vertex.

The vertex of the graph of $f(x)$ appears to be (2, –18). So, the minimum value is –18.

Find the vertex of the function $g(x) = 4x^2 + 6x - 18$.

To find the vertex of $g(x)$, use $\left(\frac{-b}{2a}, g\left(\frac{-b}{2a}\right)\right)$ with $a = 4$ and $b = 6$.

$$x = \frac{-b}{2a} = \frac{-(6)}{2(4)} = \frac{-6}{8} = -0.75$$

Substitute –0.75 for x in the original function $g(x)$ to find $g(-0.75)$:

$$\begin{aligned} g(x) &= 4x^2 + 6x - 18 \\ g(-0.75) &= 4(-0.75)^2 + 6(-0.75) - 18 \\ &\quad 2.25 - 4.5 - 18 \\ &\quad = -20.25 \end{aligned}$$

The minimum value of $g(x)$ is –20.25.

$-20.25 < -18$, so the function $g(x)$ has the lesser minimum value.

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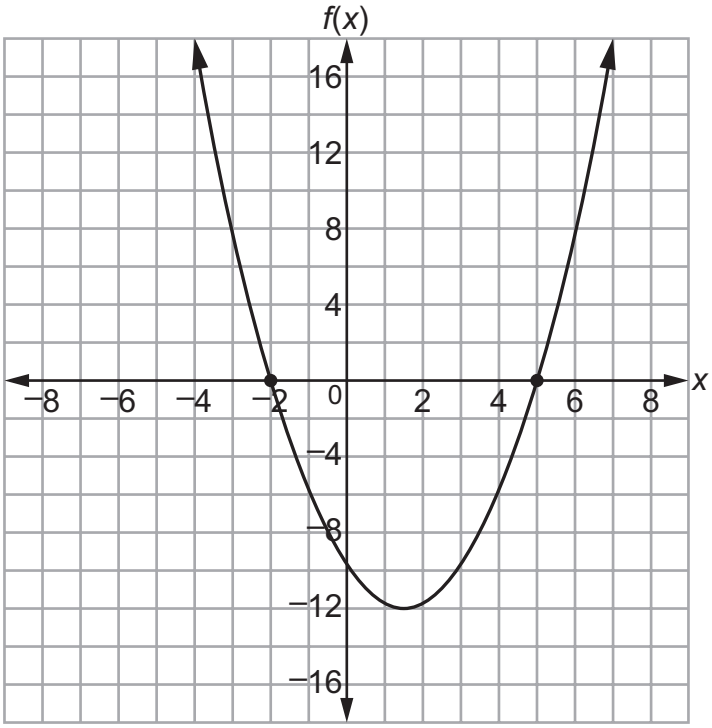
SAMPLE ITEMS

1. Use this graph to answer the question.



Which function is shown in the graph?

A. $f(x) = x^2 - 3x - 10$
B. $f(x) = x^2 + 3x - 10$
C. $f(x) = x^2 + x - 12$
D. $f(x) = x^2 - 5x - 8$

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2. The function $f(t) = -16t^2 + 64t + 5$ models the height of a ball that was hit into the air, where t is measured in seconds and h is the height in feet.

This table represents the height, $g(t)$, of a second ball that was thrown into the air.

Time, t (seconds)	Height, $g(t)$ (feet)
0	4
1	36
2	36
3	4

Which statement BEST compares the length of time each ball is in the air?

A. The ball represented by $f(t)$ is in the air for about 5 seconds, and the ball represented by $g(t)$ is in the air for about 3 seconds.

B. The ball represented by $f(t)$ is in the air for about 3 seconds, and the ball represented by $g(t)$ is in the air for about 5 seconds.

C. The ball represented by $f(t)$ is in the air for about 3 seconds, and the ball represented by $g(t)$ is in the air for about 4 seconds.

D. The ball represented by $f(t)$ is in the air for about 4 seconds, and the ball represented by $g(t)$ is in the air for about 3 seconds.

Answers to Unit 5.6 Sample Items

1. A 2. D

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5.7 Build a Function That Models a Relationship between Two Quantities

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

MGSE9-12.F.BF.1a Determine an explicit expression and the recursive process (steps for calculation) from context. *For example, if Jimmy starts out with $15 and earns $2 a day, the explicit expression "2x + 15" can be described recursively (either in writing or verbally) as "to find out how much money Jimmy will have tomorrow, you add $2 to his total today."* $J_n = J_{n-1} + 2$, $J_0 = 15$

KEY IDEAS

An ***explicit expression*** contains variables, numbers, and operation symbols and does not use an equal sign to relate the expression to another quantity.

A ***recursive process*** can show that a quadratic function has second differences that are equal to one another.

Example: Consider the function $f(x) = x^2 + 4x - 1$.

This table of values shows five values of the function.

x	$f(x)$
–2	–5
–1	–4
0	–1
1	4
2	11

The first and second differences are shown. The first differences are the differences between the consequence terms. The second differences are the differences between the consequence terms of the first differences.

x	$f(x)$	First differences	Second differences
–2	–5		
		–4 – (–5) = 1	
–1	–4		3 – 1 = 2
		–1 – (–4) = 3	
0	–1		5 – 3 = 2
		4 – (–1) = 5	
1	4		7 – 5 = 2
		11 – 4 = 7	
2	11		

A ***recursive function*** is one in which each function value is based on a previous value (or previous values) of the function.

When building a model function, functions can be added, subtracted, or multiplied together. The result will still be a function. This includes linear, quadratic, exponential, and constant functions.

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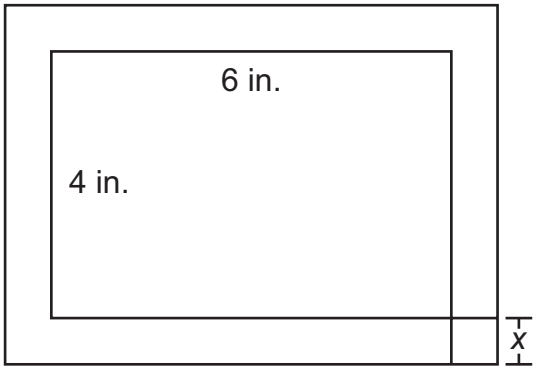
REVIEW EXAMPLES

- Annie is framing a photo with a length of 6 inches and a width of 4 inches. The distance from the edge of the photo to the edge of the frame is x inches. The combined area of the photo and frame is 63 square inches.



a. Write a quadratic function to find the distance from the edge of the photo to the edge of the frame.

b. How wide are the photo and frame together?

Solution:

a. The length of the photo and frame is $x + 6 + x = 6 + 2x$. The width of the photo and frame is $x + 4 + x = 4 + 2x$. The area of the frame is $(6 + 2x)(4 + 2x) = 4x^2 + 20x + 24$. Set this expression equal to the area: $63 = 4x^2 + 20x + 24$.

b. Solve the equation for x.

$$63 = 4x^2 + 20x + 24$$
$$0 = 4x^2 + 20x - 39$$
$$x = -6.5 \text{ or } x = 1.5$$

Length cannot be negative, so the distance from the edge of the photo to the edge of the frame is 1.5 inches. Therefore, the width of the photo and frame together is $4 + 2x = 4 + 2(1.5) = 7$ inches.

- A scuba diving company currently charges \$100 per dive. On average, there are 30 customers per day. The company performed a study and learned that for every \$20 price increase, the average number of customers per day would be reduced by 2.

a. The total revenue from the dives is the price per dive multiplied by the number of customers. What is the revenue after 4 price increases?

b. Write a quadratic equation to represent x price increases.

c. What price would give the greatest revenue?

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Solution:

a. Make a table to show the revenue after 4 price increases.

Number of Price Increases	Price per Dive ($)	Number of Customers per Day	Revenue per Day ($)
0	100	30	(100)(30) = 3,000
1	100 + 20(1) = 120	30 – 2(1) = 28	(120)(28) = 3,360
2	100 + 20(2) = 140	30 – 2(2) = 26	(140)(26) = 3,640
3	100 + 20(3) = 160	30 – 2(3) = 24	(160)(24) = 3,840
4	100 + 20(4) = 180	30 – 2(4) = 22	(180)(22) = 3,960

The revenue after 4 price increases is ($180)(22) = $3,960.

b. The table shows a pattern. The price per dive for x price increases is $100 + 20x$. The number of customers for x price increases is $30 – 2x$. So, the equation $y = (100 + 20x)(30 – 2x) = –40x^2 + 400x + 3{,}000$ represents the revenue for x price increases.

c. To find the price that gives the greatest revenue, first find the number of price increases that gives the greatest value. This occurs at the vertex.

Use $\frac{-b}{2a}$ with $a = –40$ and $b = 400$.

$$\frac{-b}{2a} = \frac{-400}{2(-40)} = \frac{-400}{-80} = 5$$

The maximum revenue occurs after 5 price increases.

100 + 20(5) = 200

The price of $200 per dive gives the greatest revenue.

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♦ Consider the sequence 2, 6, 12, 20, 30, ...

a. What explicit expression can be used to find the next term in the sequence?

b. What is the tenth term of the sequence?

Solution:

a. The difference between terms is not constant, so the operation involves multiplication. Make a table to try to determine the relationship between the number of the term and the value of the term.

Term number	Term value	Relationship
1	2	1 • 2
2	6	2 • 3
3	12	3 • 4
4	20	4 • 5
5	30	5 • 6

Notice the pattern: The value of each term is the product of the term number and one more than the term number. So, the expression is $n(n + 1)$ or $n^2 + n$.

b. The tenth term is $n^2 + n = (10)^2 + (10) = 110$.

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SAMPLE ITEMS

1. What explicit expression can be used to find the next term in this sequence?

2, 8, 18, 32, 50, . . .

A. $2n$
B. $2n + 6$
C. $2n^2$
D. $2n^2 + 1$

2. The function $s(t) = vt + h - 0.5at^2$ represents the height of an object, s, in feet, above the ground in relation to the time, t, in seconds, since the object was thrown into the air with an initial velocity of v feet per second at an initial height of h feet and where a is the acceleration due to gravity (32 feet per second squared).

A baseball player hits a baseball 4 feet above the ground with an initial velocity of 80 feet per second. About how long will it take the baseball to hit the ground?

A. 2 seconds
B. 3 seconds
C. 4 seconds
D. 5 seconds

3. A café's annual income depends on x, the number of customers. The function $I(x) = 4x^2 - 20x$ describes the café's total annual income. The function $C(x) = 2x^2 + 5$ describes the total amount the café spends in a year. The café's annual profit, $P(x)$, is the difference between the annual income and the amount spent in a year.

Which function describes $P(x)$?

A. $P(x) = 2x^2 - 20x - 5$
B. $P(x) = 4x^3 - 20x^2$
C. $P(x) = 6x^2 - 20x + 5$
D. $P(x) = 8x^4 - 40x^3 - 20x^2 - 100x$

Answers to Unit 5.7 Sample Items

1. C 2. D 3. A

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5.8 Build New Functions from Existing Functions

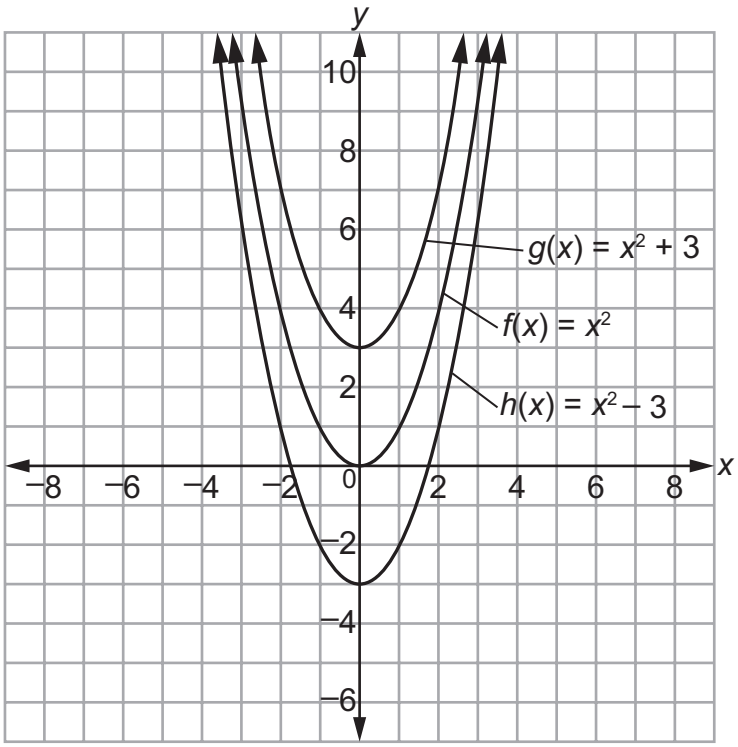
MGSE9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

KEY IDEAS

A ***parent function*** is the basic function from which all the other functions in a function family are modeled. For the quadratic function family, the parent function is $f(x) = x^2$.

For a parent function $f(x)$ and a real number k,

- the function $f(x) + k$ will move the graph of $f(x)$ up by k units.
- the function $f(x) - k$ will move the graph of $f(x)$ down by k units.



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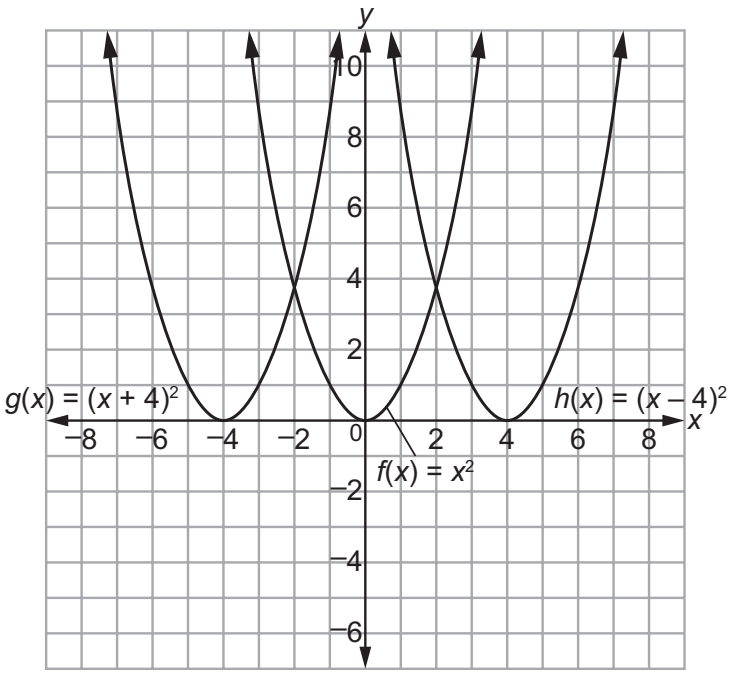
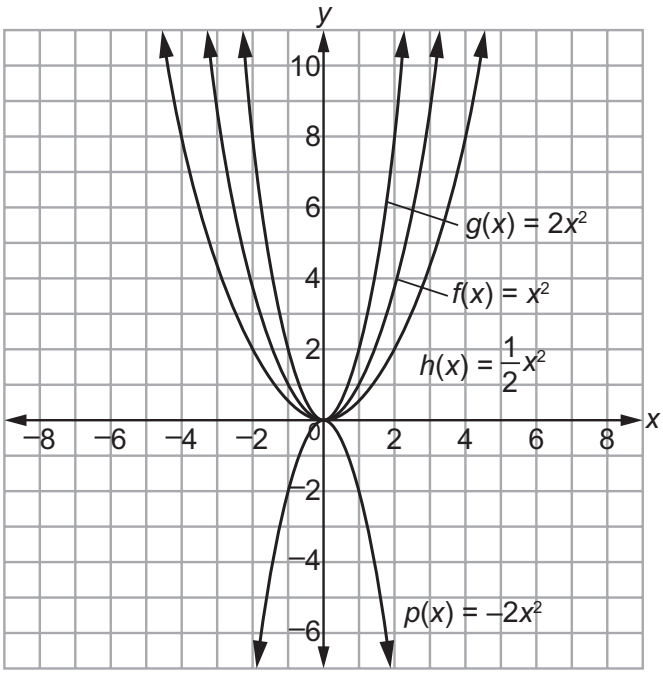
For a parent function $f(x)$ and a real number k,

- the function $f(x + k)$ will move the graph of $f(x)$ left by k units.
- the function $f(x - k)$ will move the graph of $f(x)$ right by k units.



