Georgia

Milestones

Assessment System

Study/Resource Guide for Students and Parents

Geometry

Study/Resource Guide

The Study/Resource Guides are intended to serve as a resource for parents and students. They contain practice questions and learning activities for the course. The standards identified in the Study/Resource Guides address a sampling of the state-mandated content standards.

For the purposes of day-to-day classroom instruction, teachers should consult the wide array of resources that can be found at www.georgiastandards.org.

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THE GEORGIA MILESTONES ASSESSMENT SYSTEM

Dear Student,

The **Georgia Milestones Geometry EOC Study/Resource Guide for Students and Parents** is intended as a resource for parents and students.

This guide contains information about the core content ideas and skills that are covered in the course. There are practice sample questions for every unit. The questions are fully explained and describe why each answer is either correct or incorrect. The explanations also help illustrate how each question connects to the Georgia state standards.

In addition, the guide includes activities that you can try to help you better understand the concepts taught in the course. The standards and additional instructional resources can be found on the Georgia Department of Education website, www.georgiastandards.org.

Get ready—open this guide—and get started!

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GEORGIA MILESTONES END-OF-COURSE (EOC) ASSESSMENTS

The EOC assessments serve as the final exam in certain courses. The courses are:

English Language Arts

- Ninth Grade Literature and Composition
- American Literature and Composition

Mathematics

- Algebra I
- Analytic Geometry
- Coordinate Algebra
- Geometry

Science

- Physical Science
- Biology

Social Studies

- United States History
- Economics/Business/Free Enterprise

All End-of-Course assessments accomplish the following:

- Ensure that students are learning
- Count as part of the course grade
- Provide data to teachers, schools, and school districts
- Identify instructional needs and help plan how to meet those needs
- Provide data for use in Georgia's accountability measures and reports

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HOW TO USE THIS GUIDE

Let's get started!

First, preview the entire guide. Learn what is discussed and where to find helpful information. Even though the focus of this guide is Geometry, you need to keep in mind your overall good reading habits.

- Start reading with a pencil or a highlighter in your hand and sticky notes nearby.
- Mark the important ideas, the things you might want to come back to, or the explanations you have questions about. On that last point, your teacher is your best resource.
- You will find some key ideas and important tips to help you prepare for the test.
- You can learn about the different types of items on the test.
- When you come to the sample items, don't just read them, *do* them. Think about strategies you can use for finding the right answer. Then read the analysis of the item to check your work. The reasoning behind the correct answer is explained for you. It will help you see any faulty reasoning in the ones you may have missed.
- Use the activities in this guide to get hands-on understanding of the concepts presented in each unit.
- With the Depth of Knowledge (DOK) information, you can gauge just how complex the item is. You will see that some items ask you to recall information and others ask you to infer or go beyond simple recall. The assessment will require all levels of thinking.
- Plan your studying and schedule your time.
- Proper preparation will help you do your best!

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OVERVIEW OF THE GEOMETRY EOC ASSESSMENT

ITEM TYPES

The Geometry EOC assessment consists of **selected-response** and **technology-enhanced** items.

A **selected-response** item, sometimes called a multiple-choice item, is a question, problem, or statement that is followed by four answer choices. These questions are worth one point.

A **technology-enhanced (TE)** item has a question, problem, or statement. These types of items are worth one or two points. Partial credit may be awarded on two-point items if you select some but not all of the correct answers or if you get one part of the question correct but not the other part.

- In multi-select items, you will be asked to select more than one right answer.
- In multi-part items, the items will have more than one part. You will need to provide an answer in each part.
- In drag-and-drop items, you will be asked to use a mouse, touchpad, or touchscreen to move responses to designated areas on the screen.
- In drop-down menu items, you will be asked to use a mouse, touchpad, or touchscreen to open a drop-down menu and select an option from the menu. A drop-down item may have multiple drop-down menus.
- In keypad-input items, you will be asked to use a physical keyboard or the pop-up keyboard on a touchscreen to type a number, expression, or equation into an answer box.
- In coordinate-graph items, you will be asked to use a mouse, touchpad, or touchscreen to draw lines and/or plot points on a coordinate grid on the screen.
- In line-plot items, you will be asked to use a mouse, touchpad, or touchscreen to place Xs above a number line to create a line plot.
- In bar-graph items, you will be asked to use a mouse, touchpad, or touchscreen to select the height of each bar to create a bar graph.
- In number-line items, you will be asked to use a mouse, touchpad, or touchscreen to plot a point and/or represent inequalities.
- Since some technology-enhanced items in this guide were designed to be used in an online, interactive-delivery format, some of the item-level directions will not appear to be applicable when working within the format presented in this document (for example, “Move the clocks into the graph” or “Create a scatter plot”).
- This icon ➲ identifies special directions that will help you answer technology-enhanced items as shown in the format presented within this guide. These directions do not appear in the online version of the test but explain information about how the item works that would be easily identifiable if you were completing the item in an online environment.

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To practice using technology-enhanced items in an online environment very similar to how they will appear on the online test, visit "Experience Online Testing Georgia."

1. Go to the website "Welcome to Experience Online Testing Georgia" (http://gaexperienceonline.com/).
2. Select "Test Practice."
3. Under "End-of-Course (EOC) All Assessments," select "Online Tools Training."
4. Select "EOC Test Practice."
5. Select "Technology Enhanced Items."
6. You will be taken to a login screen. Use the username and password provided on the screen to log in and practice navigating technology-enhanced items online.

Please note that Google Chrome is the only supported browser for this public version of the online testing environment.

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DEPTH OF KNOWLEDGE DESCRIPTORS

Items found on the Georgia Milestones assessments, including the Geometry EOC assessment, are developed with a particular emphasis on the kinds of thinking required to answer questions. In current educational terms, this is referred to as Depth of Knowledge (DOK). DOK is measured on a scale of 1 to 4 and refers to the level of cognitive demand (different kinds of thinking) required to complete a task, or in this case, an assessment item. The following table shows the expectations of the four DOK levels in detail.

The DOK table lists the skills addressed in each level as well as common question cues. These question cues not only demonstrate how well you understand each skill but also relate to the expectations that are part of the state standards.

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Depth of Knowledge

Level 1—Recall of Information	
Level 1 generally requires that you identify, list, or define. This level usually asks you to recall facts, terms, concepts, and trends and may ask you to identify specific information contained in documents, maps, charts, tables, graphs, or illustrations. Items that require you to "describe" and/or "explain" could be classified as Level 1 or Level 2. A Level 1 item requires that you just recall, recite, or reproduce information.	
Skills Demonstrated	**Question Cues**
• Make observations • Recall information • Recognize formulas, properties, patterns, processes • Know vocabulary, definitions • Know basic concepts • Perform one-step processes • Translate from one representation to another • Identify relationships	• Find • List • Define • Identify; label; name • Choose; select • Compute; estimate • Express • Read from data displays • Order

Level 2—Basic Reasoning	
Level 2 includes the engagement (use) of some mental processing beyond recalling or reproducing a response. A Level 2 "describe" and/or "explain" item would require that you go beyond a description or explanation of recalled information to describe and/or explain a result or "how" or "why."	
Skills Demonstrated	**Question Cues**
• Apply learned information to abstract and real-life situations • Use methods, concepts, and theories in abstract and real-life situations • Perform multi-step processes • Solve problems using required skills or knowledge (requires more than habitual response) • Make a decision about how to proceed • Identify and organize components of a whole • Extend patterns • Identify/describe cause and effect • Recognize unstated assumptions; make inferences • Interpret facts • Compare or contrast simple concepts/ideas	• Apply • Calculate; solve • Complete • Describe • Explain how; demonstrate • Construct data displays • Construct; draw • Analyze • Extend • Connect • Classify • Arrange • Compare; contrast

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Level 3—Complex Reasoning	
Level 3 requires reasoning, using evidence, and thinking on a higher and more abstract level than Level 1 and Level 2. You will go beyond explaining or describing "how and why" to justifying the "how and why" through application and evidence. Level 3 items often involve making connections across time and place to explain a concept or a "big idea."	
Skills Demonstrated	**Question Cues**
• Solve an open-ended problem with more than one correct answer • Create a pattern • Relate knowledge from several sources • Draw conclusions • Make predictions • Translate knowledge into new contexts • Compare and discriminate between ideas • Assess value of methods, concepts, theories, processes, and formulas • Make choices based on a reasoned argument • Verify the value of evidence, information, numbers, and data	• Plan; prepare • Predict • Create; design • Generalize • Justify; explain why; support; convince • Assess • Rank; grade • Test; judge • Recommend • Select • Conclude

Level 4—Extended Reasoning	
Level 4 requires the complex reasoning of Level 3 with the addition of planning, investigating, applying significant conceptual understanding, and/or developing that will most likely require an extended period of time. You may be required to connect and relate ideas and concepts *within* the content area or *among* content areas in order to be at this highest level. The Level 4 items would be a show of evidence, through a task, a product, or an extended response, that the cognitive demands have been met.	
Skills Demonstrated	**Question Cues**
• Analyze and synthesize information from multiple sources • Apply mathematical models to illuminate a problem or situation • Design a mathematical model to inform and solve a practical or abstract situation • Combine and synthesize ideas into new concepts	• Design • Connect • Synthesize • Apply concepts • Critique • Analyze • Create • Prove

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DEPTH OF KNOWLEDGE EXAMPLE ITEMS

Example items that represent the applicable DOK levels across various Geometry content domains are provided on the following pages.

All example and sample items contained in this guide are the property of the Georgia Department of Education.

Example Item 1

Selected-Response

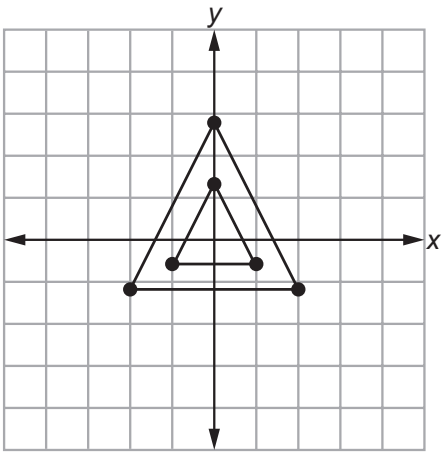
DOK Level 1: This is a DOK Level 1 item because it requires the student to demonstrate an understanding of dilations and determining the scale factor.

Geometry Content Domain: Congruence and Similarity

Standard: MGSE9-12.G.SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor.

b. The dilation of a line segment is longer or shorter according to the ratio given by the scale factor.

The smaller triangle is transformed to create the larger triangle. Which of these is the scale factor of the dilation centered at the point (0, 0)?



A. 4

B. 2

C. 1

D. $\frac{1}{2}$

Correct Answer: B

Explanation of Correct Answer: The correct answer is choice (B) 2. Since the length of each segment has doubled, the scale factor is 2. Choice (A) is incorrect because a scale factor of 4 would make the horizontal side of the image a length of 8. Choice (C) is incorrect because a scale factor of 1 does not change the size of the pre-image. Choice (D) is incorrect because it represents the scale factor when the pre-image and image are reversed.

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Example Item 2

Keypad-Input Multi-Part Technology-Enhanced

DOK Level 2: This is a DOK level 2 item because it requires students to determine conditional probabilities of independent events.

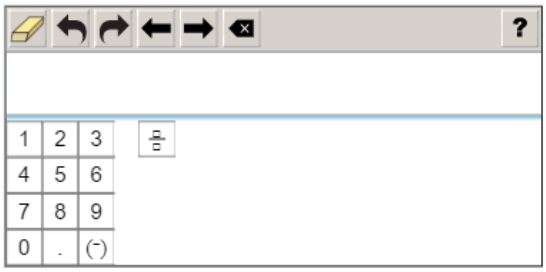
Geometry Content Domain: Statistics and Probability

Standard: MGSE9-12.S.CP.2. Understand that if two events A and B are independent, the probability of A and B occurring together is the product of their probabilities, and that if the probability of two events A and B occurring together is the product of their probabilities, the two events are independent.

Part A

Emily's bus is scheduled to arrive at 6:48 A.M. The probability that Emily arrives at the bus stop after 6:48 A.M. is 0.06. The probability that the bus arrives at the bus stop after 6:48 A.M. is 0.04. The time that Emily arrives at the bus stop and the time that the bus arrives at the bus stop are independent.

Part A What is the probability that the bus arrives at the bus stop after 6:48 A.M. given that Emily arrives at the bus stop after 6:48 A.M.?



Use a mouse, touchpad, or touchscreen to enter a response.

Go on to the next page to finish example item 2.

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Example Item 2. *Continued*.

Part B

Emily's bus is scheduled to arrive at 6:48 A.M. The probability that Emily arrives at the bus stop after 6:48 A.M. is 0.06. The probability that the bus arrives at the bus stop after 6:48 A.M. is 0.04. The time that Emily arrives at the bus stop and the time that the bus arrives at the bus stop are independent.

Part B What is the probability that both Emily and the bus will arrive at the bus stop after 6:48 A.M. on a given day?

Use a mouse, touchpad, or touchscreen to enter a response.

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Example Item 2. *Continued*.

Scoring Rubric

Points	Description
2	The student correctly answers both Part A and Part B.
1	The student correctly answers either Part A OR Part B.
0	The student does not correctly answer either part.

Exemplar Response

Part A

The correct response is shown below.

This is the correct response because the independence of the events means that the probability of the bus arriving after 6:48 given that Emily arrives after 6:48 is the same as the probability of the bus arriving after 6:48.

Part B

The correct response is shown below.

This is the correct response because the probability of two independent events occurring together is found by multiplying the probabilities of the two events.

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Example Item 3

Drop-Down Multi-Part Technology-Enhanced

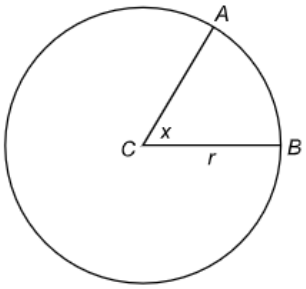
DOK Level 3: This is a DOK level 3 item because it requires students to determine the effects of changing measures in a circle with unknown dimensions.

Geometry Content Domain: Circles

Standard: MGSE9-12.G.C.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Part A

Circle *C*, with radius, *r*, and central angle measure, *x*, is shown.

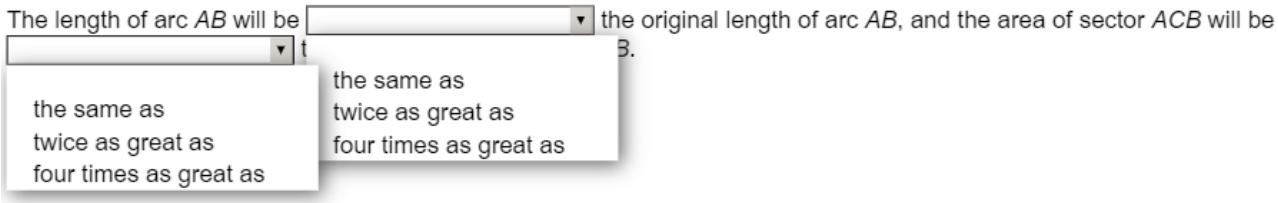


Part A What will be the effect of doubling the measure of the central angle, *x*, on the length of arc *AB* and the area of sector *ACB*?

Use the drop-down menus to complete the statement.

The length of arc *AB* will be [ ] the original length of arc *AB*, and the area of sector *ACB* will be [ ] the original area of sector *ACB*.

Use a mouse, touchpad, or touchscreen to click the arrow beside each of the two blank boxes. When you click the arrow, a drop-down menu will appear, showing you all the possible options for that blank box. Each drop-down menu with its options is shown below.



Go on to the next page to finish example item 3.

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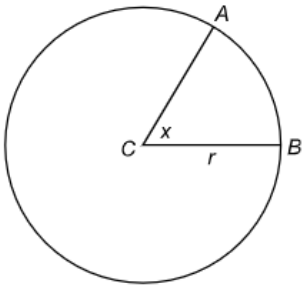
Example Item 3. *Continued*.

Part B

Circle *C*, with radius, *r*, and central angle measure, *x*, is shown.

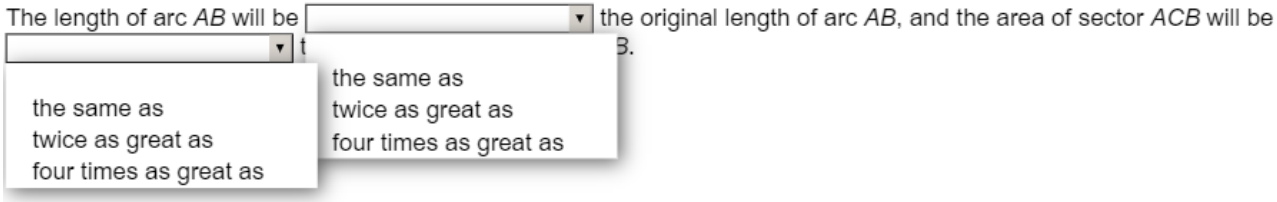


Part B What will be the effect of doubling the length of the radius, *r*, on the length of arc *AB* and the area of sector *ACB*?

Use the drop-down menus to complete the statement.

The length of arc *AB* will be [ ] the original length of arc *AB*, and the area of sector *ACB* will be [ ] the original area of sector *ACB*.

Use a mouse, touchpad, or touchscreen to click the arrow beside each of the two blank boxes. When you click the arrow, a drop-down menu will appear, showing you all the possible options for that blank box. Each drop-down menu with its options is shown below.



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Example Item 3. *Continued*.

Scoring Rubric

Points	Description
2	The student correctly completes both statements.
1	The student correctly completes one of the statements.
0	The student does not correctly complete either of the statements.

Exemplar Response

Part A

The correct response is shown below.

The length of arc *AB* will be [twice as great as] the original length of arc *AB*, and the area of sector *ACB* will be [twice as great as] the original area of sector *ACB*.

"Twice as great as" is correct for both drop-down menus because the length of an arc and the area of a sector both have a direct relationship to the measure of the central angle.

Part B

The correct response is shown below.

The length of arc *AB* will be [twice as great as] the original length of arc *AB*, and the area of sector *ACB* will be [four times as great as] the original area of sector *ACB*.

"Twice as great as" is correct for the first drop-down menu because the length of an arc has a direct relationship to the radius of the circle. "Four times as great as" is correct for the second drop-down because the area of a sector has a direct relationship to the square of the radius of the circle.

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DESCRIPTION OF TEST FORMAT AND ORGANIZATION

The Georgia Milestones Geometry EOC assessment consists of a total of 55 items. You will be asked to respond to selected-response (multiple-choice) and technology-enhanced items.

The test will be given in two sections.

- You may have up to 65 minutes per section to complete Sections 1 and 2.
- The total estimated testing time for the Geometry EOC assessment ranges from approximately 60 to 130 minutes. Total testing time describes the amount of time you have to complete the assessment. It does not take into account the time required for the test examiner to complete pre-administration and post-administration activities (such as reading the standardized directions to students).
- Sections 1 and 2 may be administered on the same day or across two consecutive days, based on the district's testing protocols for the EOC measures (in keeping with state guidance).
- During the Geometry EOC assessment, a formula sheet will be available for you to use. Another feature of the Geometry assessment is that you may use a graphing calculator in calculator-approved sections.

Effect on Course Grade

It is important that you take this course and the EOC assessment very seriously.

- For students in Grade 10 or above beginning with the 2011–2012 school year, the final grade in each course is calculated by weighing the course grade 85% and the EOC score 15%.
- For students in Grade 9 beginning with the 2011–2012 school year and later, the final grade in each course is calculated by weighing the course grade 80% and the EOC score 20%.
- A student must have a final grade of at least 70% to pass the course and to earn credit toward graduation.

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PREPARING FOR THE GEOMETRY EOC ASSESSMENT

STUDY SKILLS

As you prepare for this test, ask yourself the following questions:

- How would you describe yourself as a student?
- What are your study skills strengths and/or weaknesses?
- How do you typically prepare for a classroom test?
- What study methods do you find particularly helpful?
- What is an ideal study situation or environment for you?
- How would you describe your actual study environment?
- How can you change the way you study to make your study time more productive?

ORGANIZATION—OR TAKING CONTROL OF YOUR WORLD

- Establish a study area that has minimal distractions.
- Gather your materials in advance.
- Develop and implement your study plan.

ACTIVE PARTICIPATION

The most important element in your preparation is *you*. You and your actions are the key ingredient. Your active studying helps you stay alert and be more productive. In short, you need to interact with the course content. Here's how you do it.

- Carefully read the information and then DO something with it. Mark the important material with a highlighter, circle it with a pen, write notes on it, or summarize the information in your own words.
- Ask questions. As you study, questions often come into your mind. Write them down and actively seek the answers.
- Create sample test questions and answer them.
- Find a friend who is also planning to take the test and quiz each other.

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TEST-TAKING STRATEGIES

Part of preparing for a test is having a set of strategies you can draw from. Include these strategies in your plan:

- Read and understand the directions completely. If you are not sure, ask a teacher.
- Read each question and all of the answer choices carefully.
- If you use scratch paper, make sure you copy your work to your test accurately.
- Underline important parts of each task. Make sure that your answer goes on the answer sheet.
- Be aware of time. If a question is taking too much time, come back to it later.
- Answer all questions. Check your answers for accuracy.
- Stay calm and do the best you can.

PREPARING FOR THE GEOMETRY EOC ASSESSMENT

Read this guide to help prepare for the Geometry EOC assessment.

The section of the guide titled “Content of the Geometry EOC Assessment” provides a snapshot of the Geometry course. In addition to reading this guide, do the following to prepare to take the assessment:

- Read your resources and other materials.
- Think about what you learned, ask yourself questions, and answer them.
- Read and become familiar with the way questions are asked on the assessment.
- Answer some practice Geometry questions.
- There are additional items to practice your skills available online. Ask your teacher about online practice sites that are available for your use.

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CONTENT OF THE GEOMETRY EOC ASSESSMENT

Up to this point in the guide, you have been learning how to prepare for taking the EOC assessment. Now you will learn about the topics and standards that are assessed in the Geometry EOC assessment and will see some sample items.

- The first part of this section focuses on what will be tested. It also includes sample items that will let you apply what you have learned in your classes and from this guide.
- The second part contains additional items to practice your skills.
- The third part contains a table that shows the standard assessed for each item, the DOK level, the correct answer (key), and a rationale/explanation of the right and wrong answers.
- You can use the sample items to familiarize yourself with the item formats found on the assessment.

All example and sample items contained in this guide are the property of the Georgia Department of Education.

The Geometry EOC assessment will assess the Geometry standards documented at www.georgiastandards.org.

The content of the assessment is organized into four groupings, or domains, of standards for the purpose of providing feedback on student performance.

- A content domain is a reporting category that *broadly* describes and defines the content of the course, as measured by the EOC assessment.
- On the actual test the standards for Geometry are grouped into four domains that follow your classwork: Congruence and Similarity; Circles; Equations and Measurement; and Statistics and Probability.
- Each domain was created by organizing standards that share similar content characteristics.
- The content standards describe the level of understanding each student is expected to achieve. They include the knowledge, concepts, and skills assessed on the EOC assessment, and they are used to plan instruction throughout the course.

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SNAPSHOT OF THE COURSE

This section of the guide is organized into six units that review the material taught within the four domains of the Geometry course. The material is presented by concept rather than by category or standard. In each unit you will find sample items similar to what you will see on the EOC assessment. The next section of the guide contains additional items to practice your skills followed by a table that shows for each item the standard assessed, the DOK level, the correct answer (key), and a rationale/explanation about the key and distractors.

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The more you understand about the concepts in each unit, the greater your chances of getting a good score on the EOC assessment.

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UNIT 1: TRANSFORMATIONS IN THE COORDINATE PLANE

In this unit, students review the definitions of three types of transformations that preserve distance and angle: rotations, reflections, and translations. They investigate how these transformations are applied in the coordinate plane as functions, mapping pre-image points (inputs) to image points (outputs). Using their knowledge of basic geometric figures and special polygons, they apply these transformations to obtain images of given figures. They also specify transformations that can be applied to obtain a given image from a given pre-image, including cases in which the image and pre-image are the same figure.

1.1 Experiment with Transformations in the Plane

MGSE9-12.G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

MGSE9-12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

MGSE9-12.G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

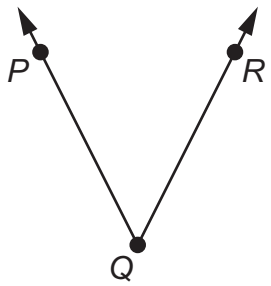
MGSE9-12.G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

MGSE9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

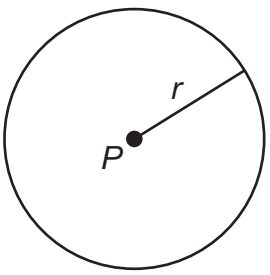
KEY IDEAS

A ***line segment*** is part of a line; it consists of two points and all points between them. An ***angle*** is formed by two rays with a common endpoint. A ***circle*** is the set of all points in a plane that are equidistant from a given point, called the center; the fixed distance is the ***radius***. ***Parallel lines*** are lines in the same plane that do not intersect. ***Perpendicular lines*** are two lines that intersect to form right angles.

Line segment *AB*



Angle *PQR*



Circle *P* with radius *r*

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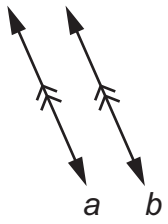
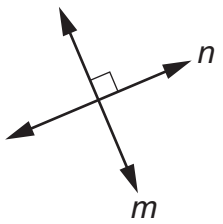
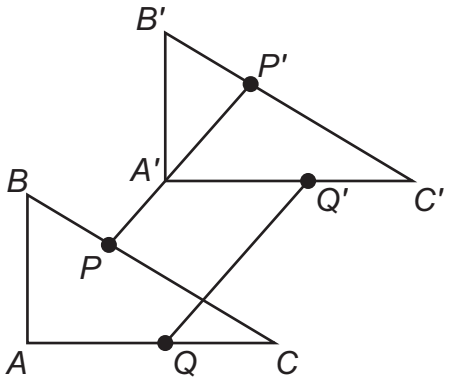


Parallel lines *a* and *b*



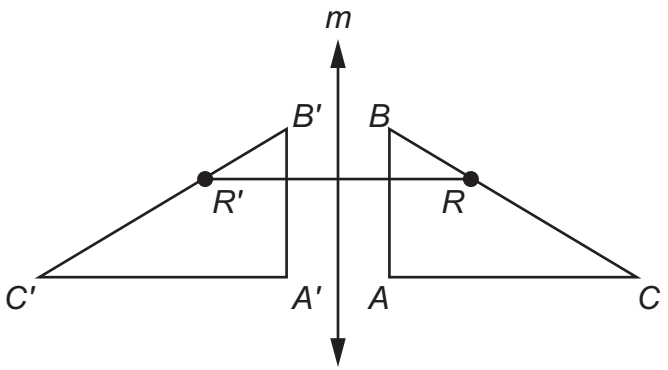
Perpendicular lines *m* and *n*

A ***transformation*** is an operation that maps, or moves, a pre-image onto an ***image***. In each transformation defined below, it is assumed that all points and figures are in one plane. In each case, $\triangle ABC$ is the pre-image and $\triangle A'B'C'$ is the image.



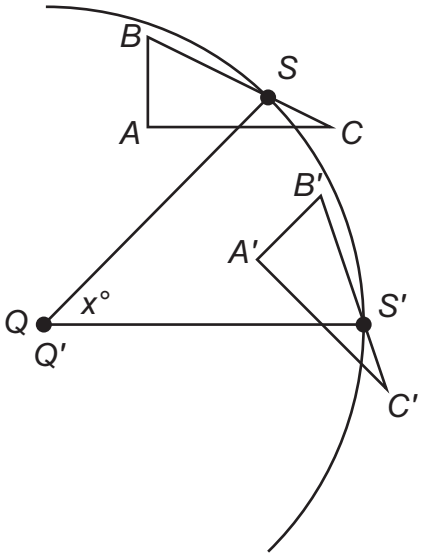
A ***translation*** maps every two points P and Q to points P' and Q' so that the following properties are true:

- $PP' = QQ'$
- $\overline{PP'} \parallel \overline{QQ'}$



A ***reflection*** across a line m maps every point R to R' so that the following properties are true:

- If R is not on m, then m is the perpendicular bisector of $\overline{RR'}$.
- If R is on m, then R and R' are the same point.



A ***rotation*** of $x°$ about a point Q maps every point S to S' so that the following properties are true:

- $SQ = S'Q$ and $m\angle SQS' = x°$.
- Pre-image point Q and image point Q' are the same.

Note: $\overline{QS}$ and $\overline{QS'}$ are radii of a circle with center Q. x is called the ***angle of rotation***.

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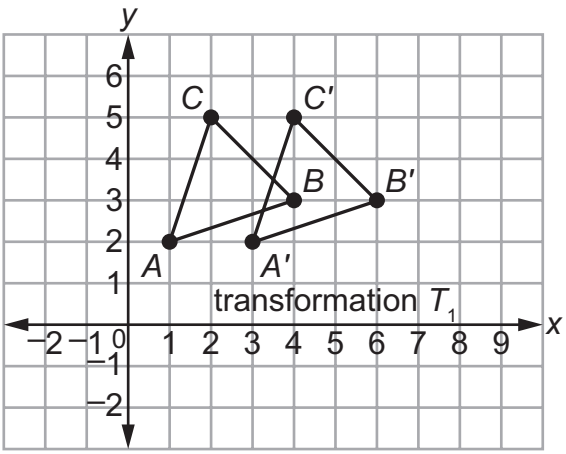
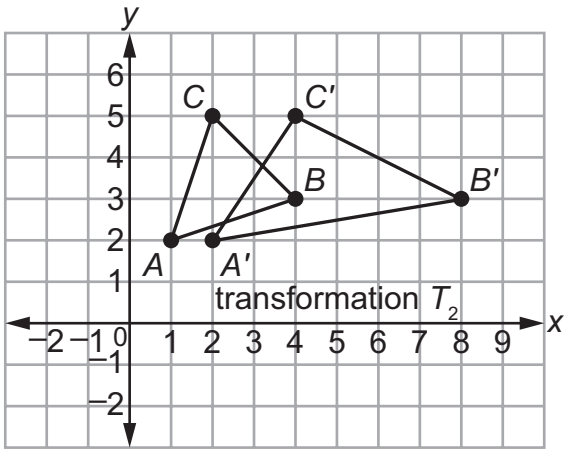
A transformation in a coordinate plane can be described as a function that maps pre-image points (inputs) to image points (outputs). Translations, reflections, and rotations all preserve distance and angle measure because, for each of those transformations, the pre-image and image are ***congruent***. But some types of transformations do not preserve distance and angle measure, because the pre-image and image are not congruent.



T_1: $(x, y) \rightarrow (x + 2, y)$

T_1 translates $\triangle ABC$ to the right 2 units.

T_1 preserves distance and angle measure because $\triangle ABC \cong \triangle A'B'C'$.



T_2: $(x, y) \rightarrow (2x, y)$

T_2 stretches $\triangle ABC$ horizontally by the factor 2.

T_2 preserves neither distance nor angle measure.

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If vertices are not named, then there might be more than one transformation that will accomplish a specified mapping. If vertices are named, then they must be mapped in a way that corresponds to the order in which they are named.