For a parent function $f(x)$ and a real number k,

- the function $kf(x)$ will vertically stretch the graph of $f(x)$ by a factor of k units for $|k| > 1$.
- the function $kf(x)$ will vertically shrink the graph of $f(x)$ by a factor of k units for $|k| < 1$.
- the function $kf(x)$ will reflect the graph of $f(x)$ over the x-axis for negative values of k.



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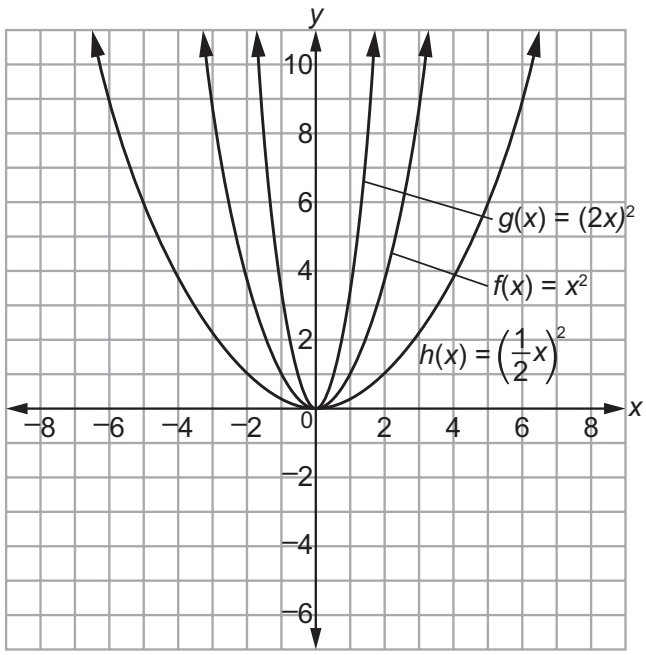
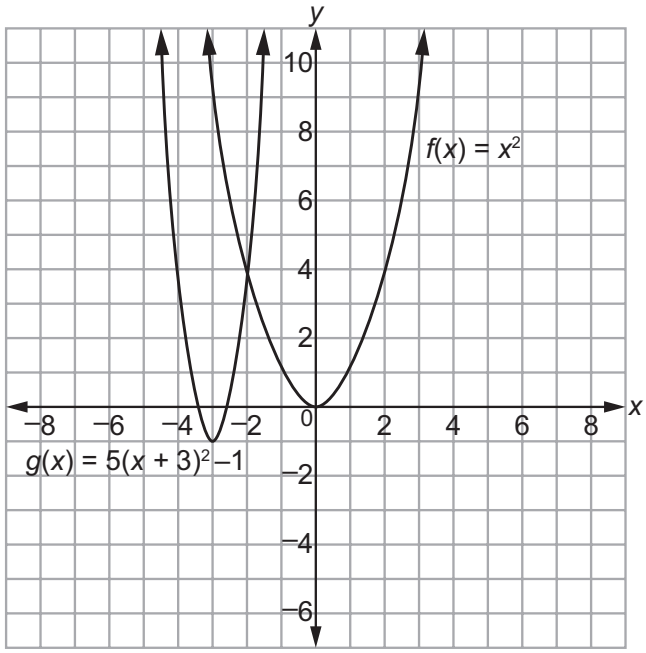
For a parent function $f(x)$ and a real number k,

- the function $f(kx)$ will horizontally shrink the graph of $f(x)$ by a factor of $\frac{1}{k}$ units for $|k| > 1$.
- the function $f(kx)$ will horizontally stretch the graph of $f(x)$ by a factor of $\frac{1}{k}$ units for $|k| < 1$.
- the function $f(kx)$ will reflect the graph of $f(x)$ over the y-axis for negative values of k.



You can apply more than one of these changes at a time to a parent function.

Example: $f(x) = 5(x + 3)^2 - 1$ will translate $f(x) = x^2$ left 3 units and down 1 unit and stretch the function vertically by a factor of 5.



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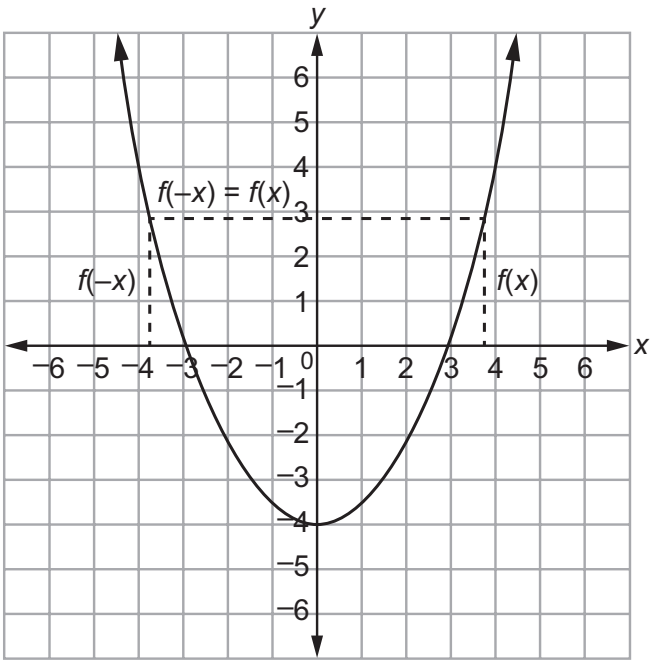
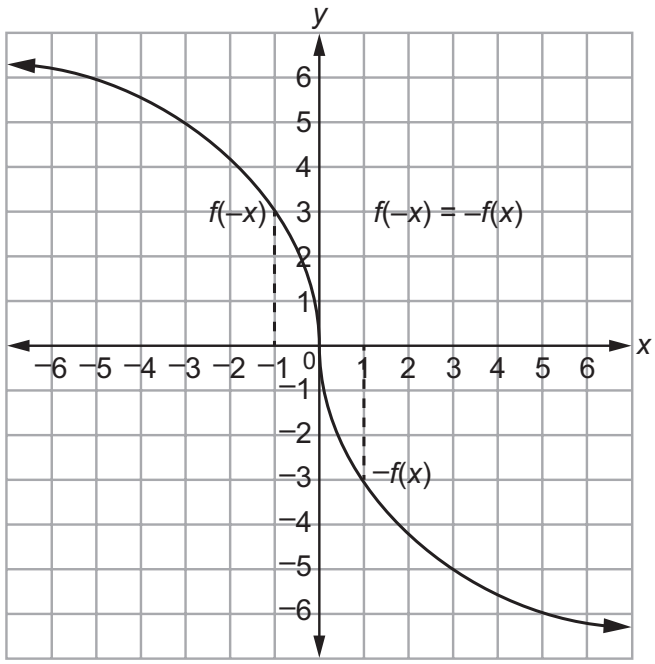
Functions can be classified as even or odd.

- If a graph is symmetric to the y-axis, then it is an ***even function***.
  That is, if $f(-x) = f(x)$, then the function is even.
- If a graph is symmetric to the origin, then it is an ***odd function***.
  That is, if $f(-x) = -f(x)$, then the function is odd.



Symmetric about the y-axis
Even Function



Symmetric about the Origin
Odd Function

Important Tip

✍ Remember that when you change $f(x)$ to $f(x + k)$, move the graph to the **left** when k is positive and to the **right** when k is negative. This may seem different from what you would expect, so be sure to understand why this occurs in order to apply the shift or translation correctly.

REVIEW EXAMPLES

♦ Compare the graphs of the following functions to $f(x)$.

a. $\frac{1}{2}\ f(x)$

b. $f(x) - 5$

c. $f(x - 2) + 1$

Solution:

a. The graph of $\frac{1}{2}\ f(x)$ is a vertical shrink of $f(x)$ by a factor of $\frac{1}{2}$.

b. The graph of $f(x) - 5$ is a shift or vertical translation of the graph of $f(x)$ down 5 units.

c. The graph of $f(x - 2) + 1$ is a shift or vertical translation of the graph of $f(x)$ right 2 units and up 1 unit.

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- Is $f(x) = 2x^3 + 6x$ even, odd, or neither? Explain how you know.

  Solution:

  Substitute $-x$ for x and evaluate:

$$\begin{aligned} f(-x) &= 2(-x)^3 + 6(-x) \\ &= 2(-x)^3 - 6x \\ &= -(2x^3 + 6x) \end{aligned}$$

  $f(-x)$ is the opposite of $f(x)$, so the function is odd.

  Substitute -3 for x and evaluate:

$$\begin{aligned} f(-3) &= 2(-3)^3 + 6(-3) \\ &= 2(-3)^3 - 18 \\ &= -(2(27) + 18) \\ &= -(72) \end{aligned} \qquad \begin{aligned} f(3) &= 2(3)^3 + 6(3) \\ &= 2(27) + 18 \\ &= 72 \end{aligned}$$

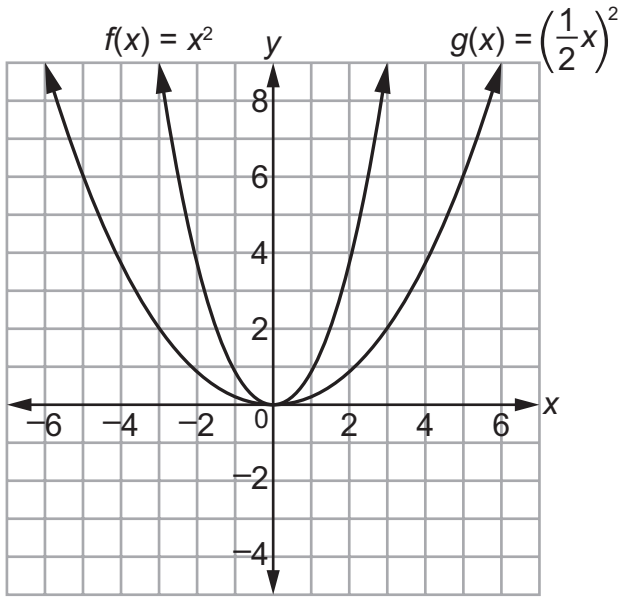
  $f(-3)$ is the opposite of $f(3)$, so the function is odd.

- How does the graph of $f(x)$ compare to the graph of $f\left(\frac{1}{2}x\right)$?

  Solution:

  The graph of $f\left(\frac{1}{2}x\right)$ is a horizontal stretch of $f(x)$ by a factor of 2. The graphs of $f(x)$ and $g(x) = f\left(\frac{1}{2}x\right)$ are shown.

  For example, at $y = 4$, the width of $f(x)$ is 4 and the width of $g(x)$ is 8. So, the graph of $g(x)$ is wider than $f(x)$ by a factor of 2.



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SAMPLE ITEMS

1. Which statement BEST describes the graph of $f(x + 6)$?

A. The graph of $f(x)$ is shifted up 6 units.
B. The graph of $f(x)$ is shifted left 6 units.
C. The graph of $f(x)$ is shifted right 6 units.
D. The graph of $f(x)$ is shifted down 6 units.

2. Which of these is an even function?

A. $f(x) = 5x^2 - x$
B. $f(x) = 3x^3 + x$
C. $f(x) = 6x^2 - 8$
D. $f(x) = 4x^3 + 2x^2$

3. Which statement BEST describes how the graph of $g(x) = -3x^2$ compares to the graph of $f(x) = x^2$?

A. The graph of $g(x)$ is a vertical stretch of $f(x)$ by a factor of 3.
B. The graph of $g(x)$ is a reflection of $f(x)$ across the x-axis.
C. The graph of $g(x)$ is a vertical shrink of $f(x)$ by a factor of $\frac{1}{3}$ and a reflection across the x-axis.
D. The graph of $g(x)$ is a vertical stretch of $f(x)$ by a factor of 3 and a reflection across the x-axis.

Answers to Unit 5.8 Sample Items

1. B 2. C 3. D

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5.9 Construct and Compare Linear, Quadratic, and Exponential Models to Solve Problems

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

KEY IDEAS

Exponential functions have a fixed number as the base and a variable number as the exponent.

The value of an exponential function with a base greater than 1 will eventually exceed the value of a quadratic function. Similarly, the value of a quadratic function will eventually exceed the value of a linear function.

Example:

Exponential	
x	$y = 2^x$
1	2
2	4
3	8
4	16
5	32
6	64

Quadratic	
x	$y = x^2 + 2$
1	3
2	6
3	11
4	18
5	27
6	38

Linear	
x	$y = x + 2$
1	3
2	4
3	5
4	6
5	7
6	8

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REVIEW EXAMPLES

- This table shows that the value of $f(x) = 5x^2 + 4$ is greater than the value of $g(x) = 2^x$ over the interval [0, 8].

x	$f(x)$	$g(x)$
0	$5(0)^2 + 4 = 4$	$2^0 = 1$
2	$5(2)^2 + 4 = 24$	$2^2 = 4$
4	$5(4)^2 + 4 = 84$	$2^4 = 16$
6	$5(6)^2 + 4 = 184$	$2^6 = 64$
8	$5(8)^2 + 4 = 324$	$2^8 = 256$

As x increases, will the value of $f(x)$ always be greater than the value of $g(x)$? Explain how you know.

Solution:

For some value of x, the value of an exponential function will eventually exceed the value of a quadratic function. To demonstrate this, find the values of $f(x)$ and $g(x)$ for another value of x, such as $x = 10$.

$$f(x) = 5(10)^2 + 4 = 504$$
$$g(x) = 2^{10} = 1{,}024$$

In fact, this means that for some value of x between 8 and 10, the value of $g(x)$ becomes greater than the value of $f(x)$ and remains greater for all subsequent values of x.

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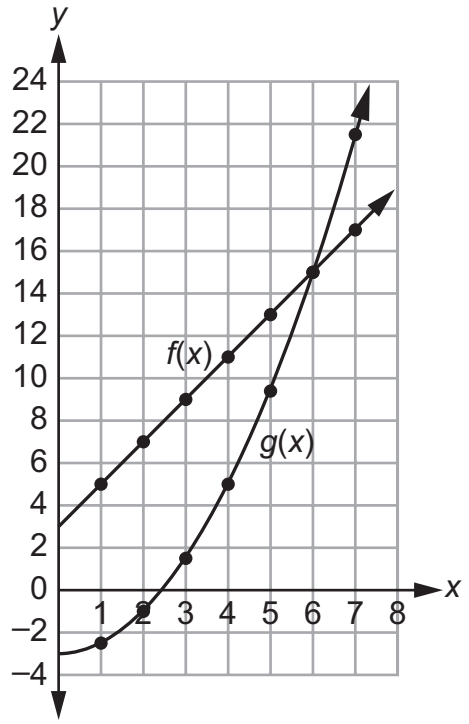
♦ How does the growth rate of the function $f(x) = 2x + 3$ compare with $g(x) = 0.5x^2 - 3$?