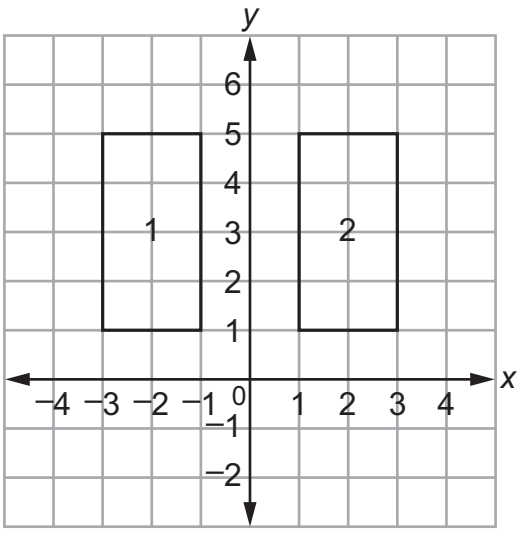
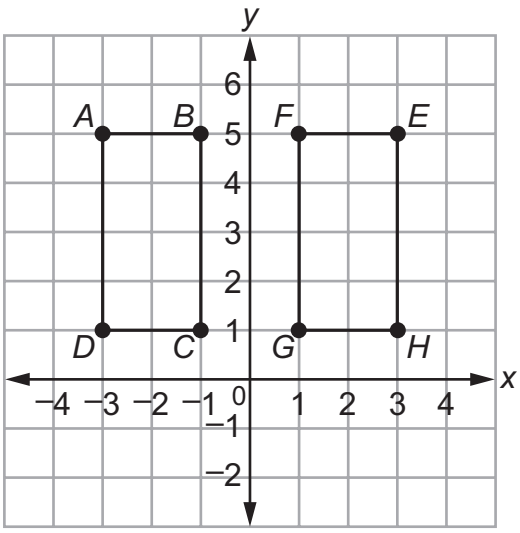


Figure 1 can be mapped to Figure 2 by either of these transformations:

- a reflection across the *y*-axis (the upper-left vertex in Figure 1 is mapped to the upper-right vertex in Figure 2)
- a translation 4 units to the right (the upper-left vertex in Figure 1 is mapped to the upper-left vertex in Figure 2)



ABCD can be mapped to *EFGH* by a reflection across the *y*-axis, but not by a translation.

The mapping of *ABCD* → *EFGH* requires these vertex mappings:
A → *E*, *B* → *F*, *C* → *G*, and *D* → *H*.

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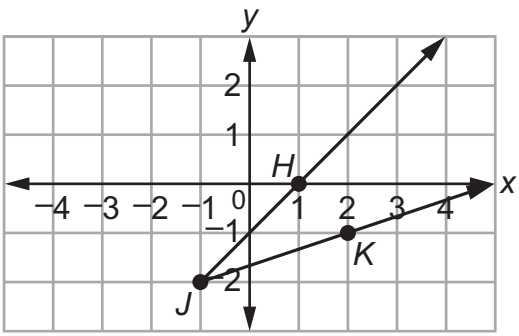
REVIEW EXAMPLES

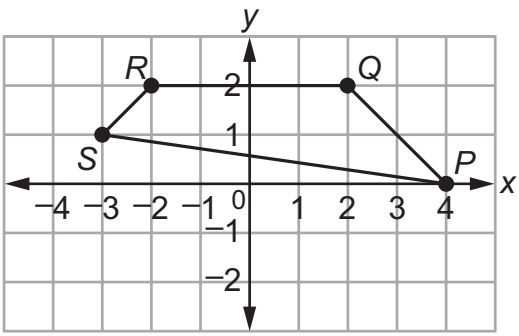
♦ Draw the image of each figure, using the given transformation.

a.



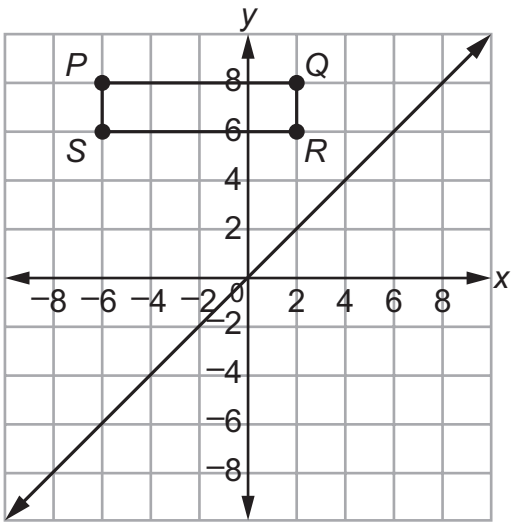
Use the translation $(x, y) \rightarrow (x - 3, y + 1)$.

b.



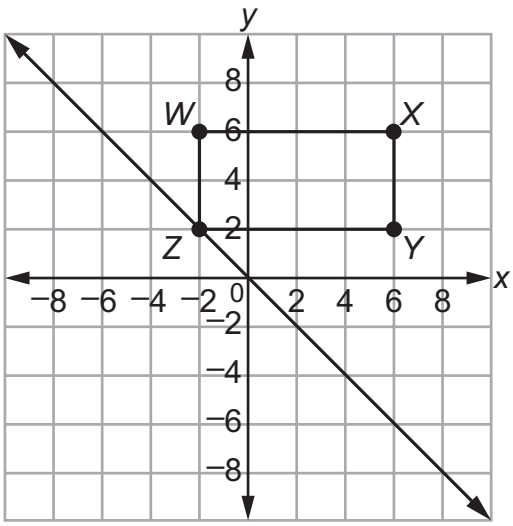
Reflect across the x-axis.

c.



Reflect across the line $y = x$.

d.



Reflect across the line $y = -x$.

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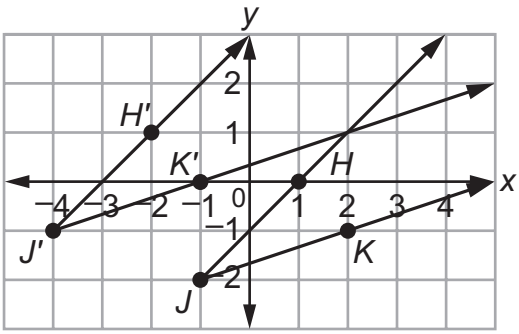
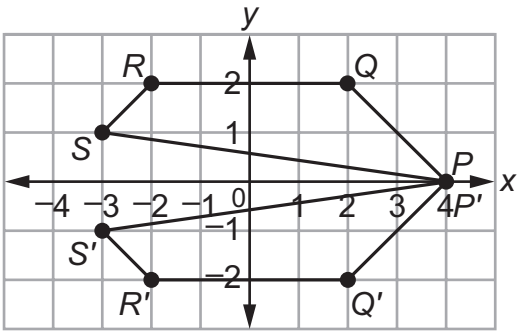
Solution:

a.



Identify the vertex and a point on each side of the angle. Translate each point 3 units left and 1 unit up. The image of given $\angle HJK$ is $\angle H'J'K'$.

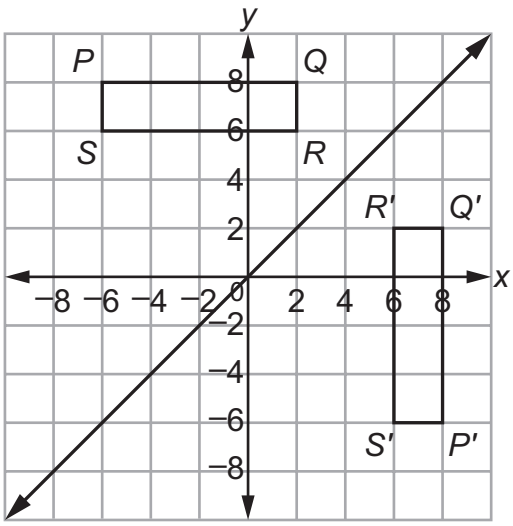
b.



Identify the vertices. The reflection image of each point (x, y) across the x-axis is $(x, -y)$.

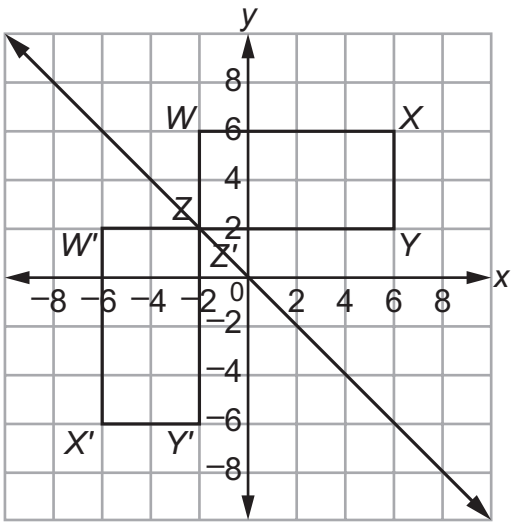
The image of given polygon $PQRS$ is $P'Q'R'S'$, where P and P' are the same.

c.



Identify the vertices. The reflection image of each point (x, y) across the line $y = x$ is (y, x).

d.



Identify the vertices. The reflection image of each point (x, y) across the line $y = -x$ is $(-y, -x)$.

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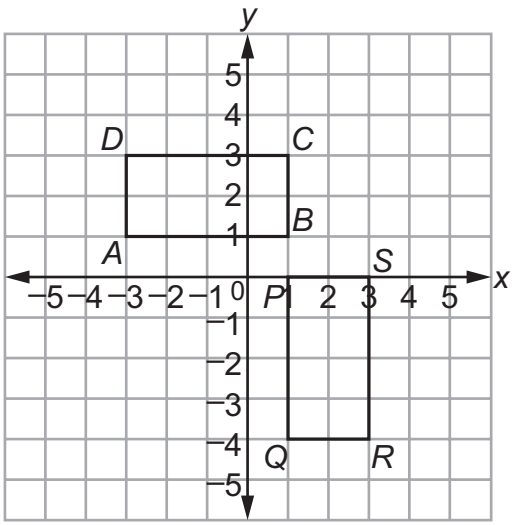
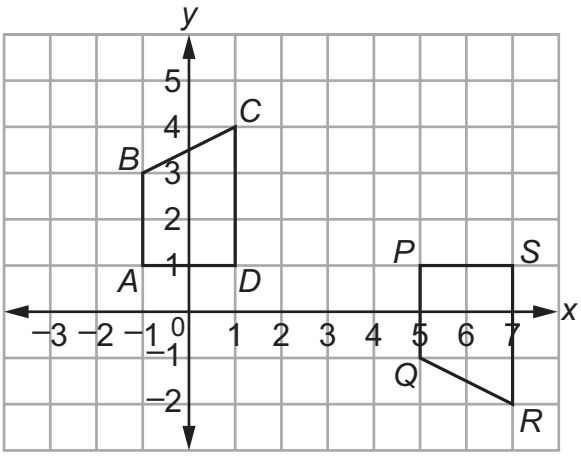
♦ Specify a sequence of transformations that will map *ABCD* to *PQRS* in each case.

a.

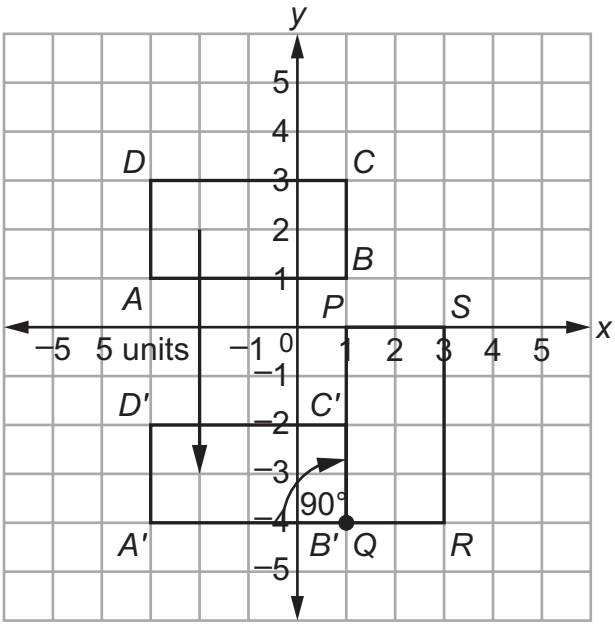


b.



Solution:

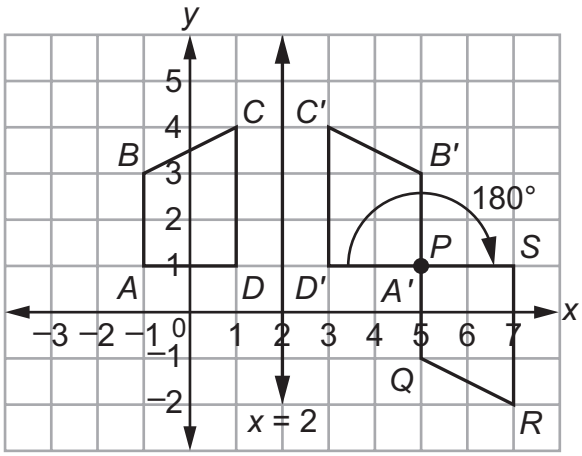
a.



Translate *ABCD* down 5 units to obtain *A′B′C′D′*. Then rotate *A′B′C′D′* clockwise 90° about point *B′* to obtain *PQRS*.

Note that *B′* and *Q* are the same point.

b.



Reflect *ABCD* across the line $x = 2$ to obtain *A′B′C′D′*. Then rotate *A′B′C′D′* 180° about point *A′* to obtain *PQRS.*

Note that *A′* and *P* are the same point.

Note that there are other sequences of transformations that will also work for each case.

Important Tip

✍ A 180° rotation clockwise is equivalent to a 180° rotation counterclockwise.

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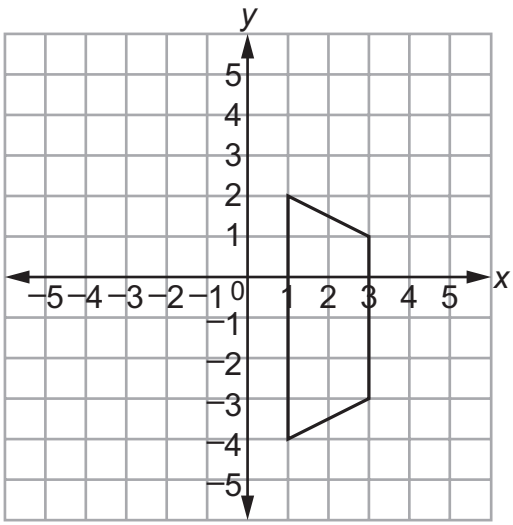
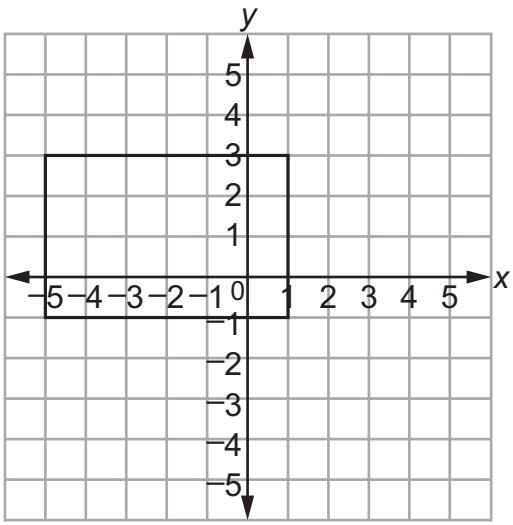
♦ Describe every transformation that maps each given figure to itself.

a.

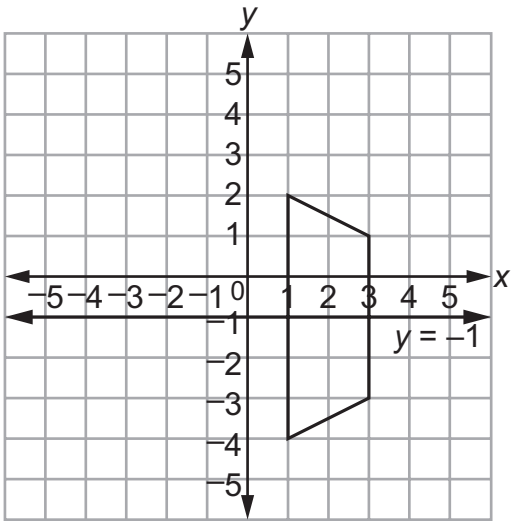


b.



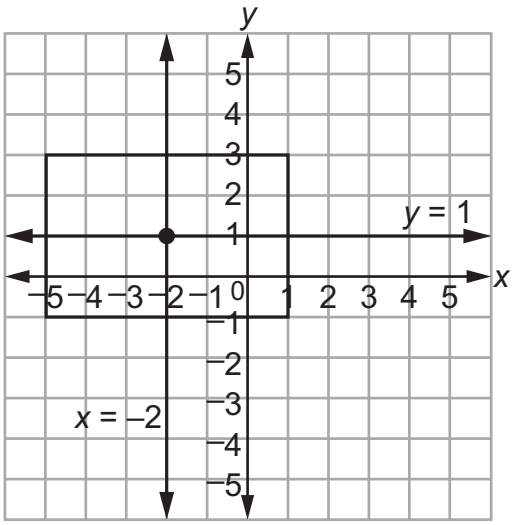
Solution:

a.



There is only one transformation: Reflect the figure across the line $y = -1$.

b.



There are three transformations:

- Reflect across the line $y = 1$
- Reflect across the line $x = -2$
- Rotate 180° about the point (–2, 1)

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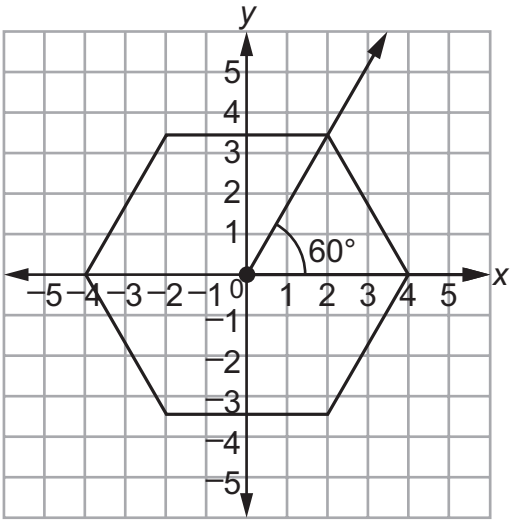
- Describe every transformation that maps this figure to itself: a regular hexagon (6 sides) that is centered about the origin and that has a vertex at (4, 0).

  Solution:

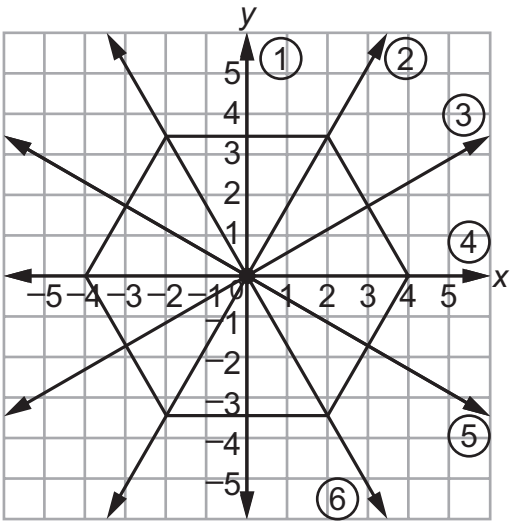
  The angle formed by any two consecutive vertices and the center of the hexagon measures 60° because $\frac{360°}{6} = 60°$. So a rotation about the origin, clockwise or counterclockwise, of 60°, 120°, or any other multiple of 60° maps the hexagon to itself.



If a reflection across a line maps a figure to itself, then that line is called a ***line of symmetry***.

A regular hexagon has 6 lines of symmetry: 3 lines through opposite vertices and 3 lines through midpoints of opposite sides.

A reflection across any of the 6 lines of symmetry maps the hexagon to itself.



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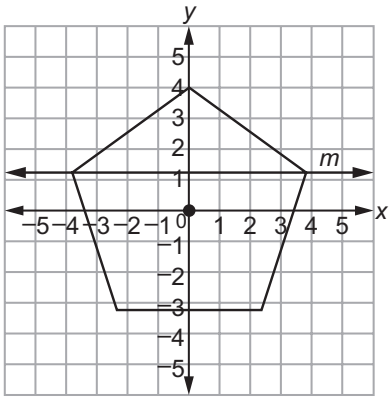
SAMPLE ITEMS

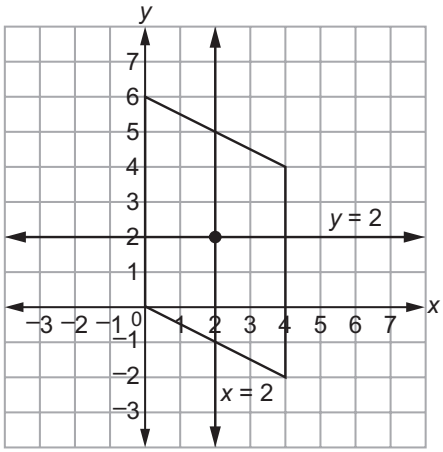
1. A regular pentagon is centered about the origin and has a vertex at (0, 4).



Which transformation maps the pentagon to itself?

A. a reflection across line *m*
B. a reflection across the *x*-axis
C. a clockwise rotation of 100° about the origin
D. a clockwise rotation of 144° about the origin

2. A parallelogram has vertices at (0, 0), (0, 6), (4, 4), and (4, –2).



Which transformation maps the parallelogram to itself?

A. a reflection across the line $x = 2$
B. a reflection across the line $y = 2$
C. a rotation of 180° about the point (2, 2)
D. a rotation of 180° about the point (0, 0)

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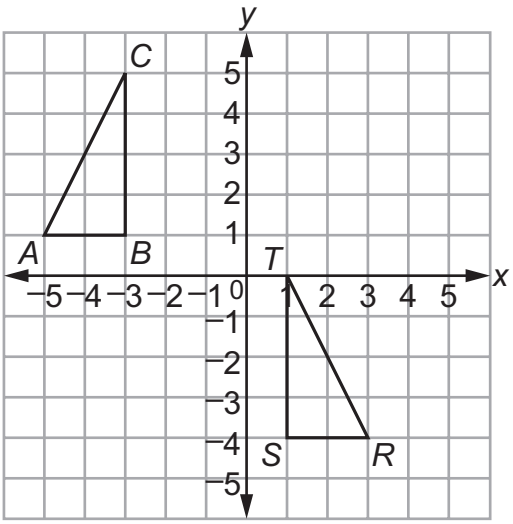
3. Which sequence of transformations maps △*ABC* to △*RST*?



A. Reflect △*ABC* across the line $x = -1$. Then translate the result 1 unit down.

B. Reflect △*ABC* across the line $x = -1$. Then translate the result 5 units down.

C. Translate △*ABC* 6 units to the right. Then rotate the result 90° clockwise about the point (1, 1).

D. Translate △*ABC* 6 units to the right. Then rotate the result 90° counterclockwise about the point (1, 1).

Answers to Unit 1 Sample Items

1. D 2. C 3. B

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UNIT 2: SIMILARITY, CONGRUENCE, AND PROOFS

This unit introduces the concepts of similarity and congruence. The definition of similarity is explored through dilation transformations. The concept of scale factor with respect to dilations allows figures to be enlarged or reduced. Rigid motions lead to the definition of congruence. Once congruence is established, various congruence criteria (e.g., ASA, SSS, SAS) can be explored. Once similarity is established, various similarity criteria (e.g., AA) can be explored. These criteria, along with other postulates and definitions, provide a framework for solving various geometric proofs. In this unit, various geometric figures are constructed. These topics allow students a deeper understanding of formal reasoning, which will be beneficial throughout the remainder of Geometry. Students are asked to prove theorems about parallelograms. Theorems include opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and, conversely, rectangles are parallelograms with congruent diagonals. The method for proving is not specified, so it could be done by using knowledge of congruency and establishing a formalized proof, it could be proven by constructions, or it could be proved algebraically by using the coordinate plane.

2.1 Understand Similarity in Terms of Similarity Transformations

MGSE9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor.

a. The dilation of a line not passing through the center of the dilation results in a parallel line and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter according to the ratio given by the scale factor.

MGSE9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain, using similarity transformations, the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

MGSE9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

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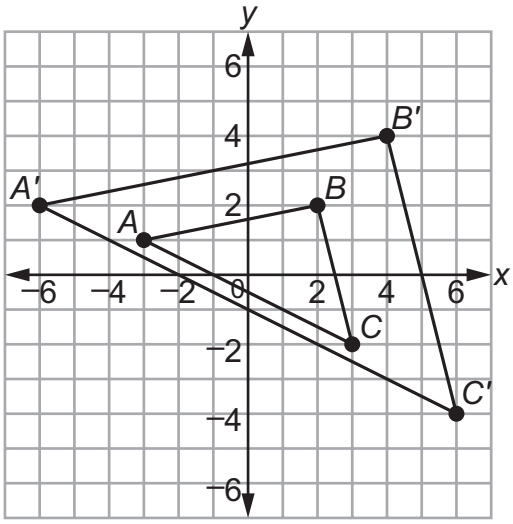
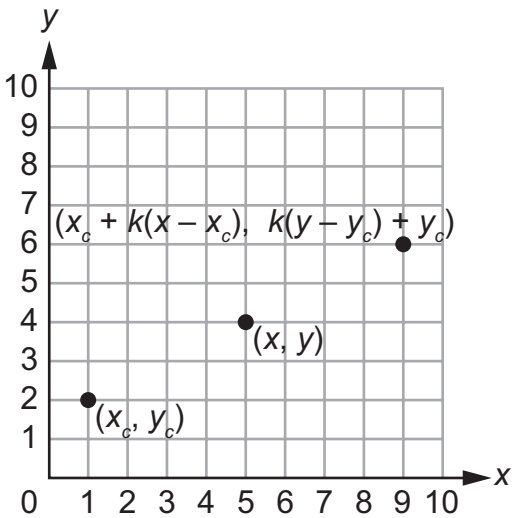
KEY IDEAS

A ***dilation*** is a transformation that changes the size of a figure, but not the shape, based on a ratio given by a ***scale factor*** with respect to a fixed point called the ***center***. When the scale factor is greater than 1, the figure is made larger. When the scale factor is between 0 and 1, the figure is made smaller. When the scale factor is 1, the figure does not change. When the center of dilation is the origin, you can multiply each coordinate of the original figure, or ***pre-image***, by the scale factor to find the coordinates of the dilated figure, or ***image***.

Example: The diagram below shows $\triangle ABC$ dilated about the origin with a scale factor of 2 to create $\triangle A'B'C'$.



When the center of dilation is not the origin, you can use a rule that is derived from shifting the center of dilation, multiplying the shifted coordinates by the scale factor, and then shifting the center of dilation back to its original location. For a point (x, y) and a center of dilation (x_c, y_c), the rule for finding the coordinates of the dilated point with a scale factor of k is $(x_c + k(x - x_c), k(y - y_c) + y_c)$.



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When a figure is transformed under a dilation, the ***corresponding angles*** of the pre-image and the image have equal measures.

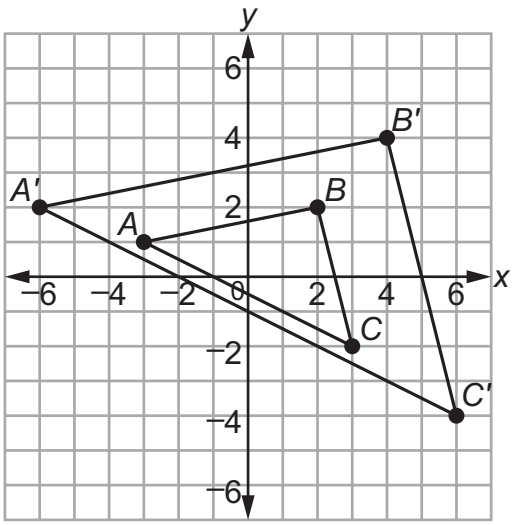
For $\triangle ABC$ and $\triangle A'B'C'$ below, $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\angle C \cong \angle C'$.

When a figure is transformed under a dilation, the ***corresponding sides*** of the pre-image and the image are proportional.

For $\triangle ABC$ and $\triangle A'B'C'$ on the coordinate grid below, $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}$.

So when a figure is under a dilation transformation, the pre-image and the image are ***similar***.

For $\triangle ABC$ and $\triangle A'B'C'$ below, $\triangle ABC \sim \triangle A'B'C'$.



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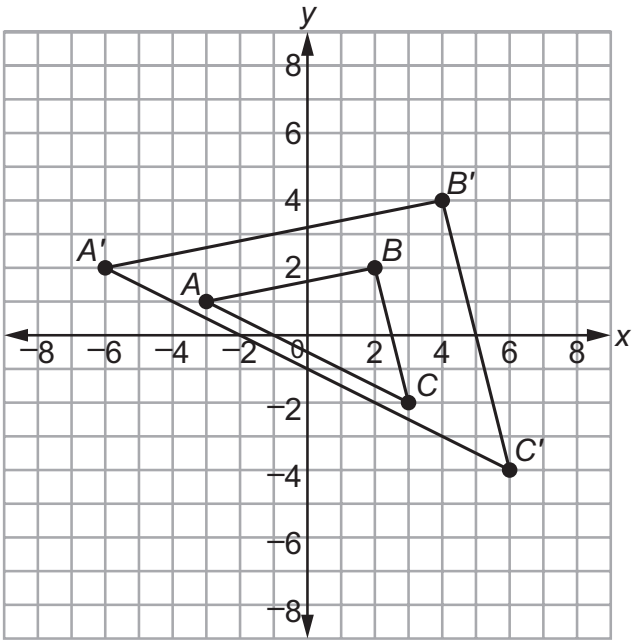
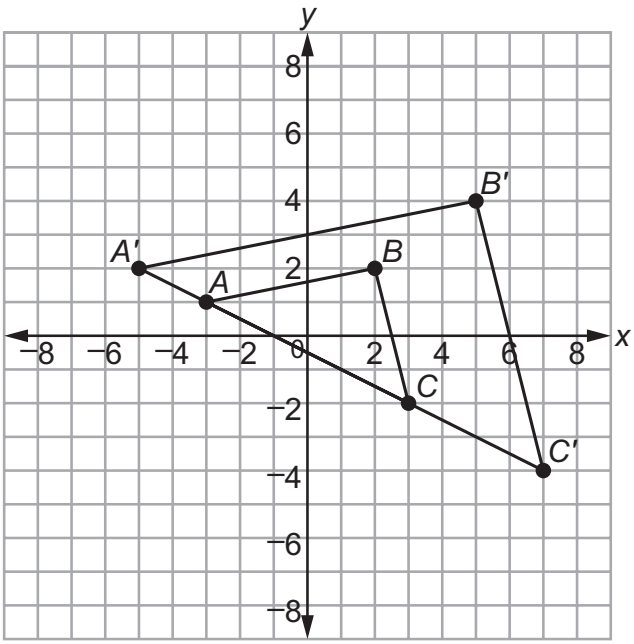
When a figure is dilated, a segment of the pre-image that does not pass through the center of dilation is parallel to its image. In the figure below, $\overline{AC} \parallel \overline{A'C'}$ since neither segment passes through the center of dilation. The same is true about $\overline{AB}$ and $\overline{A'B'}$ as well as $\overline{BC}$ and $\overline{B'C'}$.



center of dilation (0,0)
scale factor = 2

When the segment of a figure does pass through the center of dilation, the segment of the pre-image and image are on the same line. In the figure below, the center of dilation is on $\overline{AC}$, so $\overline{AC}$ and $\overline{A'C'}$ are on the same line.



center of dilation (–1,0)
scale factor = 2

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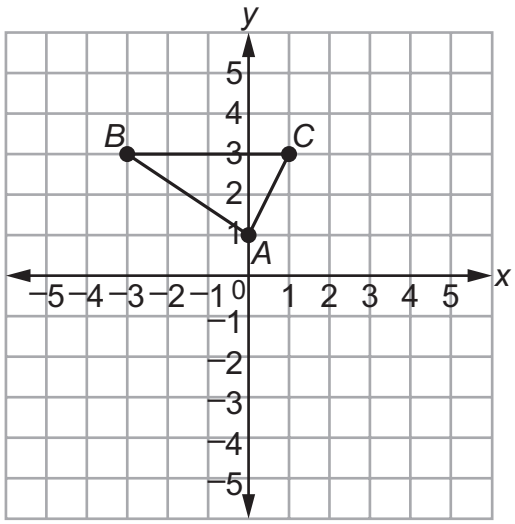
REVIEW EXAMPLES

♦ Draw a triangle with vertices at $A(0, 1)$, $B(-3, 3)$, and $C(1, 3)$. Dilate the triangle using a scale factor of 1.5 and a center of (0, 0). Sketch and name the dilated triangle $A'B'C'$.