Use a graph to explain your answer.

Solution:

Graph $f(x)$ and $g(x)$ over the interval $x \geq 0$.



The graph of $f(x)$ increases at a constant rate because it is linear.

The graph of $g(x)$ increases at an increasing rate because it is quadratic.

The graphs can be shown to intersect at (6, 15), and the value of $g(x)$ is greater than the value of $f(x)$ for $x > 6$.

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SAMPLE ITEMS

1. A table of values is shown for $f(x)$ and $g(x)$.

x	$f(x)$
0	0
1	1
2	4
3	9
4	16
5	25

x	$g(x)$
0	–2
1	–1
2	1
3	5
4	13
5	29

Which statement compares the graphs of $f(x)$ and $g(x)$ over the interval [0, 5]?

A. The graph of $f(x)$ always exceeds the graph of $g(x)$ over the interval [0, 5].
B. The graph of $g(x)$ always exceeds the graph of $f(x)$ over the interval [0, 5].
C. The graph of $g(x)$ exceeds the graph of $f(x)$ over the interval [0, 4], the graphs intersect at a point between 4 and 5, and then the graph of $f(x)$ exceeds the graph of $g(x)$.
D. The graph of $f(x)$ exceeds the graph of $g(x)$ over the interval [0, 4], the graphs intersect at a point between 4 and 5, and then the graph of $g(x)$ exceeds the graph of $f(x)$.

2. Which statement is true about the graphs of exponential functions?

A. The graphs of exponential functions never exceed the graphs of linear and quadratic functions.
B. The graphs of exponential functions always exceed the graphs of linear and quadratic functions.
C. The graphs of exponential functions eventually exceed the graphs of linear and quadratic functions.
D. The graphs of exponential functions eventually exceed the graphs of linear functions but not quadratic functions.

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3. Which statement BEST describes the comparison of the function values for $f(x)$ and $g(x)$?

x	$f(x)$	$g(x)$
0	0	–10
1	2	–9
2	4	–6
3	6	–1
4	8	6

A. The values of $f(x)$ will always exceed the values of $g(x)$.
B. The values of $g(x)$ will always exceed the values of $f(x)$.
C. The values of $f(x)$ exceed the values of $g(x)$ over the interval [0, 5].
D. The values of $g(x)$ begin to exceed the values of $f(x)$ within the interval [4, 5].

Answers to Unit 5.9 Sample Items

1. D 2. C 3. D

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5.10 Summarize, Represent, and Interpret Data on Two Categorical and Quantitative Variables

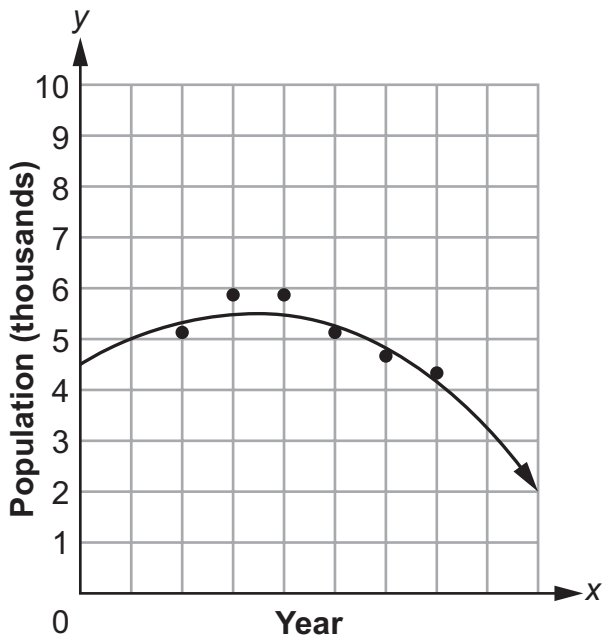
MGSE9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

> **MGSE9-12.S.ID.6a** Decide which type of function is most appropriate by observing graphed data, charted data, or by analysis of context to generate a viable (rough) function of best fit. Use this function to solve problems in context. Emphasize quadratic models.

KEY IDEAS

A ***quadratic regression*** equation is a curve of best fit for data given in a scatter plot. The curve most likely will not go through all the data points but should come close to most of them.

Example:



A quadratic regression equation can be used to make predictions about data. To do this, evaluate the equation for a given input value.

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REVIEW EXAMPLES

- Amery recorded the distance and height of a basketball when shooting a free throw.

Distance (feet), x	Height (feet), $f(x)$
0	4
2	8.4
6	12.1
9	14.2
12	13.2
13	10.5
15	9.8

The height of the basketball at a distance of x feet can be approximated by the quadratic function $f(x) = -0.118x^2 + 2.112x + 4.215$. Using this function, what is the approximate maximum height of the basketball?

Solution:

Find the vertex of the function.

$$\frac{-b}{2a} = \frac{-(2.112)}{2(-0.118)} \approx 8.949$$

Substitute 8.949 for x in the original function:

$$f(8.949) = -0.118(8.949)^2 + 2.112(8.949) + 4.215 \approx 13.665 \approx 13.7$$

The maximum height of the basketball predicted by the function is about 13.7 feet.

- This table shows the population of a city every 10 years since 1970.

Years since 1970, x	Population (thousands), y
0	489
10	801
20	1,202
30	1,998
40	2,959

a. Make a scatter plot showing the data.

b. Which type of function better models the relationship between 1970 and 2010: quadratic or linear?

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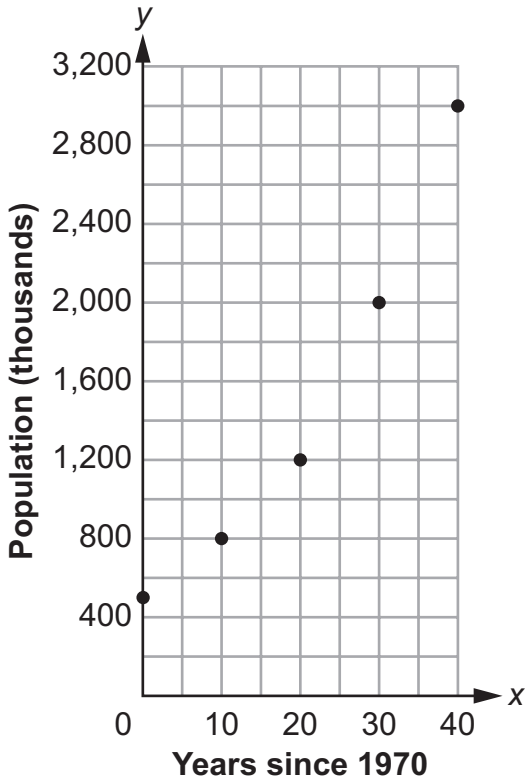
Solution:

a. Plot the points on a coordinate grid.



b. A quadratic model represents the population better than a linear model.

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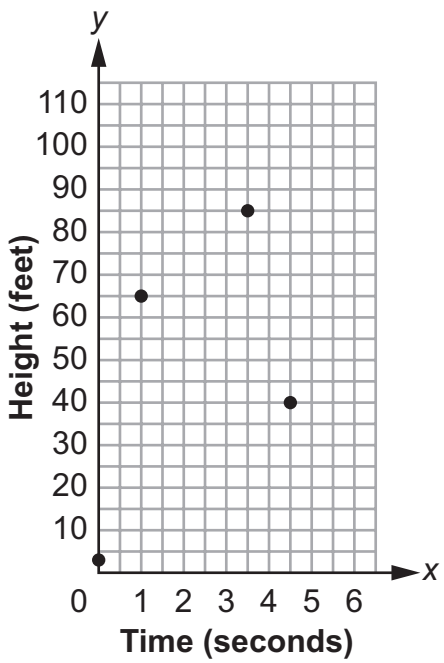
SAMPLE ITEMS

1. This scatter plot shows the height, in feet, of a ball launched in the air from an initial height of 3 feet and the time, in seconds, the ball traveled.



Based on an estimated quadratic regression curve, which height is the BEST estimate for the maximum height of the ball?

A. 75 feet
B. 85 feet
C. 100 feet
D. 120 feet

2. The quadratic function $f(x) = -45x^2 + 350x + 1{,}590$ models the population of a city, where x is the number of years after 2005 and $f(x)$ is the population of the city in thousands of people. What is the estimated population of the city in 2015?

A. 45,000
B. 77,000
C. 590,000
D. 670,000

Answers to Unit 5.10 Sample Items

1. C 2. C

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UNIT 6: GEOMETRIC AND ALGEBRAIC CONNECTIONS

This unit investigates coordinate geometry. Students look at equations for circles and parabolas and use given information to derive equations for representations of these figures on a coordinate plane. Students also use coordinates to prove simple geometric theorems using the properties of distance, slope, and midpoints. Students will verify whether a figure is a special quadrilateral by showing that sides of a figure are parallel or perpendicular. Students will model situations by using geometric shapes and their properties to describe objects. Students will also be able to use area and volume to apply concepts of density such as determining a number of units per volume.

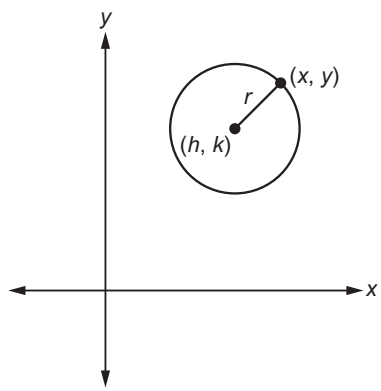
6.1 Translate between the Geometric Description and the Equation for a Conic Section

MGSE9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

KEY IDEAS

A ***circle*** is the set of points in a plane equidistant from a given point, or center, of the circle.

The standard form of the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the ***center*** of the circle and r is the ***radius*** of the circle.



The equation of a circle can be derived from the ***Pythagorean Theorem***; $a^2 + b^2 = c^2$.

Example: Given a circle with a center at (h, k) and a point (x, y) on the circle, draw a horizontal line segment from (h, k) to (x, k). Label this line segment a. Draw a vertical line segment from (x, y) to (x, k). Label this line segment b. Label the radius c. A right triangle is formed.

The length of line segment a is given by $(x - h)$.

The length of line segment b is given by $(y - k)$.

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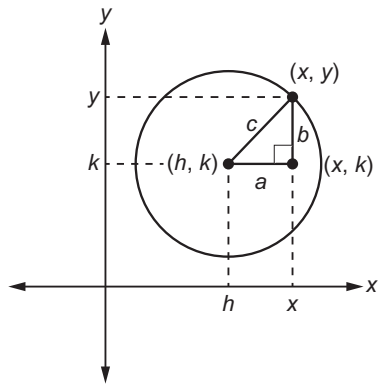


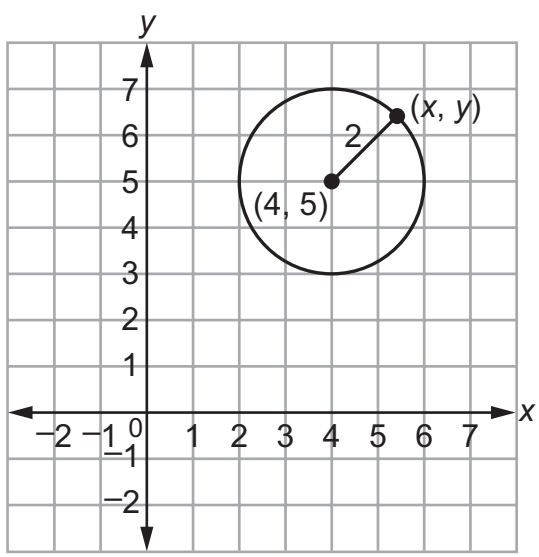
Using the Pythagorean Theorem, substitute $(x - h)$ for a, $(y - k)$ for b, and r for c in the equation.

$a^2 + b^2 = c^2$	Use the Pythagorean Theorem.
$(x - h)^2 + (y - k)^2 = r^2$	Substitution

The equation for a circle with a center at (h, k) and a radius r is $(x - h)^2 + (y - k)^2 = r^2$.

REVIEW EXAMPLES

- What is the equation of the circle with a center at (4, 5) and a radius of 2?



Solution:

Use the standard form for the equation for a circle: $(x - h)^2 + (y - k)^2 = r^2$. Substitute the values into the equation, with $h = 4$, $k = 5$, and $r = 2$.

$(x - h)^2 + (y - k)^2 = r^2$	Equation for a circle
$(x - (4))^2 + (y - (5))^2 = (2)^2$	Substitute the values in the equation of a circle.
$(x - 4)^2 + (y - 5)^2 = 4$	Evaluate.

The equation of a circle with a center at (4, 5) and a radius of 2 is $(x - 4)^2 + (y - 5)^2 = 4$, or $x^2 + y^2 - 8x - 10y + 37 = 0$ when expanded.

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- What is the center and radius of the circle given by $8x^2 + 8y^2 - 16x - 32y + 24 = 0$?

  Solution:

  Write the equation in standard form to identify the center and radius of the circle. First, write the equation so the x-terms are next to each other and the y-terms are next to each other, both on the left side of the equation, and the constant term is on the right side of the equation.

$8x^2 + 8y^2 - 16x - 32y + 24 = 0$	Original equation
$8x^2 + 8y^2 - 16x - 32y = -24$	Subtract 24 from both sides.
$8x^2 - 16x + 8y^2 - 32y = -24$	Commutative Property
$x^2 - 2x + y^2 - 4y = -3$	Divide both sides by 8.
$(x^2 - 2x) + (y^2 - 4y) = -3$	Associative Property

  Next, to write the equation in standard form, complete the square for the x-terms and the y-terms.

  Using $ax^2 + bx + c = 0$, find $\left(\frac{b}{2a}\right)^2$ for the x- and y-terms.

  x-term: $\left(\frac{b}{2a}\right)^2 = \left(\frac{-2}{2(1)}\right)^2 = (-1)^2 = 1$

  y-term: $\left(\frac{b}{2a}\right)^2 = \left(\frac{-4}{2(1)}\right)^2 = (-2)^2 = 4$

$(x^2 - 2x + 1) + (y^2 - 4y + 4) = -3 + 1 + 4$	Add 1 and 4 to each side of the equation.
$(x - 1)^2 + (y - 2)^2 = 2$	Write the trinomials as squares of binomials.

  This equation for the circle is written in standard form, where $h = 1$, $k = 2$, and $r^2 = 2$. The center of the circle is (1, 2), and the radius is $\sqrt{2}$.

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SAMPLE ITEMS

1. Which is an equation for the circle with a center at (–2, 3) and a radius of 3?

A. $x^2 + y^2 + 4x - 6y + 22 = 0$
B. $2x^2 + 2y^2 + 3x - 3y + 4 = 0$
C. $x^2 + y^2 + 4x - 6y + 4 = 0$
D. $3x^2 + 3y^2 + 4x - 6y + 4 = 0$

2. What is the center of the circle given by the equation $x^2 + y^2 - 10x - 11 = 0$?

A. (5, 0)
B. (0, 5)
C. (–5, 0)
D. (0, –5)

Answers to Unit 6.1 Sample Items

1. C 2. A

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6.2 Use Coordinates to Prove Simple Geometric Theorems Algebraically

MGSE9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

(Focus on quadrilaterals, right triangles, and circles.)