Solution:

Plot points $A(0, 1)$, $B(-3, 3)$, and $C(1, 3)$. Draw $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$.



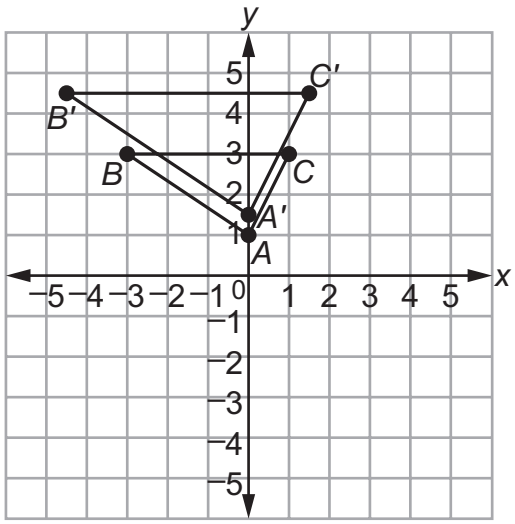
The center of dilation is the origin, so to find the coordinates of the image, multiply the coordinates of the pre-image by the scale factor, 1.5.

Point A': $(1.5 \cdot 0, 1.5 \cdot 1) = (0, 1.5)$

Point B': $(1.5 \cdot (-3), 1.5 \cdot 3) = (-4.5, 4.5)$

Point C': $(1.5 \cdot 1, 1.5 \cdot 3) = (1.5, 4.5)$

Plot points $A'(0, 1.5)$, $B'(-4.5, 4.5)$, and $C'(1.5, 4.5)$. Draw $\overline{A'B'}$, $\overline{A'C'}$, and $\overline{B'C'}$.



***Note*:** Since no part of the pre-image passes through the center of dilation, $\overline{BC} \parallel \overline{B'C'}$, $\overline{AB} \parallel \overline{A'B'}$, and $\overline{AC} \parallel \overline{A'C'}$.

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- Line segment CD is 5 inches long. If line segment CD is dilated to form line segment $C'D'$ with a scale factor of 0.6, what is the length of line segment $C'D'$?

  Solution:

  The ratio of the length of the image and the pre-image is equal to the scale factor.

$$\frac{C'D'}{CD} = 0.6$$

  Substitute 5 for CD.

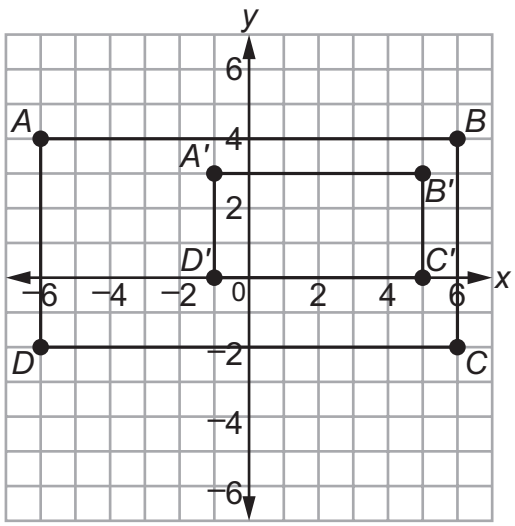
$$\frac{C'D'}{5} = 0.6$$

  Solve for $C'D'$.

$$C'D' = 0.6 \cdot 5$$
$$C'D' = 3$$

  The length of line segment $C'D'$ is 3 inches.

- Figure $A'B'C'D'$ is a dilation of figure $ABCD$.



  a. Determine the center of dilation.
  b. Determine the scale factor of the dilation.
  c. What is the relationship between the sides of the pre-image and the corresponding sides of the image?

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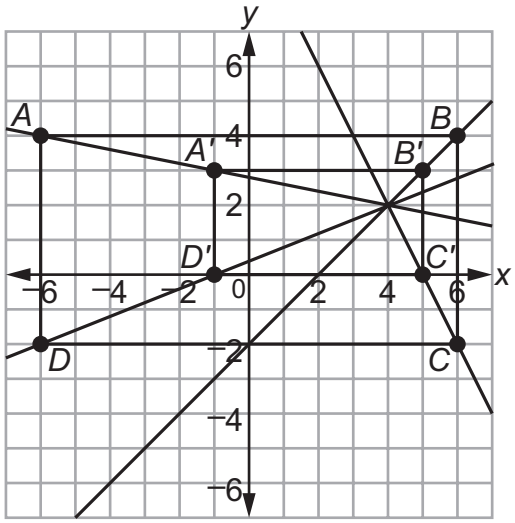
Solution:

a. To find the center of dilation, draw lines connecting each corresponding vertex from the pre-image to the image. The lines meet at the center of dilation.



The center of dilation is (4, 2).

b. Find the ratios of the lengths of the corresponding sides.

$$\frac{A'B'}{AB} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{B'C'}{BC} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{C'D'}{CD} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{A'D'}{AD} = \frac{3}{6} = \frac{1}{2}$$

The ratio for each pair of corresponding sides is $\frac{1}{2}$, so the scale factor is $\frac{1}{2}$.

c. Each side of the image is parallel to the corresponding side of its pre-image and is $\frac{1}{2}$ the length.

Note: Lines connecting corresponding points pass through the center of dilation.

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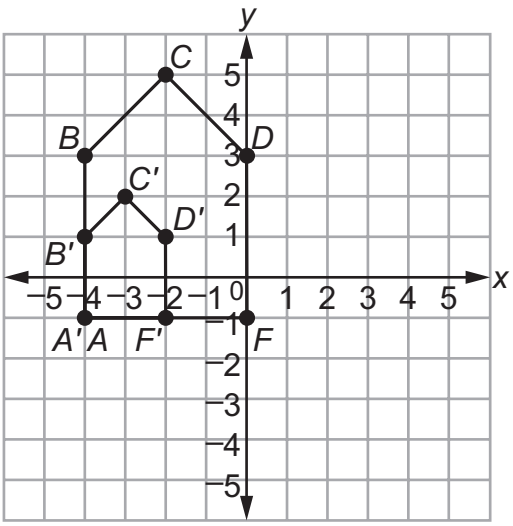
SAMPLE ITEMS

1. Figure *A′B′C′D′F′* is a dilation of figure *ABCDF* by a scale factor of $\frac{1}{2}$. The dilation is centered at (–4, –1).



Which statement is true?

A. $\frac{AB}{A'B'} = \frac{B'C'}{BC}$

B. $\frac{AB}{A'B'} = \frac{BC}{B'C'}$

C. $\frac{AB}{A'B'} = \frac{BC}{D'F'}$

D. $\frac{AB}{A'B'} = \frac{D'F'}{BC}$

2. Which transformation results in a figure that is similar to the original figure but has a greater area?

A. a dilation of triangle *QRS* by a scale factor of 0.25
B. a dilation of triangle *QRS* by a scale factor of 0.5
C. a dilation of triangle *QRS* by a scale factor of 1
D. a dilation of triangle *QRS* by a scale factor of 2

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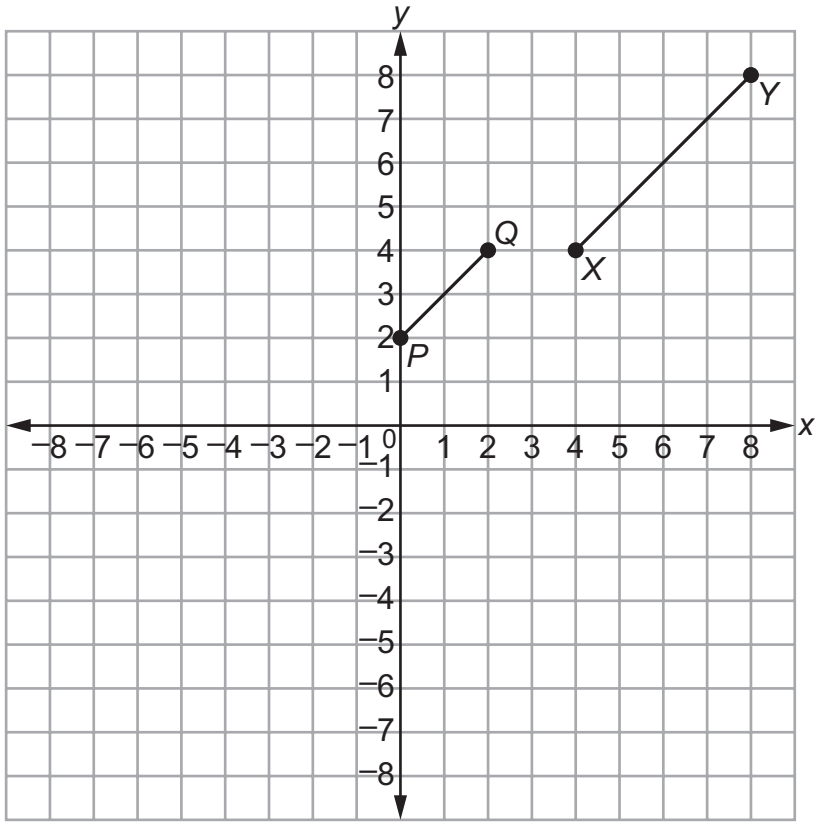
3. In the coordinate plane, segment $\overline{PQ}$ is the result of a dilation of segment $\overline{XY}$ by a scale factor of $\frac{1}{2}$.



Which point is the center of dilation?

A. (–4, 0)
B. (0, –4)
C. (0, 4)
D. (4, 0)

Note: Draw lines connecting corresponding points to determine the point of intersection (center of dilation).

Answers to Unit 2.1 Sample Items

1. B 2. D 3. A

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2.2 Prove Theorems Involving Similarity

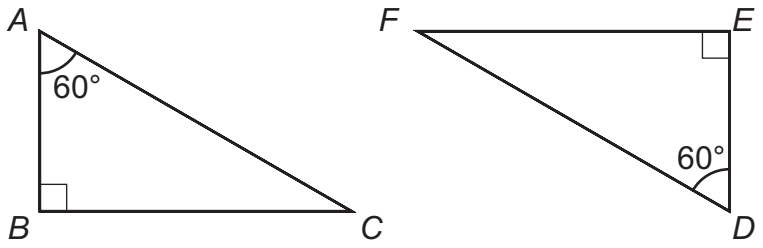
MGSE9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, (and its converse); the Pythagorean Theorem using triangle similarity.

MGSE9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

KEY IDEAS

When proving that two triangles are similar, it is sufficient to show that two pairs of corresponding angles of the triangles are congruent. This is called ***Angle-Angle* (*AA*) *Similarity***.

Example: The triangles below are similar by AA Similarity because each triangle has a 60° angle and a 90° angle. The similarity statement is written as $\triangle ABC \sim \triangle DEF$, and the order in which the vertices are written indicates which angles/sides correspond to each other.



When a triangle is dilated, the pre-image and the image are similar triangles. There are three cases of triangles being dilated:

- The image is congruent to the pre-image (scale factor of 1).
- The image is smaller than the pre-image (scale factor between 0 and 1).
- The image is larger than the pre-image (scale factor greater than 1).

When two triangles are **similar**, all corresponding pairs of angles are congruent.

When two triangles are **similar**, all corresponding pairs of sides are proportional.

When two triangles are **congruent**, the triangles are also similar.

A ***two-column proof*** is a series of statements and reasons often displayed in a chart that works from given information to the statement that needs to be proven. Reasons can be given information, can be based on definitions, or can be based on postulates or theorems.

A ***paragraph proof*** also uses a series of statements and reasons that work from given information to the statement that needs to be proven, but the information is presented as running text in paragraph form.

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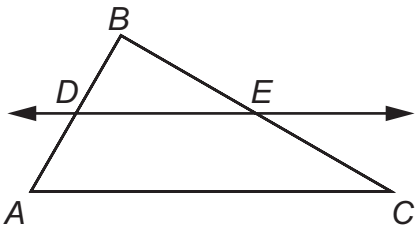
REVIEW EXAMPLES

♦ In the triangle shown, $\overline{AC} \parallel \overleftrightarrow{DE}$.



Prove that $\overleftrightarrow{DE}$ divides $\overline{AB}$ and $\overline{CB}$ proportionally.

Solution:

Step	Statement	Reason
1	$\overline{AC} \parallel \overleftrightarrow{DE}$	Given
2	$\angle BDE \cong \angle BAC$	If two parallel lines are cut by a transversal, then corresponding angles are congruent.
3	$\angle DBE \cong \angle ABC$	Reflexive Property of Congruence because they are the same angle
4	$\triangle DBE \sim \triangle ABC$	Angle-Angle (AA) Similarity
5	$\frac{BA}{BD} = \frac{BC}{BE}$	Corresponding sides of similar triangles are proportional.
6	$BD + DA = BA$ $BE + EC = BC$	Segment Addition Postulate
7	$\frac{BD + DA}{BD} = \frac{BE + EC}{BE}$	Substitution
8	$\frac{BD}{BD} + \frac{DA}{BD} = \frac{BE}{BE} + \frac{EC}{BE}$	Rewrite each fraction as a sum of two fractions.
9	$1 + \frac{DA}{BD} = 1 + \frac{EC}{BE}$	Simplify.
10	$\frac{DA}{BD} = \frac{EC}{BE}$	Subtraction Property of Equality
11	$\overleftrightarrow{DE}$ divides $\overline{AB}$ and $\overline{CB}$ proportionally.	Definition of proportionality

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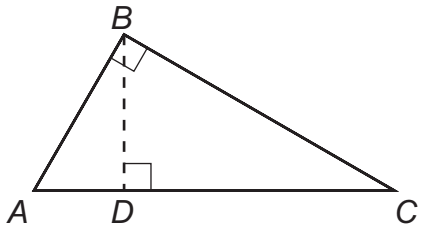
♦ Gale is trying to prove the Pythagorean Theorem using similar triangles. Part of her proof is shown below.



Step	Statement	Reason
1	$\angle ABC \cong \angle BDC$	All right angles are congruent.
2	$\angle ACB \cong \angle BCD$	Reflexive Property of Congruence
3	$\triangle ABC \sim \triangle BDC$	Angle-Angle (AA) Similarity
4	$\frac{BC}{DC} = \frac{AC}{BC}$	Corresponding sides of similar triangles are proportional.
5	$BC^2 = AC \bullet DC$	In a proportion, the product of the means equals the product of the extremes.
6	$\angle ABC \cong \angle ADB$	All right angles are congruent.
7	$\angle BAC \cong \angle DAB$	Reflexive Property of Congruence
8	$\triangle ABC \sim \triangle ADB$	Angle-Angle (AA) Similarity
9	$\frac{AB}{AD} = \frac{AC}{AB}$	Corresponding sides of similar triangles are proportional.
10	$AB^2 = AC \bullet AD$	In a proportion, the product of the means equals the product of the extremes.

What should Gale do to finish her proof?

Solution:

Step	Statement	Reason
11	$AB^2 + BC^2 = AC \bullet AD + AC \bullet DC$	Addition Property of Equality
12	$AB^2 + BC^2 = AC(AD + DC)$	Distributive Property
13	$AC = AD + DC$	Segment Addition Postulate
14	$AB^2 + BC^2 = AC \bullet AC$	Substitution
15	$AB^2 + BC^2 = AC^2$	Definition of exponent

$AB^2 + BC^2 = AC^2$ is a statement of the Pythagorean Theorem, so Gale's proof is complete.

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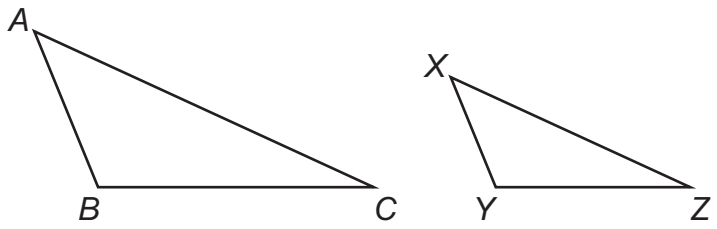
SAMPLE ITEMS

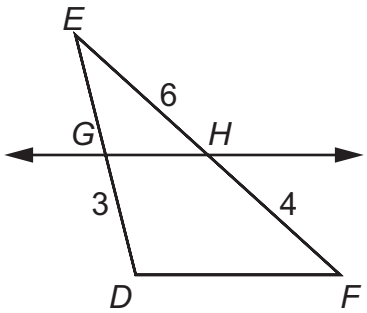
1. In the triangles shown, $\triangle ABC$ is dilated by a factor of $\frac{2}{3}$ to form $\triangle XYZ$.



Given that $m\angle A = 50°$ and $m\angle B = 100°$, what is $m\angle Z$?

A. 15°
B. 25°
C. 30°
D. 50°

2. In the triangle shown, $\overleftrightarrow{GH} \parallel \overline{DF}$.



What is the length of $\overline{GE}$?

A. 2.0
B. 4.5
C. 7.5
D. 8.0

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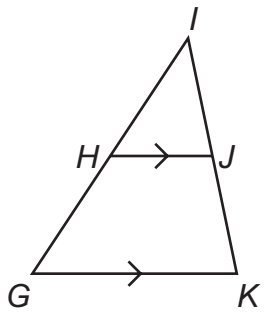
3. Use this triangle to answer the question.



This is a proof of the statement "If a line is parallel to one side of a triangle and intersects the other two sides at distinct points, then it separates these sides into segments of proportional lengths."

Step	Statement	Reason
1	$\overline{GK}$ is parallel to $\overline{HJ}$.	Given
2	$\angle IGK \cong \angle IHJ$ $\angle IKG \cong \angle IJH$	**?**
3	$\triangle GIK \sim \triangle HIJ$	AA Similarity
4	$\frac{IG}{IH} = \frac{IK}{IJ}$	Corresponding sides of similar triangles are proportional.
5	$\frac{HG + IH}{IH} = \frac{JK + IJ}{IJ}$	Segment Addition Postulate
6	$\frac{HG}{IH} = \frac{JK}{IJ}$	Subtraction Property of Equality

Which reason justifies Step 2?

A. Alternate interior angles are congruent.
B. Alternate exterior angles are congruent.
C. Corresponding angles are congruent.
D. Vertical angles are congruent.

Answers to Unit 2.2 Sample Items

1. C 2. B 3. C

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2.3 Understand Congruence in Terms of Rigid Motions

MGSE9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

MGSE9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

MGSE9-12.G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (Extend to include HL and AAS.)

KEY IDEAS

A ***rigid motion*** is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations (in any order). This transformation leaves the size and shape of the original figure unchanged.

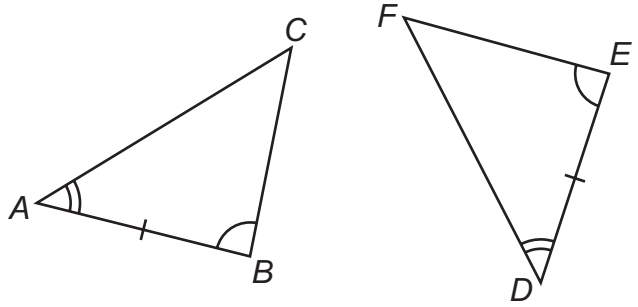
Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations). ***Congruent figures*** have the same corresponding side lengths and the same corresponding angle measures as each other.

Two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent. This is sometimes referred to as ***CPCTC***, which means **C**orresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent.

When given two congruent triangles, you can use a series of translations, reflections, and rotations to show the triangles are congruent.

You can use ***ASA (Angle-Side-Angle)*** to show two triangles are congruent. If two angles and the included side of a triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

$\triangle ABC \cong \triangle DEF$ by ASA.



You can use ***SSS (Side-Side-Side)*** to show two triangles are congruent. If three sides of a triangle are congruent to three sides of another triangle, then the triangles are congruent.

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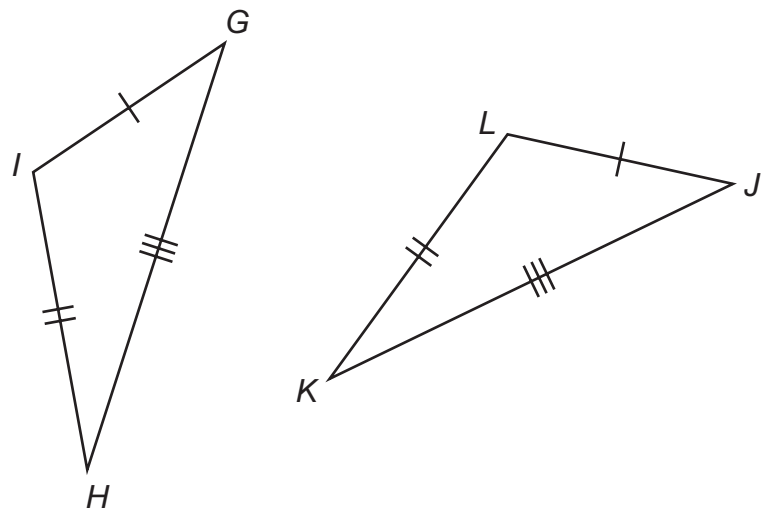
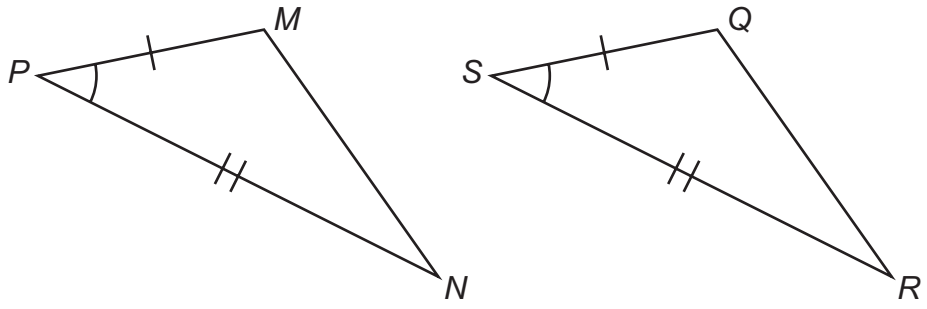
$\triangle GIH \cong \triangle JLK$ by SSS.



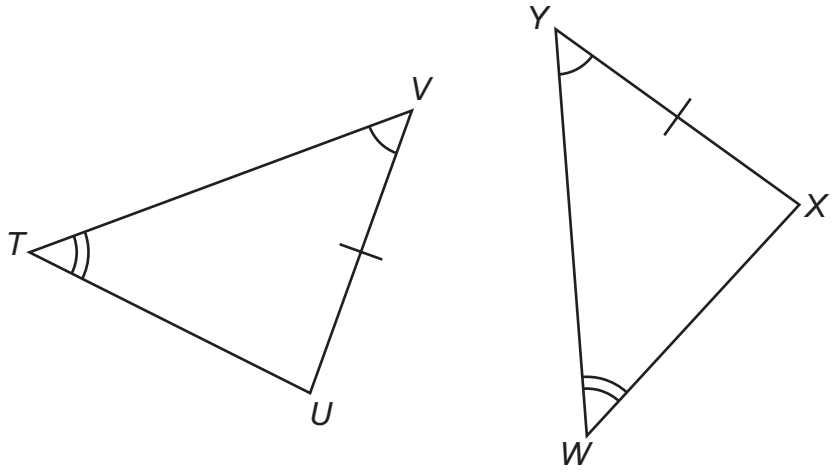
You can use ***SAS (Side-Angle-Side)*** to show two triangles are congruent. If two sides and the included angle of a triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

$\triangle MPN \cong \triangle QSR$ by SAS.



You can use ***AAS (Angle-Angle-Side)*** to show two triangles are congruent. If two angles and a non-included side of a triangle are congruent to two angles and the corresponding non-included side of another triangle, then the triangles are congruent.

$\triangle VTU \cong \triangle YWX$ by AAS.



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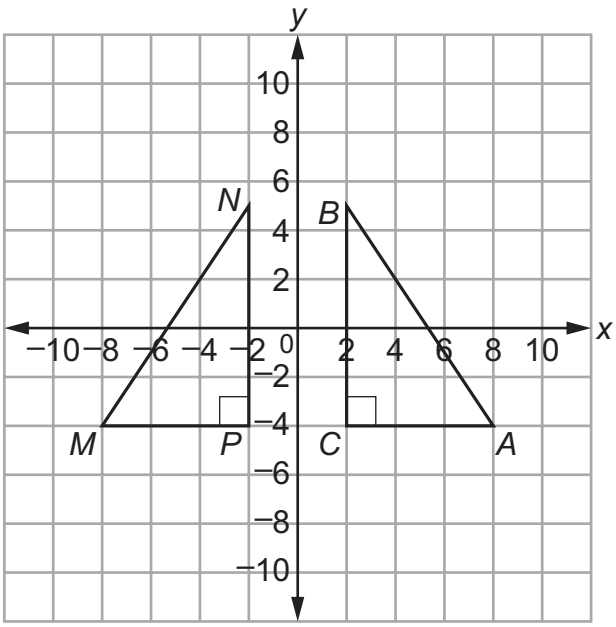
Important Tips

- If two sides and a ***non-included angle*** of one triangle are congruent to two sides and a non-included angle of a second triangle, the triangles are not necessarily congruent. Therefore, there is no way to show triangle congruency by Side-Side-Angle (SSA).
- If two triangles have all three angles congruent to each other, the triangles are similar, but not necessarily congruent. Thus, you can show similarity by Angle-Angle-Angle (AAA), but you cannot show congruence by AAA.

REVIEW EXAMPLES

- Is $\triangle ABC$ congruent to $\triangle MNP$? Explain.



(scale unit = 2)

Solution:

$\overline{AC}$ corresponds to $\overline{MP}$. Both segments are 6 units long. $\overline{BC}$ corresponds to $\overline{NP}$. Both segments are 9 units long. Angle C (the included angle of $\overline{AC}$ and $\overline{BC}$) corresponds to angle P (the included angle of $\overline{MP}$ and $\overline{NP}$). Both angles measure 90°. Because two sides and an included angle are congruent, the triangles are congruent by SAS.

Or, $\triangle ABC$ is a reflection of $\triangle MNP$ over the y-axis. This means that all of the corresponding sides and corresponding angles are congruent, so the triangles are congruent. (Reflections preserve angle measurement and lengths; therefore, corresponding angles and sides are congruent.)

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♦ Rectangle *WXYZ* has coordinates *W*(1, 2), *X*(3, 2), *Y*(3, –3), and *Z*(1, –3).

a. Graph the image of rectangle *WXYZ* after a rotation of 90° clockwise about the origin. Label the image *W′X′Y′Z′*.

b. Translate rectangle *W′X′Y′Z′* 2 units left and 3 units up. Label the image *W″X″Y″Z″*.

c. Is rectangle *WXYZ* congruent to rectangle *W″X″Y″Z″* ? Explain.

Solution:

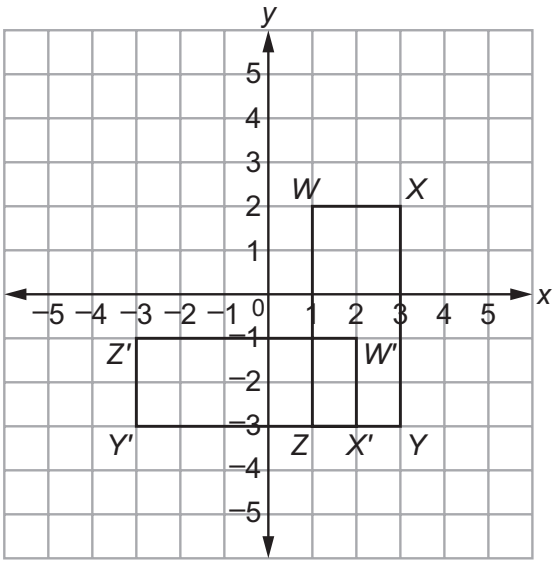
a. For a 90° clockwise rotation about the origin, use the rule $(x, y) \rightarrow (y, -x)$.

$W(1, 2) \rightarrow W'(2, -1)$

$X(3, 2) \rightarrow X'(2, -3)$

$Y(3, -3) \rightarrow Y'(-3, -3)$

$Z(1, -3) \rightarrow Z'(-3, -1)$



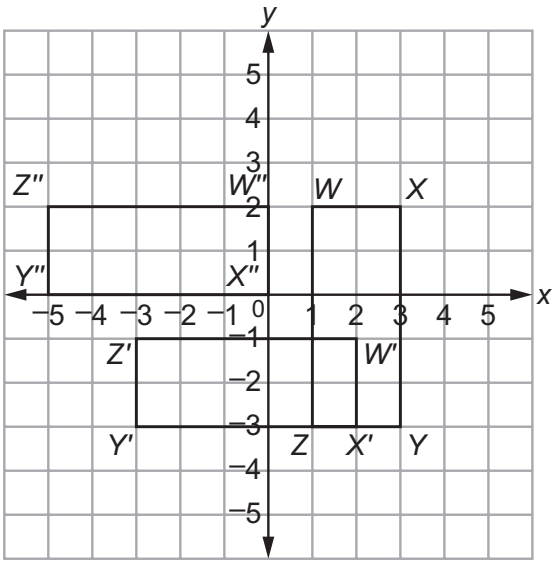
b. To translate rectangle *W′X′Y′Z′* 2 units left and 3 units up, use the rule $(x, y) \rightarrow (x - 2, y + 3)$.

$W'(2, -1) \rightarrow W''(0, 2)$

$X'(2, -3) \rightarrow X''(0, 0)$

$Y'(-3, -3) \rightarrow Y''(-5, 0)$

$Z'(-3, -1) \rightarrow Z''(-5, 2)$



c. Rectangle *W″X″Y″Z″* is the result of a rotation and a translation of rectangle *WXYZ*. These are both rigid transformations, so the shape and the size of the original figure are unchanged. All of the corresponding parts of *WXYZ* and *W″X″Y″Z″* are congruent, so *WXYZ* and *W″X″Y″Z″* are congruent.