KEY IDEAS

Given the equation of a circle, you can verify whether a point lies on the circle by substituting the coordinates of the point into the equation. If the resulting equation is true, then the point lies on the figure. If the resulting equation is not true, then the point does not lie on the figure.

Given the center and radius of a circle, you can verify whether a point lies on the circle by determining whether the distance between the given point and the center is equal to the radius.

REVIEW EXAMPLES

- Circle *C* has a center of (–2, 3) and a radius of 4. Does point (–4, 6) lie on circle *C*?

  Solution:

  The distance from any point on the circle to the center of the circle is equal to the radius. Use the distance formula to find the distance from (–4, 6) to the center (–2, 3). Then see if it is equal to the radius, 4.

$\sqrt{(-4-(-2))^2+(6-3)^2}$	Substitute the coordinates of the points in the distance formula.
$\sqrt{(-2)^2+(3)^2}$	Evaluate within parentheses.
$\sqrt{4+9}$	Evaluate the exponents.
$\sqrt{13}$	Add.

  The distance from (–4, 6) to (–2, 3) is not equal to the radius, so (–4, 6) does not lie on the circle. (In fact, since $\sqrt{13} < 4$, the distance is less than the radius, so the point lies inside the circle.)

- Circle *C* has a diameter of 10 and a center at (2, 2). Point *A* is located at (–1, 6) and point *B* is located at (5, –2). Is segment *AB* a diameter of the circle?

  Solution:

  Both points *A* and *B* must be a distance of 5 units from the center and the length of segment *AB* must be 10 units. We can use the distance formula to determine this.

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Determine if points *A* and B are a distance of 5 units from (2, 2).

$\sqrt{(-1-2)^2+(6-2)^2}$	Substitute the coordinates of the points into the distance formula.
$\sqrt{(-3)^2+(4)^2}$	Evaluate within parentheses.
$\sqrt{9+16}$	Evaluate the exponents.
$\sqrt{25}$	Add.
5	Find the square root.

$\sqrt{(5-2)^2+(-2-2)^2}$	Substitute the coordinates of the points into the distance formula.
$\sqrt{(3)^2+(-4)^2}$	Evaluate within parentheses.
$\sqrt{9+16}$	Evaluate the exponents.
$\sqrt{25}$	Add.
5	Find the square root.

Now we will determine if *AB* = 10 to determine if it is a diameter of the circle.

$\sqrt{(-1-5)^2+(6-(-2))^2}$	Substitute the coordinates of the points in the distance formula.
$\sqrt{(-6)^2+(8)^2}$	Evaluate within parentheses.
$\sqrt{36+64}$	Evaluate the exponents.
$\sqrt{100}$	Add.
10	Find the square root.

The distance from (–1, 6) to (5, –2) is 10 units and both points are 5 units away from the center of the circle. This means both points lie on the circle and the measure of segment *AB* equals twice the radius. This makes segment *AB* a diameter of the circle.

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SAMPLE ITEMS

1. Which point is on a circle with a center of (3, –9) and a radius of 5?

A. (–6, 5)
B. (–1, 6)
C. (1, 6)
D. (6, –5)

2. Which two points can form the diameter of a circle with a center at the origin and a radius of 10?

A. (6, –8) and (–6, 8)
B. (–1, 0) and (9, 0)
C. (0, 5) and (0, –5)
D. (0, 0) and (10, 0)

Answers to Unit 6.2 Sample Items

1. D 2. A

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6.3 Apply Geometric Concepts in Modeling Situations

MGSE9-12.G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

MGSE9-12.G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

MGSE9-12.G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

KEY IDEAS

Modeling can be applied to describe real-life objects with geometric shapes.

Density is the mass of an object divided by its volume.

Population density can be determined by calculating the quotient of the number of people in an area and the area itself.

Apply constraints to maximize or minimize the cost of a cardboard box used to package a product that represents a geometric figure. Apply volume relationships of cylinders, pyramids, cones, and spheres.

REVIEW EXAMPLE

- A city has a population of 6,688 people. The area of the city is approximately 7.267 square miles.

  How many people per square mile live in the city?

  Solution:

  Find the quotient of 6,688 and 7.267 to find the number of people per square mile in the city.

$$\text{People per square mile} = \frac{6{,}688}{7.267} \approx 920 \text{ people per square mile}$$

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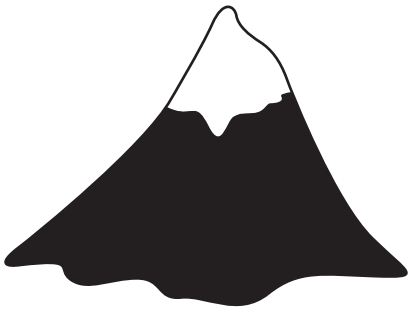
- This is a hand drawing of a mountain.



Explain which geometric shape could be used to estimate the total amount of earth the mountain is made of.

Solution:

The most accurate shape that could be used to model the mountain is a cone because, to determine the total amount of earth the mountain is made from, a three-dimensional shape is needed, which is why a triangle is not as accurate as a cone.

- A construction company is preparing 10 acres of land for a new housing community. The land contains large rocks that need to be removed. A machine removes 10 rocks from 360 square feet of land.

1 acre = 43,560 square feet

About how many rocks will need to be removed from the 10 acres of land?

Solution:

If there are 10 rocks in 360 square feet, then we can predict that there will be about 10 rocks every 360 square feet of land.

We will need to determine how many 360 square feet are in 10 acres.

10(43,560) = 435,600, so 435,600 square feet are in 10 acres.

$\frac{435{,}600}{360}$ = 1,210, so 1,210 parcels of 360 square feet are on the 10 acres.

(1,210)(10) = 12,100, so there should be about 12,100 rocks on the 10 acres of land.

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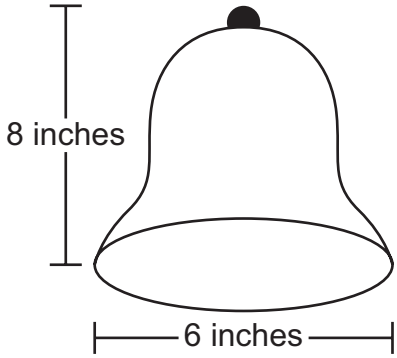
- A company needs to package this bell in a rectangular box.



What are the smallest dimensions (length, width, and height) the rectangular box can have so that the lid of the box can also close?

Solution:

Since the diameter of the base of the bell is 6 inches, the width and length of the box cannot be smaller than 6 inches. Since the height of the bell is 8 inches, then the height of the box cannot be smaller than 8 inches.

This gives us a rectangular box with these dimensions:

Length = 6 inches
Width = 6 inches
Height = 8 inches

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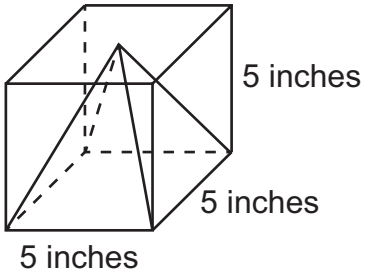
SAMPLE ITEMS

1. Joe counts 250 peach trees on 25% of the land he owns. He determines that there are 10 trees for every 1,000 square feet of land. About how many acres of land does Joe own?

1 acre = 43,560 square feet

A. 2.3
B. 10
C. 43.56
D. 2,500

2. A square pyramid is packaged inside a box.



The space inside the box around the pyramid is then filled with protective foam. About how many cubic inches of foam is needed to fill the space around the pyramid?

A. 8
B. 41
C. 83
D. 125

Answers to Unit 6.3 Sample Items

1. A 2. C

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UNIT 7: APPLICATIONS OF PROBABILITY

This unit investigates the concept of probability. Students look at sample spaces and identify unions, intersections, and complements. They identify ways to tell whether events are independent. The concept of conditional probability is related to independence, and students use the concepts to solve real-world problems, including those that are presented in two-way frequency tables. Students find probabilities of compound events using the rules of probability.

7.1 Understand Independence and Conditional Probability and Use Them to Interpret Data

MGSE9-12.S.CP.1 Describe categories of events as subsets of a sample space using unions, intersections, or complements of other events (*or, and, not*).

MGSE9-12.S.CP.2 Understand that if two events A and B are independent, the probability of A and B occurring together is the product of their probabilities, and that if the probability of two events A and B occurring together is the product of their probabilities, the two events are independent.

MGSE9-12.S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B). Interpret independence of A and B in terms of conditional probability; that is, the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

MGSE9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, use collected data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

MGSE9-12.S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

KEY IDEAS

In probability, a ***sample space*** is the set of all possible outcomes. Any subset from the sample space is an ***event***.

If the outcome of one event does not change the possible outcomes of the other event, the events are ***independent***. If the outcome of one event does change the possible outcomes of the other event, the events are ***dependent***.

The ***intersection*** of two or more events is all of the outcomes shared by both events. The intersection is denoted with the word “and” or with the $\cap$ symbol. For example, the intersection of A and B is shown as $A \cap B$.

The ***union*** of two or more events is all of the outcomes for either event. The union is denoted with the word “or” or with the $\cup$ symbol. For example, the union of A and B is shown as $A \cup B$. The probability of the union of two events that have no outcomes in common is the sum of each individual probability.

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The ***complement*** of an event is the set of outcomes in the same sample space that are not included in the outcomes of the event. The complement is denoted with the word "not" or with the $'$ symbol. For example, the complement of A is shown as A'. The set of outcomes and its complement make up the entire sample space.

Conditional probabilities are found when one event has already occurred and a second event is being analyzed. Conditional probability is denoted $P(A \mid B)$ and is read as "the probability of A given B."

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Two events—A and B—are independent if the probability of the intersection is the same as the product of each individual probability. That is, $P(A \text{ and } B) = P(A) \bullet P(B)$. This is called the ***Multiplication Rule for Independent Events***.

If two events are independent, then $P(A \mid B) = P(A)$ *and* $P(B \mid A) = P(B)$.

Two-way frequency tables summarize data in two categories. These tables can be used to show whether the two events are independent and to approximate conditional probabilities.

Example: A random survey was taken to gather information about grade level and car ownership status of students at a school. This table shows the results of the survey.

Car Ownership by Grade

	Owns a Car	**Does Not Own a Car**	**Total**
Junior	6	10	16
Senior	12	8	20
Total	18	18	36

Estimate the probability that a randomly selected student will be a junior given that the student owns a car.

Let $P(J)$ be the probability that the student is a junior. Let $P(C)$ be the probability that the student owns a car.

$$P(J \mid C) = \frac{P(J \text{ and } C)}{P(C)} = \frac{\frac{6}{36}}{\frac{18}{36}} = \frac{6}{18} = \frac{1}{3}$$

The probability that a randomly selected student will be a junior given that the student owns a car is $\frac{1}{3}$.

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REVIEW EXAMPLES

- This chart shows the names of students in Mr. Leary's class sorted by bicycle and skateboard ownership.

Bicycle and Skateboard Ownership

Owns a Bicycle	Owns a Skateboard	Owns a Bicycle AND a Skateboard	Does NOT Own a Bicycle OR a Skateboard
Ryan	Brett	Joe	Amy
Sarah	Juan	Mike	Gabe
Mariko	Tobi	Linda	Abi
Nina		Rose	
Dion			

Let set *A* be the names of students who own bicycles, and let set *B* be the names of students who own skateboards.

a. Find *A* and *B.* What does the set represent?

b. Find *A* or *B.* What does the set represent?

c. Find (*A* or *B*)′. What does the set represent?

Solution:

a. The intersection is the set of elements that are common to both set *A* and set *B*, so *A* and *B* is {Joe, Mike, Linda, Rose}. This set represents the students who own both a bicycle and a skateboard.

b. The union is the set of elements that are in set *A* or set *B* or in both set *A* and set *B*. You need to list the names in the intersection only one time, so *A* or *B* is {Ryan, Sarah, Mariko, Nina, Dion, Brett, Juan, Tobi, Joe, Mike, Linda, Rose}. This set represents the students who own a bicycle, a skateboard, or both.

c. The complement of *A* or *B* is the set of names that are not in *A* or *B*. So the complement of *A* or *B* is {Amy, Gabe, Abi}. This set represents the students who own neither a bicycle nor a skateboard.

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- In a certain town, the probability that a person plays sports is 65%. The probability that a person is between the ages of 12 and 18 is 40%. The probability that a person plays sports and is between the ages of 12 and 18 is 25%. Are the events independent? How do you know?

  Solution:

  Let $P(S)$ be the probability that a person plays sports.

  Let $P(A)$ be the probability that a person is between the ages of 12 and 18.

  If the two events are independent, then $P(S \text{ and } A) = P(S) \bullet P(A)$.

  Because $P(S \text{ and } A)$ is given as 25%, find $P(S) \bullet P(A)$ and then compare.

  $$\begin{aligned} P(S) \bullet P(A) &= 0.65 \bullet 0.4 \\ &= 0.26 \end{aligned}$$

  Because $0.26 \neq 0.25$, the events are not independent.

- A random survey was conducted to gather information about employment status and age. This table shows the data that were collected.

Employment Survey Results

		Age (years)		
		Younger Than 18	**18 or Older**	**Total**
Status	**Has Job**	20	587	607
	Does Not Have Job	245	92	337
	Total	265	679	944

a. What is the probability that a randomly selected person surveyed has a job given that the person is younger than 18 years old?

b. What is the probability that a randomly selected person surveyed has a job given that the person is 18 years old or older?

c. Are having a job (A) and being 18 years old or older (B) independent events? Explain.
   - $P(A)$ = has a job
   - $P(A')$ = does not have a job
   - $P(B)$ = 18 years old or older
   - $P(B')$ = younger than 18 years old

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Solution:

a. Find the total number of people surveyed younger than 18 years old: 265. Divide the number of people who have a job and are younger than 18 years old, 20, by the number of people younger than 18 years old, 265: $\frac{20}{265} \approx 0.08$. The probability that a person surveyed has a job given that the person is younger than 18 years old is about 0.08.

$$P(A \mid B') = \frac{P(A \text{ and } B')}{P(B')} = \frac{\frac{20}{944}}{\frac{265}{944}} = \frac{20}{265} = 0.08$$

b. Find the total number of people surveyed 18 years old or older: 679. Divide the number of people who have a job and are 18 years old or older, 587, by the number of people 18 years old or older, 679: $\frac{587}{679} \approx 0.86$. The probability that a person surveyed has a job given that the person is 18 years old or older, is about 0.86.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{\frac{587}{944}}{\frac{679}{944}} = \frac{587}{679} = 0.86$$

c. The events are independent if $P(A \mid B) = P(A)$ *and* $P(B \mid A) = P(B)$.
From part (b), $P(A \mid B) \approx 0.86$.

$P(A) = \frac{607}{944} \approx 0.64$

$P(A \mid B) \neq P(A)$, so the events are not independent.

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SAMPLE ITEMS

1. In a particular state, the first character on a license plate is always a letter. The last character is always a digit from 0 to 9.

If *V* represents the set of all license plates beginning with a vowel and *O* represents the set of all license plates that end with an odd number, which license plate belongs to the set *V* and O′?

A. E23 PC8

B. MG4 3F5

C. AR8 8X9

D. P7M Z56

2. For which set of probabilities would events *A* and *B* be independent?

A. $P(A) = 0.25$; $P(B) = 0.25$; $P(A \text{ and } B) = 0.5$
B. $P(A) = 0.08$; $P(B) = 0.4$; $P(A \text{ and } B) = 0.12$
C. $P(A) = 0.16$; $P(B) = 0.24$; $P(A \text{ and } B) = 0.32$
D. $P(A) = 0.3$; $P(B) = 0.15$; $P(A \text{ and } B) = 0.045$

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3. Assume that the following events are independent:

- The probability that a high school senior will go to college is 0.72.
- The probability that a high school senior will go to college and live on campus is 0.46.