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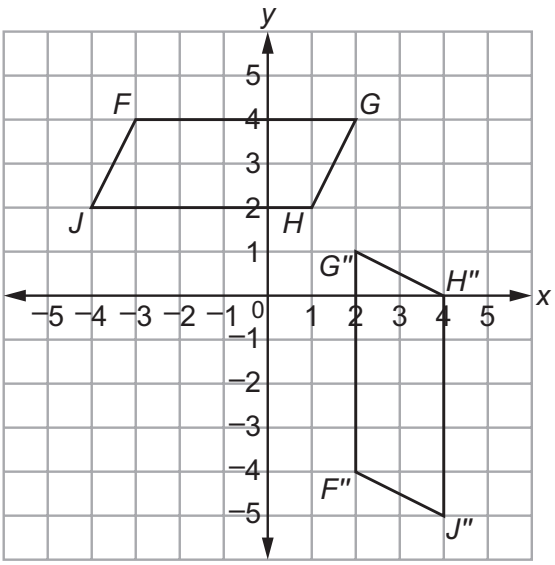
SAMPLE ITEMS

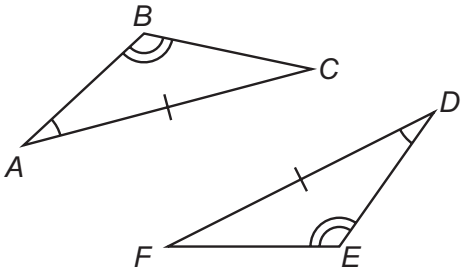
1. Parallelogram *FGHJ* was translated 3 units down to form parallelogram *F′G′H′J′*. Parallelogram *F′G′H′J′* was then rotated 90° counterclockwise about point *G′* to obtain parallelogram *F″G″H″J″*.



Which statement is true about parallelogram *FGHJ* and parallelogram *F″G″H″J″*?

A. The figures are both similar and congruent.
B. The figures are neither similar nor congruent.
C. The figures are similar but not congruent.
D. The figures are congruent but not similar.

2. Consider the triangles shown.



Which can be used to prove the triangles are congruent?

A. SSS
B. ASA
C. SAS
D. AAS

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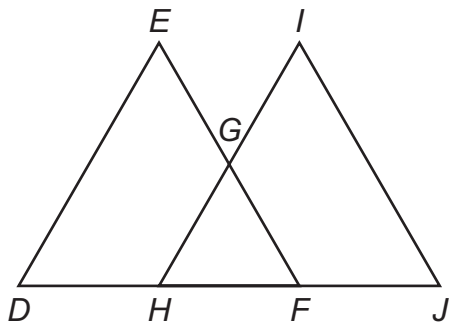
3. In this diagram, $\overline{DE} \cong \overline{JI}$ and $\angle D \cong \angle J$.



Which additional information is sufficient to prove that $\triangle DEF$ is congruent to $\triangle JIH$?

A. $\overline{ED} \cong \overline{IH}$

B. $\overline{DH} \cong \overline{JF}$

C. $\overline{HG} \cong \overline{GI}$

D. $\overline{HF} \cong \overline{JF}$

Answers to Unit 2.3 Sample Items

1. A 2. D 3. B

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2.4 Prove Geometric Theorems

MGSE9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

MGSE9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

MGSE9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

KEY IDEAS

A ***two-column proof*** is a series of statements and reasons often displayed in a chart that works from given information to the statement that needs to be proven. Reasons can be given information, can be based on definitions, or can be based on postulates or theorems.

A ***paragraph proof*** also uses a series of statements and reasons that work from given information to the statement that needs to be proven, but the information is presented as running text in paragraph form.

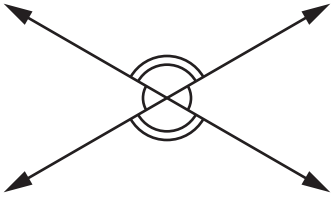
It is important to plan a geometric proof logically. Think through what needs to be proven and decide how to get to that statement from the given information. Often a diagram or a flow chart will help organize your thoughts.

An ***auxiliary line*** is a line drawn in a diagram that makes other figures, such as congruent triangles or angles formed by a transversal. Many times, an auxiliary line is needed to help complete a proof.

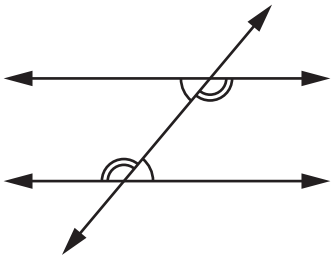
Once a theorem in geometry has been proven, that theorem can be used as a reason in future proofs.

Some important key ideas about lines and angles include the following:

- ***Vertical Angle Theorem***: Vertical angles are congruent.



- ***Alternate Interior Angles Theorem***: If two parallel lines are cut by a transversal, then alternate interior angles formed by the transversal are congruent.



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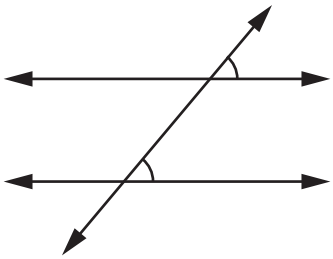
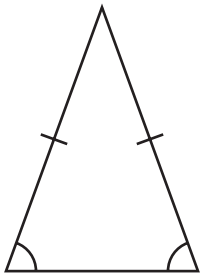
- ***Corresponding Angles Postulate***: If two parallel lines are cut by a transversal, then corresponding angles formed by the transversal are congruent.



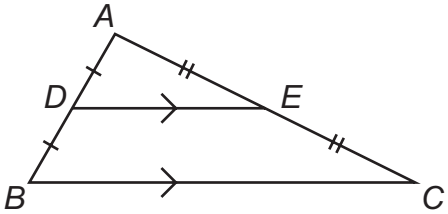
- Points on a perpendicular bisector of a line segment are equidistant from both of the segment's endpoints.

Some important key ideas about triangles include the following:

- ***Triangle Angle-Sum Theorem***: The sum of the measures of the angles of a triangle is 180°.
- ***Isosceles Triangle Theorem***: If two sides of a triangle are congruent, then the angles opposite those sides are also congruent.



- ***Triangle Midsegment Theorem***: If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and half its length.



$$m\overline{DE} = \frac{1}{2}m\overline{BC}$$

- ***Points of Concurrency***: The point where three or more lines intersect. There are 4 points of concurrency: incenter, centroid, orthocenter, and circumcenter.

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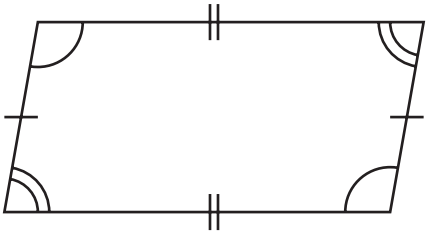
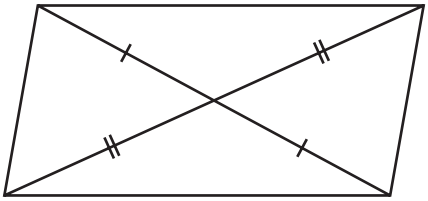
Some important key ideas about parallelograms include the following:

- Opposite sides are congruent and opposite angles are congruent.



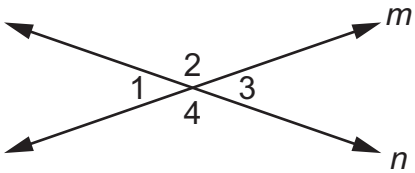
- The diagonals of a parallelogram bisect each other.



- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- A rectangle is a parallelogram with congruent diagonals.

REVIEW EXAMPLES

♦ In this diagram, line *m* intersects line *n*.



Write a two-column proof to show that the vertical angles ∠1 and ∠3 are congruent.

Solution:

Construct a proof using intersecting lines.

Step	Statement	Reason
1	Line *m* intersects line *n.*	Given
2	∠1 and ∠2 form a linear pair. ∠2 and ∠3 form a linear pair.	Definition of a linear pair
3	$m\angle 1 + m\angle 2 = 180°$ $m\angle 2 + m\angle 3 = 180°$	Angles that form a linear pair have measures that sum to 180°.
4	$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	Substitution
5	$m\angle 1 = m\angle 3$	Subtraction Property of Equality
6	$\angle 1 \cong \angle 3$	Definition of congruent angles

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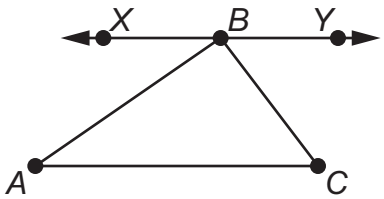
- In this diagram, $\overleftrightarrow{XY}$ is parallel to $\overline{AC}$, and point B lies on $\overleftrightarrow{XY}$.



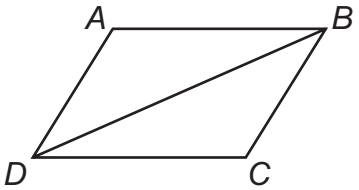
Write a paragraph to prove that the sum of the angles in a triangle is 180°.

Solution:

$\overline{AC}$ and $\overleftrightarrow{XY}$ are parallel, so $\overline{AB}$ is a transversal. The alternate interior angles formed by the transversal are congruent. So, $m\angle A = m\angle ABX$. Similarly, $\overline{BC}$ is a transversal, so $m\angle C = m\angle CBY$. The sum of the angle measures that make a straight line is 180°.

So, $m\angle ABX + m\angle ABC + m\angle CBY = 180°$. Now, substitute $m\angle A$ for $m\angle ABX$ and $m\angle C$ for $m\angle CBY$ to get $m\angle A + m\angle ABC + m\angle C = 180°$.

- In this diagram, $ABCD$ is a parallelogram and $\overline{BD}$ is a diagonal.



Write a two-column proof to show that $\overline{AB}$ and $\overline{CD}$ are congruent.

Solution:

Construct a proof using properties of the parallelogram and its diagonal.

Step	Statement	Reason
1	$ABCD$ is a parallelogram.	Given
2	$\overline{BD}$ is a diagonal.	Given
3	$\overline{AB}$ is parallel to $\overline{DC}$. $\overline{AD}$ is parallel to $\overline{BC}$.	Definition of a parallelogram
4	$\angle ABD \cong \angle CDB$ $\angle DBC \cong \angle BDA$	Alternate interior angles are congruent.
5	$\overline{BD} \cong \overline{BD}$	Reflexive Property of Congruence
6	$\triangle ADB \cong \triangle CBD$	ASA
7	$\overline{AB} \cong \overline{CD}$	CPCTC

Note: Corresponding parts of congruent triangles are congruent.

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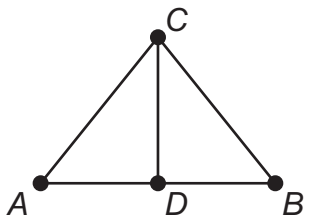
SAMPLE ITEMS

1. In this diagram, $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$. The two-column proof shows that $\overline{AC}$ is congruent to $\overline{BC}$.



Step	Statement	Reason
1	$\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.	Given
2	$\overline{AD} \cong \overline{BD}$	Definition of a bisector
3	$\overline{CD} \cong \overline{CD}$	Reflexive Property of Congruence
4	$\angle ADC$ and $\angle BDC$ are right angles.	Definition of perpendicular lines
5	$\angle ADC \cong \angle BDC$	All right angles are congruent.
6	$\triangle ADC \cong \triangle BDC$	**?**
7	$\overline{AC} \cong \overline{BC}$	CPCTC

Which of the following would justify Step 6?

A. AAS
B. ASA
C. SAS
D. SSS

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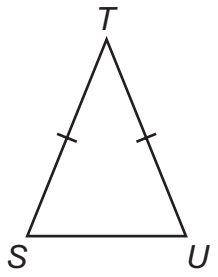
2. In this diagram, *STU* is an isosceles triangle where $\overline{ST}$ is congruent to $\overline{UT}$. The two-column proof shows that $\angle S$ is congruent to $\angle U$.



Step	Statement	Reason
1	$\overline{ST} \cong \overline{UT}$	Given
2	Construct $\overline{TV}$, the angle bisector for $\angle T$, where V is on $\overline{SU}$.	Every angle has a bisector.
3	$\angle STV \cong \angle UTV$	Definition of an angle bisector
4	$\overline{TV} \cong \overline{TV}$	Reflexive Property of Congruence
5	$\triangle STV \cong \triangle UTV$	SAS
6	$\angle S \cong \angle U$	**?**

Which reason is missing in the proof?

A. CPCTC
B. Reflexive Property of Congruence
C. Definition of right angles
D. Angle Congruence Postulate

Answers to Unit 2.4 Sample Items

1. C 2. A

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2.5 Make Geometric Constructions

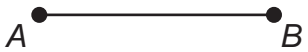
MGSE9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

MGSE9-12.G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon, each inscribed in a circle.

KEY IDEAS

To ***copy a segment***, follow the steps given:

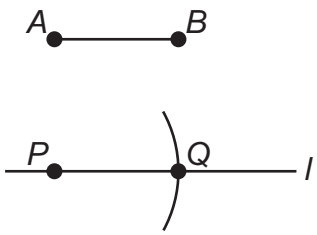
Given: $\overline{AB}$



Construct: $\overline{PQ}$ congruent to $\overline{AB}$

Procedure:

1. Use a straightedge to draw a line, *l*.
2. Choose a point on line *l* and label it point *P*.
3. Place the point of a compass on point *A*.
4. Adjust the compass width to the length of $\overline{AB}$.
5. Without changing the compass, place the compass point on point *P* and draw an arc intersecting line *l*. Label the point of intersection as point *Q*.
6. $\overline{PQ} \cong \overline{AB}$



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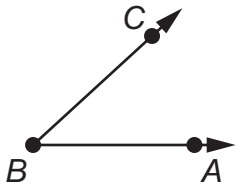
To ***copy an angle***, follow the steps given:

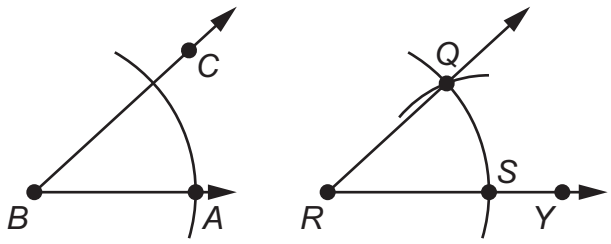


Given: $\angle ABC$

Construct: $\angle QRY$ congruent to $\angle ABC$

Procedure:

1. Draw a point, R, that will be the vertex of the new angle.
2. From point R, use a straightedge to draw $\overrightarrow{RY}$, which will become one side of the new angle.
3. Place the point of a compass on vertex B and draw an arc through point A.
4. Without changing the compass, place the compass point on point R, draw an arc intersecting $\overrightarrow{RY}$, and label the point of intersection point S.
5. Place the compass point on point A and adjust its width to where the arc intersects $\overrightarrow{BC}$.
6. Without changing the compass width, place the compass point on point S and draw another arc across the first arc. Label the point where both arcs intersect as point Q.
7. Use a straightedge to draw $\overrightarrow{RQ}$.
8. $\angle QRY \cong \angle ABC$



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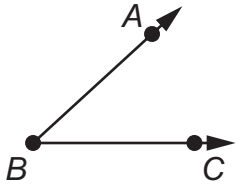
To ***bisect an angle***, follow the steps given:

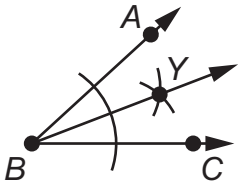


Given: $\angle ABC$

Construct: $\overrightarrow{BY}$, the bisector of $\angle ABC$

Procedure:

1. Place the point of a compass on vertex B.
2. Open the compass and draw an arc that crosses both sides of the angle.
3. Set the compass width to more than half the distance from point B to where the arc crosses $\overrightarrow{BA}$. Place the compass point where the arc crosses $\overrightarrow{BA}$ and draw an arc in the angle's interior.
4. Without changing the compass width, place the compass point where the arc crosses $\overrightarrow{BC}$ and draw an arc so that it crosses the previous arc. Label the intersection point Y.
5. Using a straightedge, draw a ray from vertex B through point Y.
6. $\overrightarrow{BY}$ is the bisector of $\angle ABC$, and $\angle ABY \cong \angle YBC$.