What is the probability that a high school senior will live on campus given that the high school senior will go to college?

A. 0.26
B. 0.33
C. 0.57
D. 0.64

4. A random survey was conducted about gender and hair color. This table records the data.

Hair Color

	Brown	Blond	Red	Total
Male	548	876	82	1,506
Female	612	716	66	1,394
Total	1,160	1,592	148	2,900

What is the probability that a randomly selected person has blond hair given that the person selected is male?

A. 0.51
B. 0.55
C. 0.58
D. 0.63

Answers to Unit 7.1 Sample Items

1. A 2. D 3. D 4. C

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7.2 Use the Rules of Probability to Compute Probabilities of Compound Events in a Uniform Probability Model

MGSE9-12.S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in context.

MGSE9-12.S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answers in context.

KEY IDEAS

Two events are ***mutually exclusive*** if the events cannot occur at the same time.

When two events A and B are mutually exclusive, the probability that event A or event B will occur is the sum of the probabilities of each event: $P(A \text{ or } B) = P(A) + P(B)$.

When two events A and B are not mutually exclusive, the probability that event A or B will occur is the sum of the probability of each event minus the intersection of the two events. That is, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. This is called the ***Addition Rule***.

You can find the conditional probability, $P(A \mid B)$, by finding the fraction of B's outcomes that also belong to A.

Example: Event A is choosing a heart card from a standard deck of cards.

Event B is choosing a face card from a standard deck of cards.

$P(A \mid B)$ is the probability that a card is a heart given that the card is a face card. You can look at B's outcomes and determine what fraction belongs to A; there are 12 face cards, 3 of which are also hearts:

$$P(A \mid B) = \frac{3}{12} = \frac{1}{4}$$

	Heart	**Not a Heart**	**Total**
Face Card	3	9	12
Not a Face Card	10	30	40
Total	13	39	52

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REVIEW EXAMPLES

- In Mr. Mabry's class, there are 12 boys and 16 girls. On Monday, 4 boys and 5 girls were wearing white shirts.

  a. If a student is chosen at random from Mr. Mabry's class, what is the probability of choosing a boy or a student wearing a white shirt?

  b. If a student is chosen at random from Mr. Mabry's class, what is the probability of choosing a girl or a student not wearing a white shirt?

  Solution:

  a. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, where A is the set of boys and B is the set of students wearing a white shirt.

     (A and B) is the set of boys wearing a white shirt. There are 12 + 16 = 28 students in Mr. Mabry's class.

     So $P(A) = \frac{12}{28}$, $P(B) = \frac{4+5}{28} = \frac{9}{28}$, and $P(A \text{ and } B) = \frac{4}{28}$.

     $P(\text{a boy or a student wearing a white shirt}) = \frac{12}{28} + \frac{9}{28} - \frac{4}{28} = \frac{17}{28}$

  b. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, where A is the set of girls and B is the set of students not wearing a white shirt.

     (A and B) is the set of girls not wearing a white shirt. There are 12 + 16 = 28 students in Mr. Mabry's class.

     So $P(A) = \frac{16}{28}$, $P(B) = \frac{8+11}{28} = \frac{19}{28}$, and $P(A \text{ and } B) = \frac{11}{28}$.

     $P(\text{a girl or a student not wearing a white shirt}) = \frac{16}{28} + \frac{19}{28} - \frac{11}{28} = \frac{24}{28} = \frac{6}{7}$

- Terry has a number cube with sides labeled 1 through 6. He rolls the number cube twice.

  a. What is the probability that the sum of the two rolls is a prime number given that at least one of the rolls is a 3?

  b. What is the probability that the sum of the two rolls is a prime number or at least one of the rolls is a 3?

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Solution:

a. This is an example of a mutually exclusive event. Make a list of the combinations where at least one of the rolls is a 3. There are 11 such pairs.

1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
3, 1	3, 2	3, 4	3, 5	3, 6	

Then identify the pairs that have a prime sum.

2, 3	3, 2	3, 4	4, 3

Of the 11 pairs of outcomes, there are 4 pairs whose sum is prime. Therefore, the probability that the sum is prime of those that show a 3 on at least one roll is $\frac{4}{11}$.

b. This is an example of events that are NOT mutually exclusive. There are 36 possible outcomes when rolling a number cube twice.

List the combinations where at least one of the rolls is a 3.

1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
3, 1	3, 2	3, 4	3, 5	3, 6	

$P(\text{at least one roll is a 3}) = \frac{11}{36}$

List the combinations that have a prime sum.

1, 1	1, 2	1, 4	1, 6
2, 1	2, 3	2, 5	
3, 2	3, 4		
4, 1	4, 3		
5, 2	5, 6		
6, 1	6, 5		

$P(\text{a prime sum}) = \frac{15}{36}$

Identify the combinations that are in both lists.

2, 3	3, 2	3, 4	4, 3

The combinations in both lists represent the intersection. The probability of the intersection is the number of outcomes in the intersection divided by the total possible outcomes.

$P(\text{at least one roll is a 3 } \textbf{and} \text{ a prime sum}) = \frac{4}{36}$

If two events share outcomes, then outcomes in the intersection are counted twice when the probabilities of the events are added. So you must subtract the probability of the intersection from the sum of the probabilities.

$P(\text{at least one roll is a 3 } \textbf{or} \text{ a prime sum}) = \frac{11}{36} + \frac{15}{36} - \frac{4}{36} = \frac{22}{36} = \frac{11}{18}$

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SAMPLE ITEMS

1. **Mrs. Klein surveyed 240 men and 285 women about their vehicles. Of those surveyed, 155 men and 70 women said they own a red vehicle. If a person is chosen at random from those surveyed, what is the probability of choosing a woman or a person who does NOT own a red vehicle?**

   A. $\frac{14}{57}$

   B. $\frac{71}{105}$

   C. $\frac{74}{105}$

   D. $\frac{88}{105}$

2. **Bianca spins two spinners that have four equal sections numbered 1 through 4. If she spins a 4 on at least one spin, what is the probability that the sum of her two spins is an odd number?**

   A. $\frac{1}{4}$

   B. $\frac{7}{16}$

   C. $\frac{4}{7}$

   D. $\frac{11}{16}$

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3. Each letter of the alphabet is written on separate cards in red ink. The cards are placed in a container. Each letter of the alphabet is also written on separate cards in black ink. The cards are placed in the same container. What is the probability that a card randomly selected from the container has a letter written in black ink or the letter is A or Z?

A. $\frac{1}{2}$

B. $\frac{7}{13}$

C. $\frac{15}{26}$

D. $\frac{8}{13}$

Answers to Unit 7.2 Sample Items

1. C 2. C 3. B

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ANALYTIC GEOMETRY ADDITIONAL PRACTICE ITEMS

This section has two parts. The first part is a set of 26 sample items for Analytic Geometry. The second part contains a table that shows for each item the standard assessed, the DOK level, the correct answer (key), and a rationale/explanation about the key and distractors. The sample items can be utilized as a mini-test to familiarize students with the item formats found on the assessment.

All example and sample items contained in this guide are the property of the Georgia Department of Education.

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Analytic Geometry Formula Sheet

Below are the formulas you may find useful as you take the test. However, you may find that you do not need to use all of the formulas. You may refer to this formula sheet as often as needed.

Quadratic Formulas

Quadratic Equations

Standard Form: $y = ax^2 + bx + c$

Vertex Form: $y = a(x - h)^2 + k$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Average Rate of Change

The change in the y-value divided by the change in the x-value for two distinct points on a graph.

Geometry Formulas

Pythagorean Theorem

$a^2 + b^2 = c^2$

Trigonometric Relationships

$\sin\theta = \frac{opp}{hyp}$; $\cos\theta = \frac{adj}{hyp}$; $\tan\theta = \frac{opp}{adj}$

Equation of a Circle

$(x - h)^2 + (y - k)^2 = r^2$

Circumference of a Circle

$C = \pi d$ or $C = 2\pi r$

$\pi \approx 3.14$

Arc Length of a Circle

Arc Length $= \frac{2\pi r\theta}{360}$

Area of a Circle

$A = \pi r^2$

Area of a Sector of a Circle

$$\text{Area of Sector} = \frac{\pi r^2\theta}{360}$$

Volume

Cylinder $V = \pi r^2 h$

Pyramid $V = \frac{1}{3}Bh$

Cone $V = \frac{1}{3}\pi r^2 h$

Sphere $V = \frac{4}{3}\pi r^3$

Statistics Formulas

Conditional Probability

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Multiplication Rule for Independent Events

$P(A \text{ and } B) = P(A) \cdot P(B)$

Addition Rule

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

You can find mathematics formula sheets on the Georgia Milestones webpage at http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Milestones-EOC-Resources.aspx.

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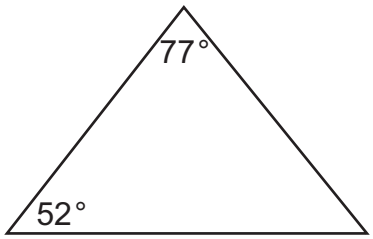
Item 1

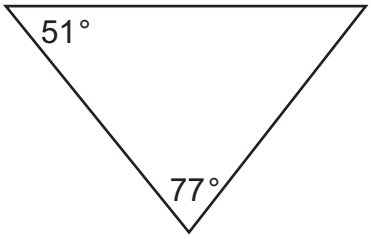
Selected-Response

Look at the triangle.

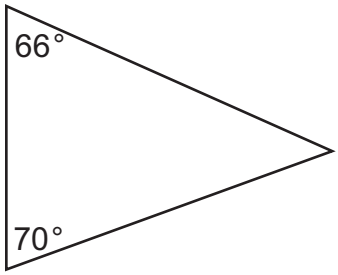


Which triangle is similar to the given triangle?

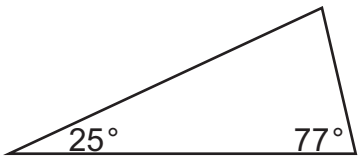
A.



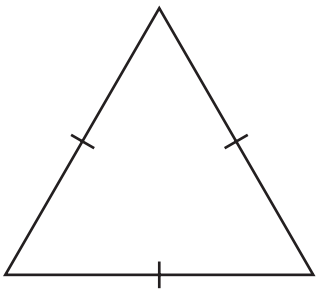
B.



C.



D.



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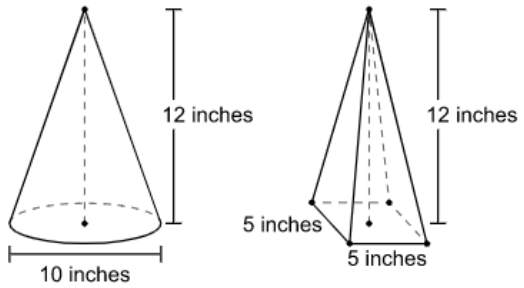
Item 2

Keypad-Input Technology-Enhanced

A cone and a pyramid are shown.



How many times the volume of the pyramid is the volume of the cone? Use 3.14 for π and round your answer to the nearest tenth.

Use a mouse, touchpad, or touchscreen to enter a response.

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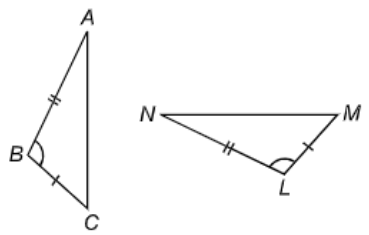
Item 3

Drop-Down Technology-Enhanced

Two triangles are shown.



Use the drop-down menus to complete the congruence statement.

$\triangle ABC \cong \triangle$ [ ] by [ ] congruence.

Use a mouse, touchpad, or touchscreen to click the arrow beside each of the two blank boxes. When you click the arrow, a drop-down menu will appear, showing you all the possible options for that blank box. Each drop-down menu with its options is shown below.

$\triangle ABC \cong \triangle$ [ ] by [ ] congruence.

LMN	AA
LNM	AAS
MLN	ASA
MNL	SAS
NLM	SSA
NML	SSS

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Item 4

Selected-Response

Which equation is true?

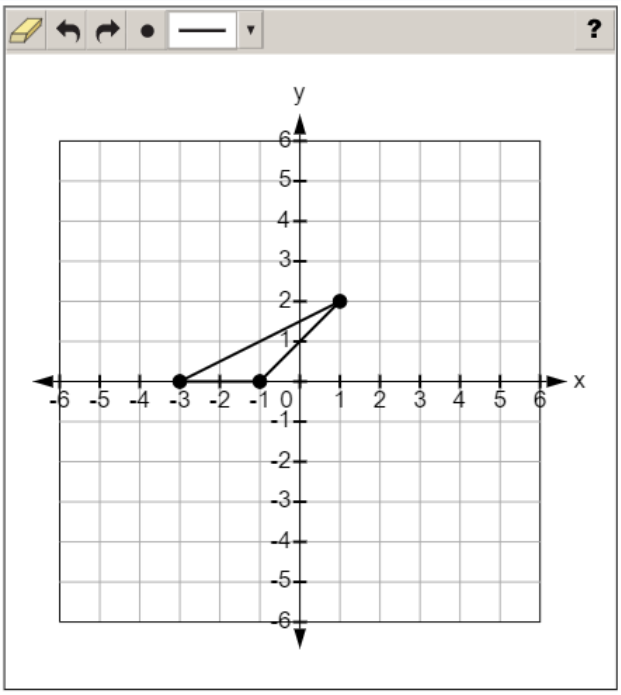
A. sin 40° = tan 50°
B. cos 40° = cos 50°
C. sin 40° = sin 50°
D. cos 40° = sin 50°

Item 5

Coordinate-Graph Technology-Enhanced

The triangle shown is rotated 180° counterclockwise around the origin and reflected across the *x*-axis.

Graph the image of the triangle after the transformations by plotting the vertices and line segments.



Use a mouse, touchpad, or touchscreen to graph the image of the triangle on the coordinate grid. At most 3 points and 3 line segments can be graphed.

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Item 6

Selected-Response

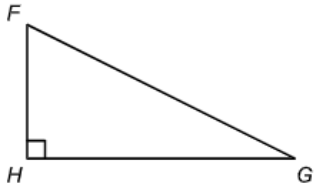
Which point is NOT on a circle with a center of (0, 0) and a radius of 10?

A. (0, 5)
B. (10, 0)
C. (0, –10)
D. (–8, 6)

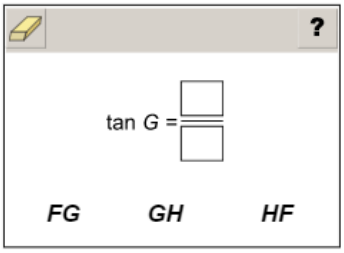
Item 7

Drag-and-Drop Technology-Enhanced

Right triangle *FGH* is shown.



Move the sides of triangle *FGH* into the boxes to complete the trigonometric ratio.



Use a mouse, touchpad, or touchscreen to move the side labels into the boxes. Each side label may be used 2 times.

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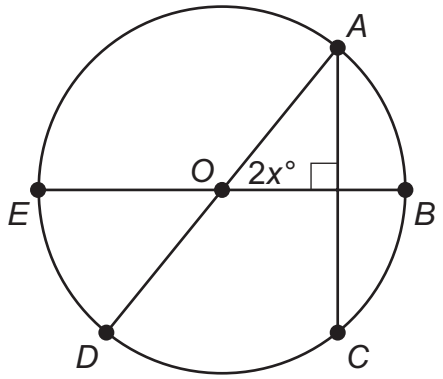
Item 8

Selected-Response

Points *A*, *B*, *C*, *D*, and *E* are located on circle *O*, as shown in this figure.



The measure of $\overset{\frown}{CD}$ is 80°. What is the value of *x*?

A. 50
B. 40
C. 35
D. 25

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Item 9

Selected-Response

Which expression is equivalent to $-4\sqrt{28x} \cdot \sqrt{7x^3}$?

A. $-56x^2$
B. $4x^2\sqrt{7}$
C. $-4x\sqrt{196}$
D. $-28x$

Item 10

Selected-Response

Which value is an irrational number?

A. $4 + \sqrt{7}$

B. $\sqrt{2}\ \sqrt{8}$

C. $\dfrac{\sqrt{3}\ \sqrt{12}}{5}$

D. $\sqrt{3} - \sqrt{3}$

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Item 11

Drag-and-Drop Technology-Enhanced

Move each expression to the correct column in the table.

Equivalent to $3\sqrt{6}$	Equivalent to $6\sqrt{3}$	Equivalent to $6\sqrt{6}$

$2\sqrt{54}$ $\sqrt{18} \cdot \sqrt{3}$ $3\sqrt{2} \cdot \sqrt{3}$ $\sqrt{3} \cdot \sqrt{6} \cdot \sqrt{6}$ $3\sqrt{3} + 3\sqrt{3}$

Use a mouse, touchpad, or touchscreen to move expressions into the columns. Each expression may be used once.

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Item 12

Multi-Select Technology-Enhanced

Select THREE equations that are true.

A. $(3x^2 + 7x - 4) + (x^2 + x + 3) = 4x^2 + 8x - 1$
B. $(5x^2 - 6x + 2) - (2x^2 - 4x + 1) = 3x^2 - 2x + 1$
C. $(3x^2 + 5x + 4) + (x^2 + 2x - 5) = 3x^2 + 7x - 1$
D. $(6x^2 - 3x - 8) - (3x^2 + 5x - 4) = 3x^2 - 2x - 4$
E. $(x + 8)(x - 9) = x^2 + x - 72$
F. $(x - 3)(x - 7) = x^2 - 10x + 21$

Item 13

Selected-Response

A weather balloon is 10 yards in diameter. It is in the shape of a sphere. What is the volume of the weather balloon to the nearest cubic yard?

A. 59 cubic yards
B. 105 cubic yards
C. 294 cubic yards
D. 523 cubic yards

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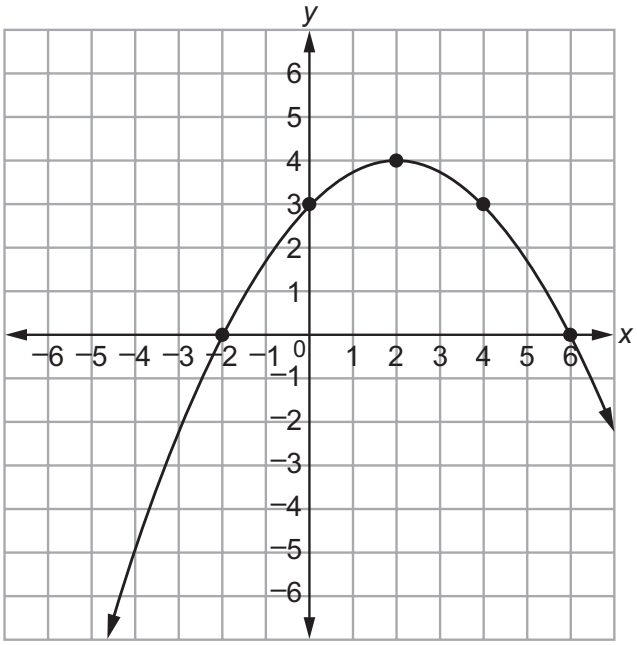
Item 14

Multi-Part Technology-Enhanced

The coordinate grid shows the graph of the quadratic function *f*(*x*).



The equation of *f*(*x*) can be written in the form $f(x) = a(x - h)^2 + k$, where *a*, *h*, and *k* are rational numbers.

Part A

What are the values of *h* and *k*?

A. $h = -2$ and $k = 3$
B. $h = -2$ and $k = 6$
C. $h = 2$ and $k = 3$
D. $h = 2$ and $k = 4$

Part B

Which statement describes the value of *a* for the function *f*(*x*)?

A. The value of *a* is a number less than –1.
B. The value of *a* is a number between –1 and 0.
C. The value of *a* is a number between 0 and 1.
D. The value of *a* is a number greater than 1.

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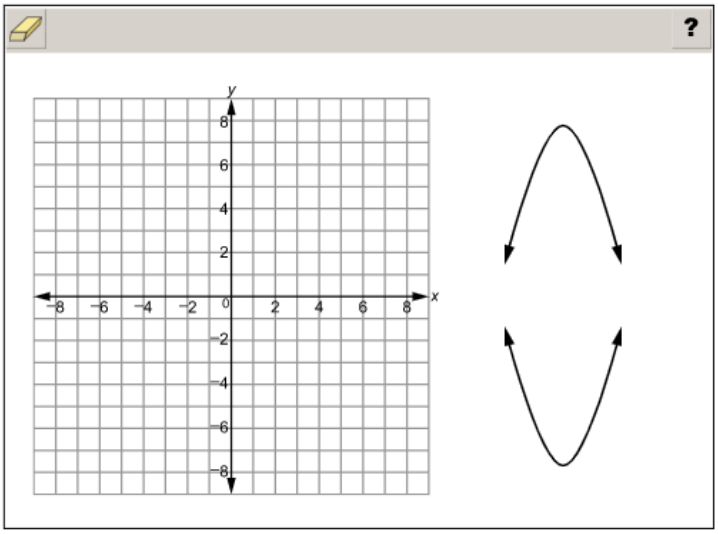
Item 15

Drag-and-Drop Technology-Enhanced

A quadratic equation is shown.

$$y = -x^2 + 4$$

Move the graph that represents the key features of the given quadratic equation onto the coordinate grid in the appropriate position.



Use a mouse, touchpad, or touchscreen to move the curve onto the grid. Only 1 curve may be placed on the grid.

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Item 16

Multi-Part Technology-Enhanced

Part A

Which situation could be modeled by a function with a domain of all positive integers?

A. the distance a runner has moved during a race as a function of time since the race started
B. the amount of fish food required in a fish tank as a function of the number of fish in the tank
C. the amount of power required to operate a computer as a function of the length of time the computer is on
D. the amount of water required by an animal as a function of the mass of the animal

Part B

Select the situation that could be modeled by a function with a domain that includes positive and negative real numbers.

A. the height of a plant as a function of time since the seed was planted
B. the elevation of a hiker as a function of the number of steps taken
C. the temperature as a function of the time of day
D. the amount of time required to read a book as a function of the number of words in the book
E. the amount of precipitation as a function of the outdoor temperature

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Item 17

Selected-Response

The table defines a quadratic function.

x	y
–1	5
0	1
1	–1
3	1

What is the average rate of change between $x = -1$ and $x = 1$?

A. undefined

B. $-\frac{1}{3}$

C. –3

D. –4

Item 18

Selected-Response

Study this equation of a circle.

$$x^2 - 6x + y^2 + 2y + 6 = 0$$

Which of these represents the center and radius of the circle?

A. center: (3, –1), radius: 4
B. center: (–3, 1), radius: 4
C. center: (3, –1), radius: 2
D. center: (–3, 1), radius: 2

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Item 19

Drop-Down Technology-Enhanced

A function is shown.

$$f(x) = x^2 - 6x - 27$$

Use the drop-down menus to make a true statement about $f(x)$.

The graph of $f(x)$ has a minimum value of [ ] and has zeros at [ ] and [ ].

Use a mouse, touchpad, or touchscreen to click the arrow beside each of the three blank boxes. When you click the arrow, a drop-down menu will appear, showing you all the possible options for that blank box. Each drop-down menu with its options is shown below.

The graph of $f(x)$ has a minimum value of [ ] and has zeros at [ ] and [ ].

–42	–9	–9
–36	–3	–3
–33	0	0
–27	3	3
	9	9

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Item 20

Drop-Down Technology-Enhanced

Some probabilities are listed below.

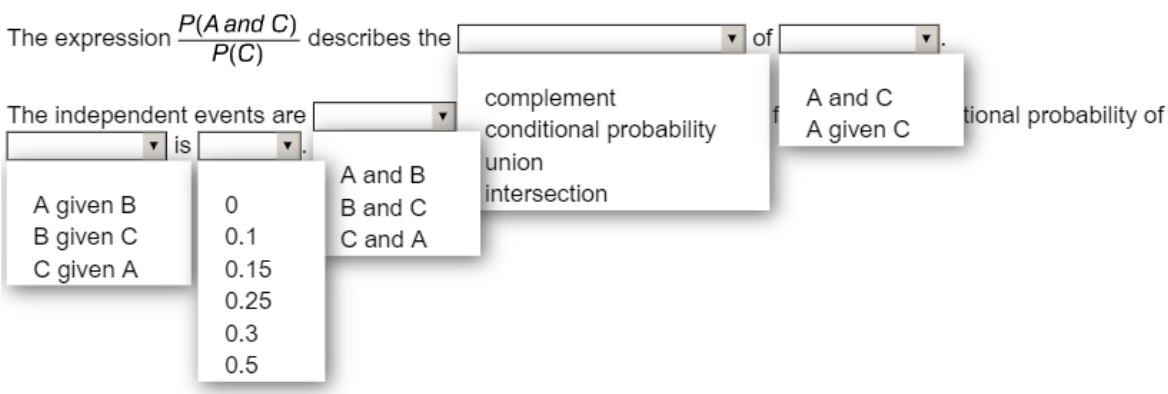
- $P(A) = 0.3$
- $P(B) = 0.5$
- $P(C) = 0.25$
- $P(A \text{ and } B) = 0.15$
- $P(B \text{ and } C) = 0$
- $P(C \text{ and } A) = 0.1$

Use the drop-down menus to complete the statements.

The expression $\frac{P(A \text{ and } C)}{P(C)}$ describes the [ ] of [ ].

The independent events are [ ]. This is demonstrated by the fact that the conditional probability of [ ] is [ ].

Use a mouse, touchpad, or touchscreen to click the arrow beside each of the five blank boxes. When you click the arrow, a drop-down menu will appear, showing you all the possible options for that blank box. Each drop-down menu with its options is shown below.



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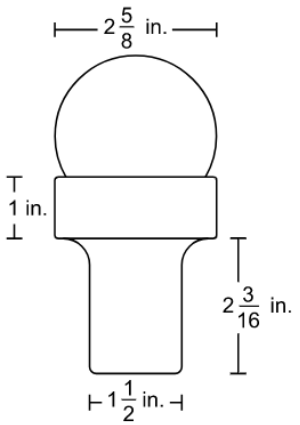
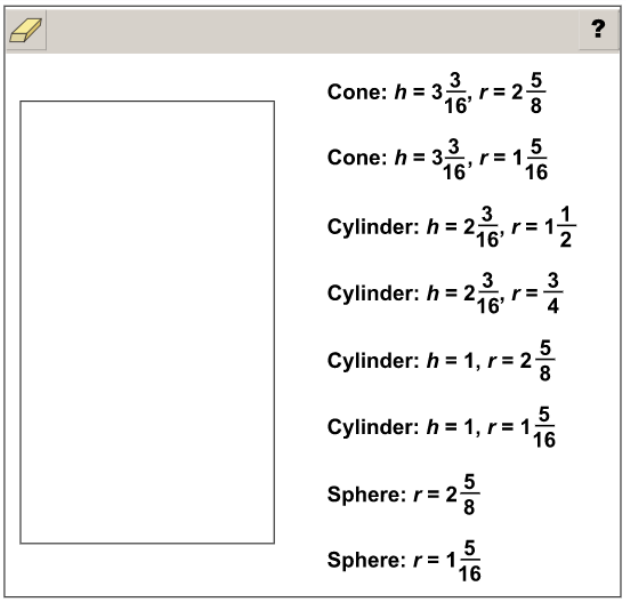
Item 21

Drag-and-Drop Technology-Enhanced

An ice-cream shop is finding the amount of ice cream used for each serving. The ice-cream shop fills the serving container completely with ice cream and then places a scoop on top. A diagram with measurements of a typical ice-cream serving from the shop is shown.



Move shapes into the box to create a combination that could be used to estimate the total volume of the ice cream in a typical ice-cream serving from the shop. Each measurement is in inches.



Use a mouse, touchpad, or touchscreen to move shapes into the box. Each shape may be used once. Not all shapes will be used.

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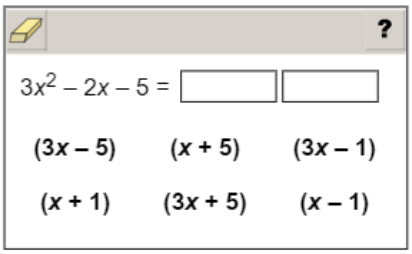
Item 22

Drag-and-Drop Technology-Enhanced

A quadratic expression is shown.

$$3x^2 - 2x - 5$$

Move an expression into each box to show the factored form of the given quadratic expression.



Use a mouse, touchpad, or touchscreen to move the expressions into the boxes. Each expression may be used 2 times.

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Item 23

Selected-Response

One bag of lawn fertilizer can cover approximately 5,000 square feet. Mike's lawn is about 500 square feet. Mike fertilizes his lawn an average of 4 times per year. How many full years will he be able to fertilize his lawn with one bag of fertilizer?

A. 2 years
B. 3 years
C. 9 years
D. 10 years

Item 24

Selected-Response

When rolling a number cube with sides labeled 1 through 6, what is the probability of rolling an even number or a number less than 3?

A. $\frac{5}{6}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

Item 25

Selected-Response

What is the probability of having rolled a 5 on a number cube with sides labeled 1 through 6 if you know that you rolled an odd number?

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

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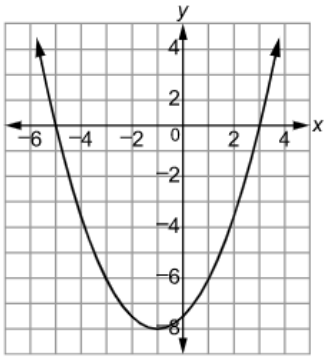
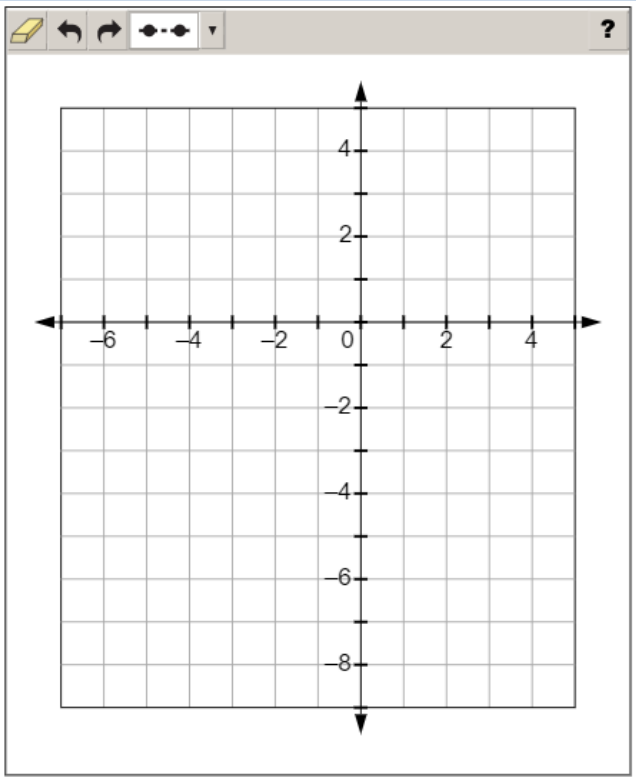
Item 26

Coordinate-Graph Technology-Enhanced

The graph of the function $y = 0.5x^2 + x - 7.5$ is shown.



Draw the axis of symmetry for this function.



Use a mouse, touchpad, or touchscreen to graph a line on the coordinate grid. At most 1 line can be graphed.

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ADDITIONAL PRACTICE ITEMS ANSWER KEY

Item	Standard/ Element	DOK Level	Correct Answer	Explanation
1	MGSE9-12.G.SRT.3	1	A	The correct answer is choice (A). The missing angle of the triangle in choice (A) is 52°, making it similar to the triangle given. Choices (B), (C), and (D) are incorrect because they have angle measures that are different than the original triangle.
2	MGSE9-12.G.GMD.3	2	N/A	See scoring rubric and exemplar response on page 216.
3	MGSE9-12.G.CO.8	2	N/A	See scoring rubric and exemplar response on page 217.
4	MGSE9-12.G.SRT.7	1	D	The correct answer is choice (D) cos 40° = sin 50°. The sine of an angle is equal to the cosine of the angle's complement. Choices (A), (B), and (C) are incorrect because they do not correspond to any trigonometric identities.
5	MGSE9-12.G.CO.6	2	N/A	See scoring rubric and exemplar response on page 218.
6	MGSE9-12.G.GPE.4	2	A	The correct answer is choice (A) (0, 5). The point is only 5 units away from the center of the circle. Choices (B), (C), and (D) are incorrect because they are 10 units away from the center of the circle.
7	MGSE9-12.G.SRT.6	1	N/A	See scoring rubric and exemplar response on page 219.
8	MGSE9-12.G.C.2	2	D	The correct answer is choice (D) 25. The measure of arc *CD* is 80° and the measure of angle *DAC* is 40°. Since the angles in a triangle add to 180°, the measure of angle *AOB* is 50°; if $2x = 50$, then $x = 25$. Choice (A) is incorrect because it is the measure of angle *AOB*. Choice (B) is incorrect because the answer is true only if the triangle is isosceles. Choice (C) is incorrect because a computation error was made when determining the value of x.
9	MGSE9-12.N.RN.2	2	A	The correct answer is choice (A) $-56x^2$. The given expression can be rewritten as $-4 \cdot \sqrt{28 \cdot 7 \cdot x^4}$, which simplifies to $-56x^2$. Choices (B), (C), and (D) are incorrect because the perfect squares are not factored out of the radical correctly.

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Item	Standard/ Element	DOK Level	Correct Answer	Explanation
10	MGSE9-12.N.RN.3	1	A	The correct answer is choice (A) $4 + \sqrt{7}$. Choice (B) is incorrect because it simplifies to 4. Choice (C) is incorrect because it simplifies to $\frac{6}{5}$. Choice (D) is incorrect because it simplifies to 0.
11	MGSE9-12.N.RN.2	2	N/A	See scoring rubric and exemplar response on page 220.
12	MGSE9-12.A.APR.1	2	A/B/F	The correct answers are choices (A), (B), and (F). Choices (A) and (B) are correct because they combine like terms correctly. Choice (F) is correct because the binomials are multiplied correctly. Choices (C) and (D) are incorrect because they do not combine like terms correctly. Choice (E) is incorrect because after the binomials are multiplied, the *x* terms are not combined correctly.
13	MGSE9-12.G.GMD.3	2	D	The correct answer is choice (D) 523 cubic yards. Choice (A) is incorrect because the value is squared and not cubed. Choice (B) is incorrect because the radius is squared instead of cubed. Choice (C) is incorrect because $\frac{3}{4}$ was used instead of $\frac{4}{3}$.
14	MGSE9-12.F.IF.7a	2	Part A: D Part B: B	Part A: The correct answer is choice (D) $h = 2$ and $k = 4$. The maximum of the function is at point (2, 4), so the value of *h* is 2 and the value of *k* is 4. Choices (A), (B), and (C) are incorrect because they all give incorrect values for the maximum of the function, (*h*, *k*). Part B: The correct answer is choice (B) The value of *a* is a number between −1 and 0. The function must have a negative number for *a*, and the shape of the graph indicates that it will be between −1 and 0. Choice (A) is incorrect because the graph with a value of *a* less than −1 would be a less wide parabola. Choices (C) and (D) are incorrect because the graph would have a minimum and an increasing end behavior.
15	MGSE9-12.A.CED.2	2	N/A	See scoring rubric and exemplar response on page 221.

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Item	Standard/ Element	DOK Level	Correct Answer	Explanation
16	MGSE9-12.F.IF.5	2	Part A: B Part B: E	Part A: The correct answer is choice (B) the amount of fish food required in a fish tank as a function of the number of fish in the tank. It is the only choice that models the domain (the number of fish in a tank) as using only positive integers. Part B: The correct answer is choice (E) the amount of precipitation as a function of the outdoor temperature. It is the only choice that models the domain (the outdoor temperature) as using positive and negative real numbers.
17	MGSE9-12.F.IF.6	2	C	The correct answer is choice (C) –3. Rate of change is found by finding the slope of the line containing the indicated points. Choice (A) is incorrect because it incorrectly computes the slopes. Choice (B) is incorrect because it reverses the numerator and denominator in the slope formula. Choice (D) is incorrect because it incorrectly computes the slope.
18	MGSE9-12.G.GPE.1	2	C	The correct answer is choice (C) center: (3, –1), radius: 2. When the equation is changed to standard form using completing the square, the *h*- and *k*-values are 3 and –1 and $r^2 = 4$, so $r = 2$. Choices (A) and (B) are incorrect because the radius comes from taking the square root of the constant in standard form. Choice (D) is incorrect because the signs of the center are opposite.
19	MGSE9-12.F.IF.8a	2	N/A	See scoring rubric and exemplar response on page 222.
20	MGSE9-12.S.CP.3	3	N/A	See scoring rubric and exemplar response on page 223.
21	MGSE9-12.G.MG.1	2	N/A	See scoring rubric and exemplar response on page 224.
22	MGSE9-12.A.SSE.3	2	N/A	See scoring rubric and exemplar response on page 225.
23	MGSE9-12.G.MG.2	2	A	The correct answer is choice (A) 2 years. The fertilizer will run out halfway into the third year, lasting only 2 full years. Choice (B) is incorrect because the fertilizer will not last through 3 years. Choice (C) is incorrect because the number of times the fertilizer is applied is subtracted from the total amount instead of divided. Choice (D) is incorrect because the number of times a year the fertilizer is applied is not divided by the total amount of fertilizer.

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Item	Standard/ Element	DOK Level	Correct Answer	Explanation
24	MGSE9-12.S.CP.7	1	B	The correct answer is choice (B) $\frac{2}{3}$. An even number or a number less than 3 includes the outcomes of 1, 2, 4, and 6 and there are 6 outcomes in the sample space; $\frac{4}{6}$ simplifies to $\frac{2}{3}$. Choice (A) is incorrect because the probability of rolling a 1 and the probability of rolling a number less than 3 were added together without subtracting the overlap. Choice (C) is incorrect because it is the probability of an even number only. Choice (D) is incorrect because it is the probability of a number less than 3 only.
25	MGSE9-12.S.CP.3	2	B	The correct answer is choice (B) $\frac{1}{3}$. With the conditional probability we assume that an odd number was rolled, which reduces our sample space to 1, 3, and 5. Out of those possibilities, the probability of rolling a 5 is $\frac{1}{3}$, 1 successful outcome out of 3 total outcomes. Choice (A) is incorrect because it is the probability of rolling 5 without knowing an odd number was rolled. Choice (C) is incorrect because it is the probability of rolling an odd number. Choice (D) is incorrect because it is the complement of the correct answer.
26	MGSE9-12.F.IF.8a	1	N/A	See scoring rubric and exemplar response on page 226.

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ADDITIONAL PRACTICE ITEMS SCORING RUBRICS AND EXEMPLAR RESPONSES

Item 2

Scoring Rubric

Points	Description
1	The student correctly answers the question.
0	The student does not correctly answer the question.

Exemplar Response

The correct response is shown below.

The volume of a square pyramid is found by multiplying one-third times the height of the pyramid times the square of the side length of the base. For this pyramid, the volume is $\frac{1}{3} \times 12 \times 5 \times 5$. The volume of a cone is found by multiplying one-third times the height of the cone times pi times the square of the radius of the base. For this cone, the volume is $\frac{1}{3} \times 12 \times \text{pi} \times 5 \times 5$. Both volumes have factors of $\frac{1}{3}$, 12, 5, and 5, so the volume of the cone is pi times the volume of the pyramid.

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Item 3

Scoring Rubric

Points	Description
1	The student selects the correct options in both drop-down menus.
0	The student does not select the correct options in both drop-down menus.

Exemplar Response

The correct response is shown below.

$\triangle ABC \cong \triangle$ [NLM ▾] by [SAS ▾] congruence.

"*NLM*" is correct because of the correspondence between the markings of the two triangles: the double tick mark is on side *AB* and on side *NL*, an angle mark is on angle *B* and on angle *L*, and the single tick mark is on side *BC* and on side *LM*. "SAS" is correct because the markings show corresponding congruent angles between pairs of corresponding congruent sides.

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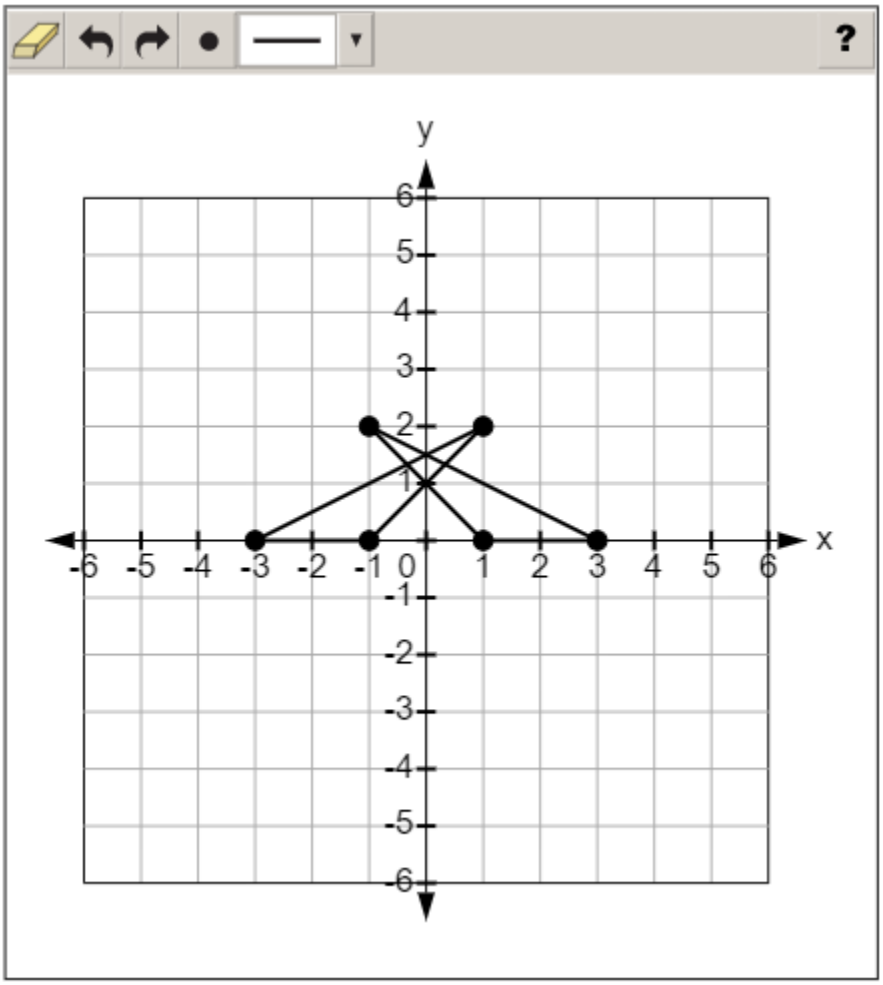
Item 5

Scoring Rubric

Points	Description
2	The student correctly graphs the image of the triangle.
1	The student correctly graphs one or two vertices of the triangle.
0	The student does not correctly graph at least one vertex of the triangle.

Exemplar Response

The correct response is shown below.



A rotation of 180° about the origin can be modeled by the rule $(x, y) \rightarrow (-x, -y)$. Following a 180° rotation about the origin with a reflection across the x-axis can be modeled by the rule $(-x, -y) \rightarrow (-x, y)$. Applying this rule to the coordinates of the original triangle yields the results (1, 0), (3, 0), and (–1, 2) for the vertices of the image of the triangle.

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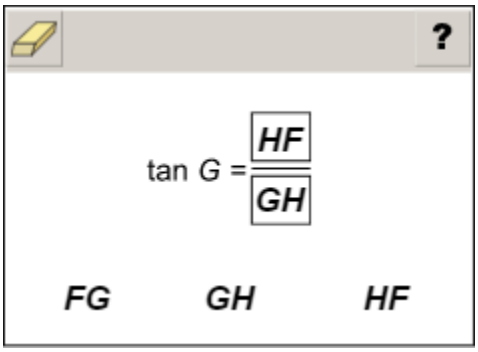
Item 7

Scoring Rubric

Points	Description
1	The student correctly completes the ratio.
0	The student does not correctly complete the ratio.

Exemplar Response

The correct response is shown below.



This is the correct response because the tangent of an angle is the ratio of the opposite side length to the adjacent side length. In this triangle, the side opposite angle *G* is side *HF* and the side adjacent to angle *G* is side *GH*.

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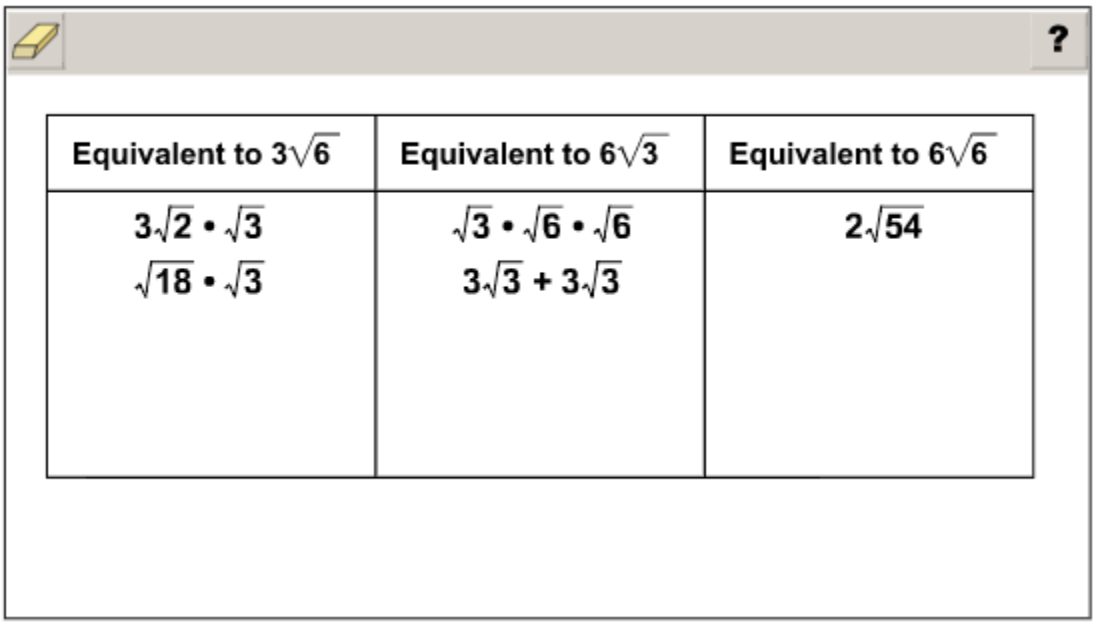
Item 11

Scoring Rubric

Points	Description
2	The student correctly places all five expressions.
1	The student correctly places three or four of the five expressions.
0	The student does not correctly place at least three expressions.

Exemplar Response

The correct response is shown below.



Equivalent to $3\sqrt{6}$	Equivalent to $6\sqrt{3}$	Equivalent to $6\sqrt{6}$
$3\sqrt{2} \cdot \sqrt{3}$ $\sqrt{18} \cdot \sqrt{3}$	$\sqrt{3} \cdot \sqrt{6} \cdot \sqrt{6}$ $3\sqrt{3} + 3\sqrt{3}$	$2\sqrt{54}$

Radical expressions are simplified the same way as variable expressions. If the expressions have like terms, such as "$3\sqrt{3} + 3\sqrt{3}$" in the second column, the expression can be simplified by adding the coefficients, so $3\sqrt{3} + 3\sqrt{3} = 6\sqrt{3}$. If the expression is a product, then the values under the radicals can be multiplied together, and if there are perfect square factors in the value under the radical, those can be separated out and calculated. The expression "$\sqrt{3} \cdot \sqrt{6} \cdot \sqrt{6}$" in the second column can be simplified to $\sqrt{3} \cdot \sqrt{36}$, which equals $6\sqrt{3}$. Similarly, in the first column, the expression "$3\sqrt{2} \cdot \sqrt{3}$" equals $3\sqrt{6}$ and the expression "$\sqrt{18} \cdot \sqrt{3}$" can be rewritten as $\sqrt{9} \cdot \sqrt{2} \cdot \sqrt{3}$, which equals $3\sqrt{6}$. It is also possible to separate out perfect square factors from a larger radical to simplify, such as in the third column response, "$2\sqrt{54}$," which can be separated into $2\sqrt{9} \cdot \sqrt{6}$ and simplified to $2 \cdot 3 \cdot \sqrt{6}$, which equals $6\sqrt{6}$.

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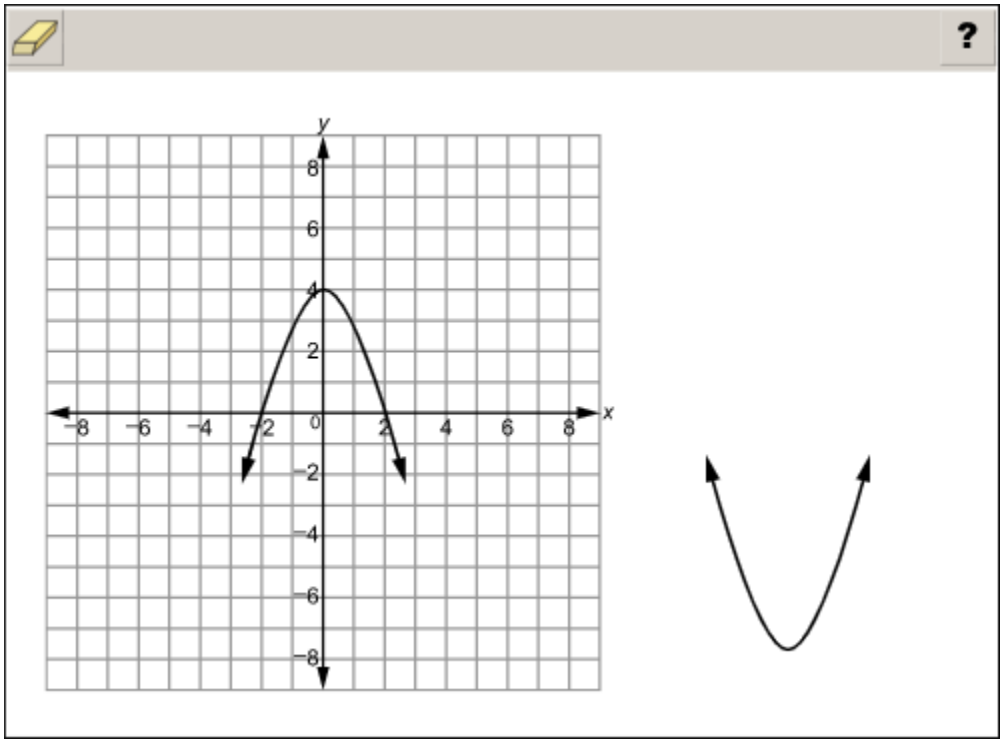
Item 15

Scoring Rubric

Points	Description
1	The student chooses the correct graph and correctly places it on the grid.
0	The student does not choose the correct graph and/or does not correctly place the graph on the grid.

Exemplar Response

The correct response is shown below.



This is the correct response because the coefficient of the x^2 term being negative indicates that the parabola opens downward. When $x = 0$, then $y = 4$; when $x = -2$ or $x = 2$, then $y = 0$.

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Item 19

Scoring Rubric

Points	Description
2	The student correctly selects the response to all of the drop-down menus.
1	The student correctly selects the response to one of the drop-down menus.
0	The student does not correctly select the response to any of the drop-down menus.

Exemplar Response

The two correct responses are shown below.

The graph of $f(x)$ has a minimum value of [–36] and has zeros at [–3] and [9].

The graph of $f(x)$ has a minimum value of [–36] and has zeros at [9] and [–3].

This is the correct response because the vertex of the function is at (3, –36) and the function crosses the x-axis at (–3, 0) and (9, 0).

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Item 20

Scoring Rubric

Points	Description
2	The student correctly completes both paragraphs.
1	The student correctly completes either the first paragraph or the second paragraph.
0	The student does not correctly complete either paragraph.

Exemplar Response

The correct response is shown below.

The expression $\frac{P(A \text{ and } C)}{P(C)}$ describes the [conditional probability] of [A given C].

The independent events are [A and B]. This is demonstrated by the fact that the conditional probability of [A given B] is [0.3].

"Conditional probability" and "A given C" correctly complete the first paragraph because the expression is the definition of conditional probability. The conditional probability of A given B can be found by using the formula in the first paragraph and replacing C with B. The events are independent because the conditional probability is equal to the probability of A.

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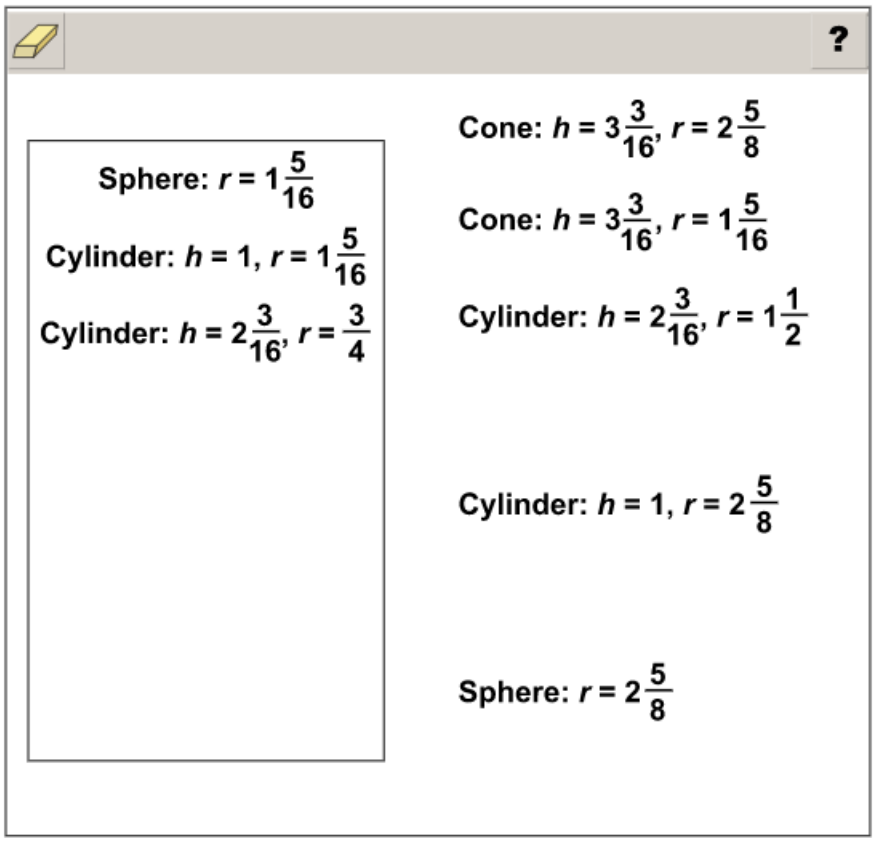
Item 21

Scoring Rubric

Points	Description
2	The student correctly places all three of the shapes needed.
1	The student correctly places two of the shapes needed.
0	The student does not correctly place at least two of the shapes needed.

Exemplar Response

The correct response is shown below.



The shape of the scoop of ice cream on top is approximately a sphere whose diameter is $2\frac{5}{8}$ inches, so the radius is $1\frac{5}{16}$ inches. The top of the serving container is close to a cylinder with a radius equal to the radius of the sphere ($1\frac{5}{16}$ inches) and a height of 1 inch. The bottom of the serving container is close to a cylinder with a diameter of $1\frac{1}{2}$ inches, which would be a radius of $\frac{3}{4}$ inch, and a height of $2\frac{3}{16}$ inches.

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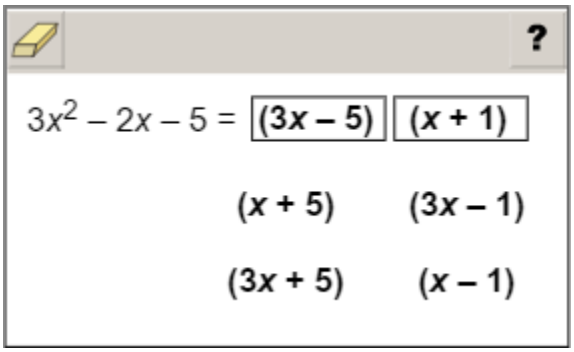
Item 22

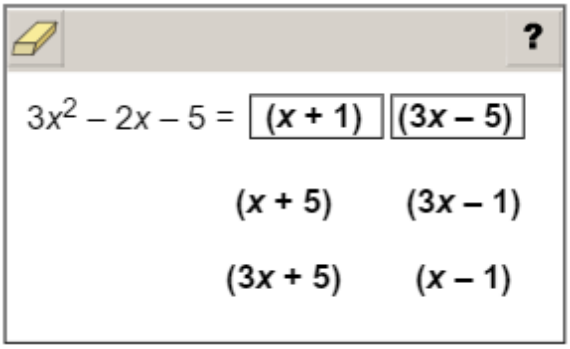
Scoring Rubric

Points	Description
1	The student correctly completes the equation.
0	The student does not correctly complete the equation.

Exemplar Response

The two correct responses are shown below.





To factor the expression $3x^2 - 2x - 5$, the leading coefficient of the x terms will be 3 and 1 because $3x^2$ has a leading coefficient that is prime. The constant term in the factors will be +1 and –5 because the constant term of the original expression is a negative prime number, and since the middle term is negative, the 5 must also be negative. That leads to the solution "$(3x - 5)$" and "$(x + 1)$."

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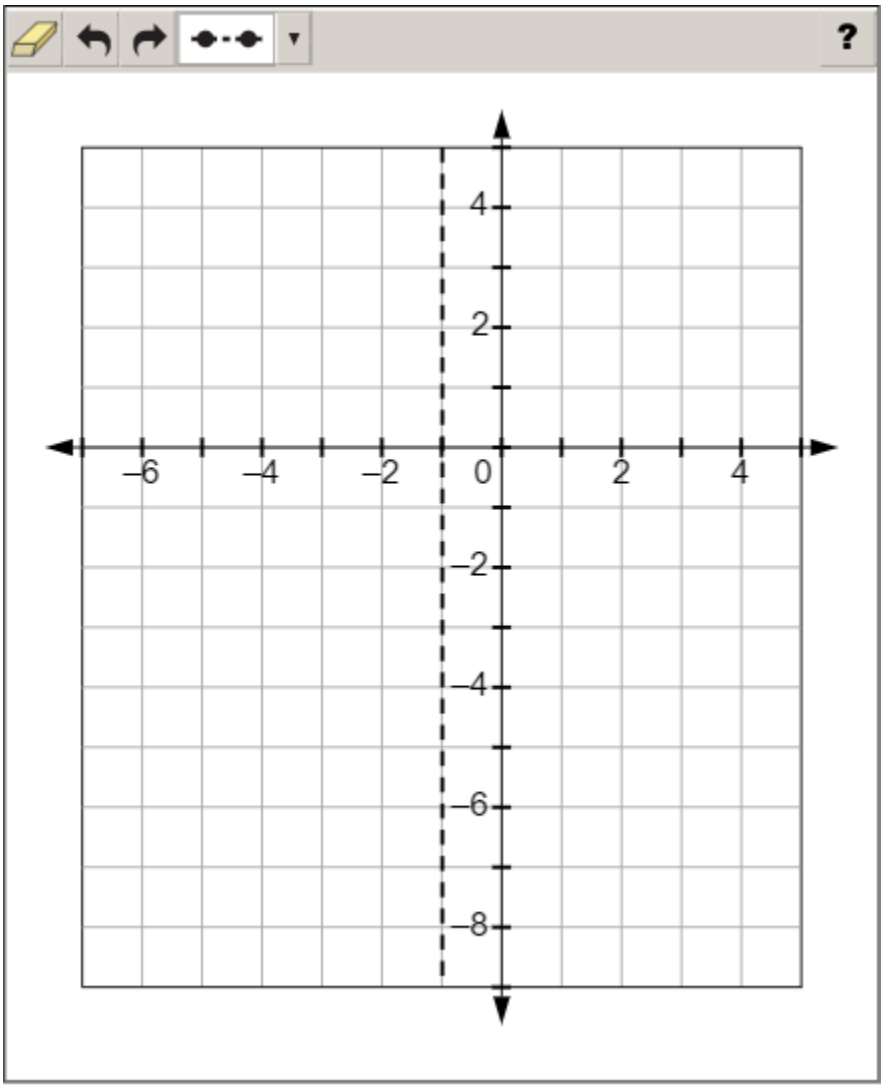
Item 26

Scoring Rubric

Points	Description
1	The student correctly graphs the line of symmetry.
0	The student does not correctly graph the line of symmetry.

Exemplar Response

The correct response is shown below.



This is the correct response because the function is a quadratic function with zeros at –5 and 3. All quadratic functions have a vertical line of symmetry half way between their zeros.

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END OF ANALYTIC GEOMETRY

EOC STUDY/RESOURCE GUIDE
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Analytic Geometry
End-of-Course