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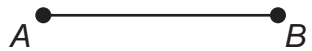
To ***construct a perpendicular bisector of a line segment***, follow the steps given:

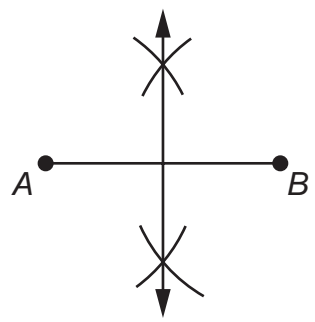
Given: $\overline{AB}$



Construct: The perpendicular bisector of $\overline{AB}$

Procedure:

1. Adjust a compass to a width greater than half the length of $\overline{AB}$.
2. Place the compass point on point *A* and draw an arc passing above $\overline{AB}$ and an arc passing below $\overline{AB}$.
3. Without changing the compass width, place the compass point on point *B* and draw an arc passing above $\overline{AB}$ and an arc passing below $\overline{AB}$.
4. Use a straightedge to draw a line through the points of intersection of these arcs.
5. The segment is the perpendicular bisector of $\overline{AB}$.



Note: To bisect $\overline{AB}$, follow the same steps listed above to construct the perpendicular bisector. The point where the perpendicular bisector intersects $\overline{AB}$ is the midpoint of $\overline{AB}$.

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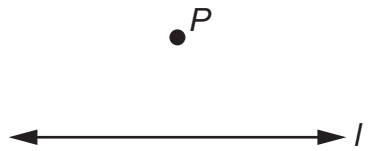
To ***construct a line perpendicular to a given line through a point not on the line***, follow the steps given:

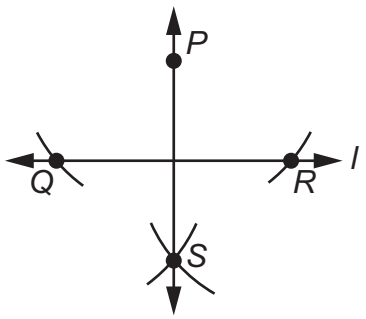


Given: Line *l* and point *P* that is not on line *l*

Construct: The line perpendicular to line *l* through point *P*

Procedure:

1. Place the point of a compass on point *P*.
2. Open the compass to a distance that is wide enough to draw two arcs across line *l*, one on each side of point *P*. Label these points *Q* and *R*.
3. From points *Q* and *R*, draw arcs on the opposite side of line *l* from point *P* so that the arcs intersect. Label the intersection point *S*.
4. Using a straightedge, draw $\overleftrightarrow{PS}$.
5. $\overline{PS}$ is perpendicular to line *l*.



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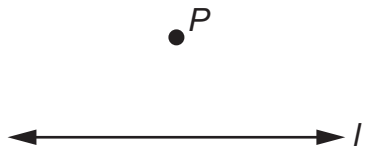
To ***construct a line parallel to a given line through a point not on the line***, follow the steps given:

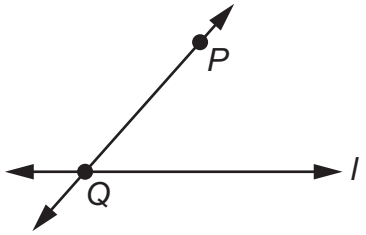


Given: Line *l* and point *P* that is not on line *l*

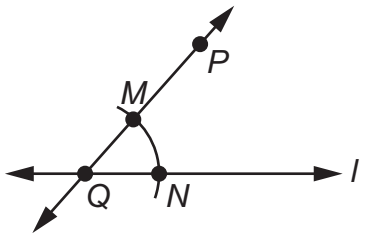
Construct: The line parallel to line *l* through point *P*

Procedure:

1. Draw a transversal line through point *P* crossing line *l* at a point. Label the point of intersection *Q*.



2. Open a compass to a width about half the distance from point *P* to point *Q*. Place the compass point on point *Q* and draw an arc that intersects both lines. Label the intersection of the arc and $\overline{PQ}$ as point *M* and the intersection of the arc and line *l* as point *N*.



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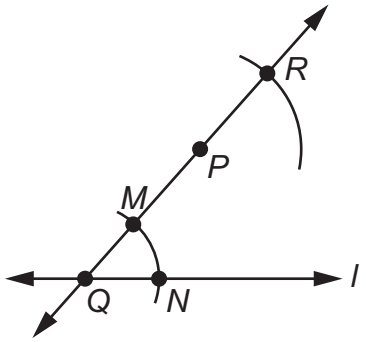
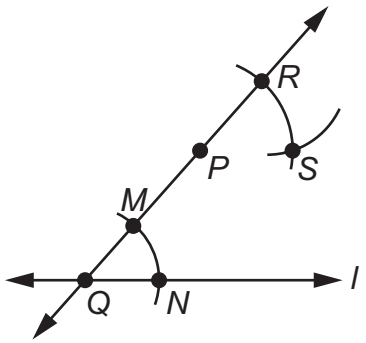
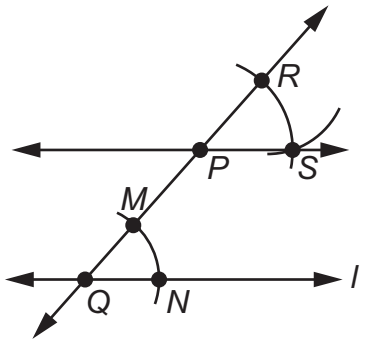
3. Without changing the compass width, place the compass point on point *P* and draw an arc that crosses $\overline{PQ}$ above point *P*. Note that this arc must have the same orientation as the arc drawn from point *M* to point *N*. Label the point of intersection *R*.



4. Set the compass width to the distance from point *M* to point *N*.
5. Place the compass point on point *R* and draw an arc that crosses the upper arc. Label the point of intersection *S*.



6. Using a straightedge, draw a line through points *P* and *S*.
7. $\overleftrightarrow{PS} \parallel l$



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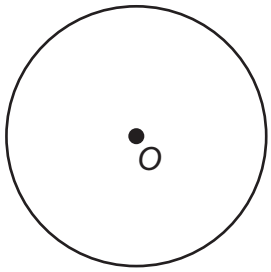
To ***construct an equilateral triangle inscribed in a circle***, follow the steps given:

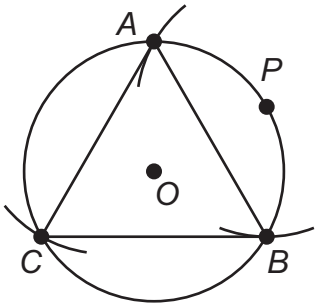


Given: Circle *O*

Construct: Equilateral $\triangle ABC$ inscribed in circle *O*

Procedure:

1. Mark a point anywhere on the circle and label it point *P*.
2. Open a compass to the radius of circle *O*.
3. Place the compass point on point *P* and draw an arc that intersects the circle at two points. Label the points *A* and *B*.
4. Using a straightedge, draw $\overline{AB}$.
5. Open the compass to the length of $\overline{AB}$.
6. Place the compass point on point *A*. Draw an arc from point *A* that intersects the circle. Label this point *C*.
7. Using a straightedge, draw $\overline{AC}$ and $\overline{BC}$.
8. Equilateral $\triangle ABC$ is inscribed in circle *O*.



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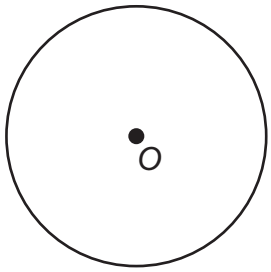
To ***construct a square inscribed in a circle***, follow the steps given:

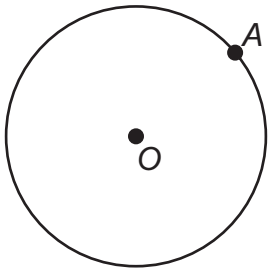


Given: Circle *O*

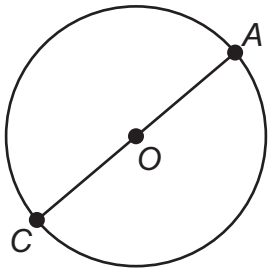
Construct: Square *ABCD* inscribed in circle *O*

Procedure:

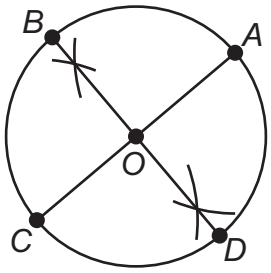
1. Mark a point anywhere on the circle and label it point *A*.



2. Using a straightedge, draw a diameter from point *A*. Label the other endpoint of the diameter as point *C*. This is diameter $\overline{AC}$.



3. Construct a perpendicular bisector of $\overline{AC}$ through the center of circle *O*. Label the points where it intersects the circle as point *B* and point *D*.



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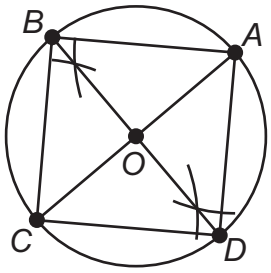
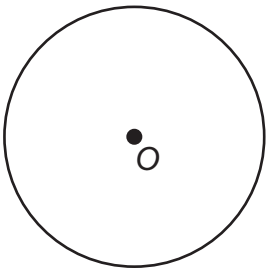
4. Using a straightedge, draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$.



5. Square *ABCD* is inscribed in circle *O*.

To ***construct a regular hexagon inscribed in a circle***, follow the steps given:

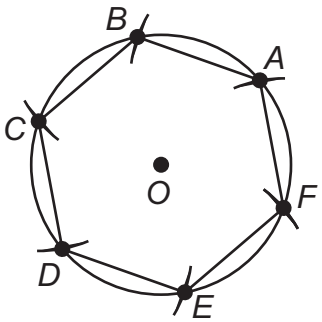


Given: Circle *O*

Construct: Regular hexagon *ABCDEF* inscribed in circle *O*

Procedure:

1. Mark a point anywhere on the circle and label it point *A*.
2. Open a compass to the radius of circle *O*.
3. Place the compass point on point *A* and draw an arc across the circle. Label this point *B*.
4. Without changing the width of the compass, place the compass point on point *B* and draw another arc across the circle. Label this point *C*.
5. Repeat this process from point *C* to a point *D*, from point *D* to a point *E*, and from point *E* to a point *F*.
6. Use a straightedge to draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, and $\overline{AF}$.
7. Regular hexagon *ABCDEF* is inscribed in circle *O*.



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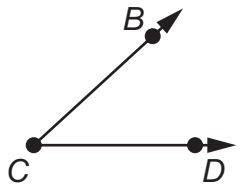
REVIEW EXAMPLES

♦ Allan drew angle *BCD*.

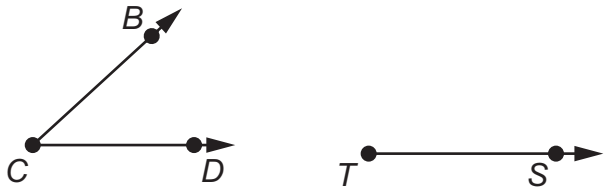


a. Copy angle *BCD*. List the steps you used to copy the angle. Label the copied angle *RTS*.

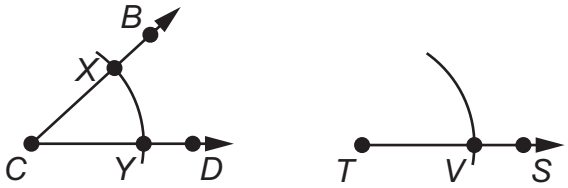
b. Without measuring the angles, how can you show they are congruent to one another?

Solution:

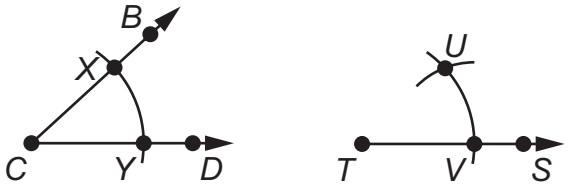
a. Draw point *T*. Draw $\overrightarrow{TS}$.



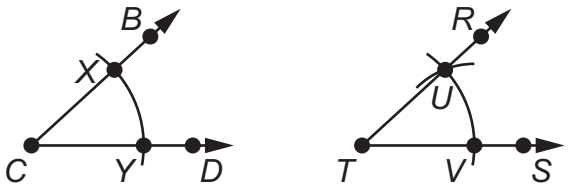
Place the point of a compass on point *C*. Draw an arc. Label the intersection points *X* and *Y*. Keep the compass width the same, and place the point of the compass on point *T*. Draw an arc and label the intersection point *V*.



Place the point of the compass on point *Y* and adjust the width to point *X*. Then place the point of the compass on point *V* and draw an arc that intersects the first arc. Label the intersection point *U*.



Draw $\overrightarrow{TU}$ and point *R* on $\overrightarrow{TU}$. Angle *BCD* has now been copied to form angle *RTS*.



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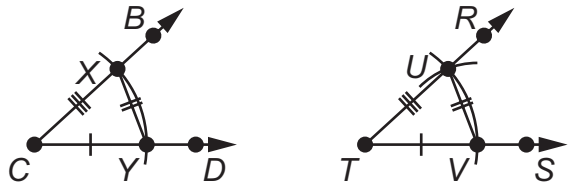
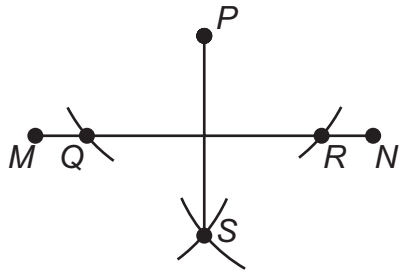
b. Connect points X and Y and points U and V to form $\triangle XCY$ and $\triangle UTV$. $\overline{CY}$ and $\overline{TV}$, $\overline{XY}$ and $\overline{UV}$, and $\overline{CX}$ and $\overline{TU}$ are congruent because they were drawn with the same compass width. So $\triangle XCY \cong \triangle UTV$ by SSS, and $\angle C \cong \angle T$ because congruent parts of congruent triangles are congruent.



♦ Construct a line segment perpendicular to $\overline{MN}$ from a point not on $\overline{MN}$. Explain the steps you used to make your construction.

Solution:

Draw a point P that is not on $\overline{MN}$. Place the point of a compass on point P. Draw an arc that intersects $\overline{MN}$ at two points. Label the intersection points Q and R. Without changing the width of the compass, place the compass point on point Q and draw an arc under $\overline{MN}$. Place the compass point on point R and draw another arc under $\overline{MN}$. Label the intersection point S. Draw $\overline{PS}$. Segment PS is perpendicular to and bisects $\overline{MN}$.



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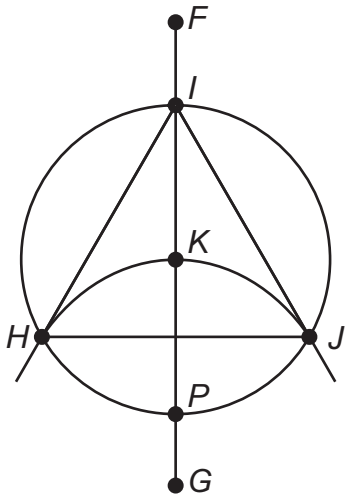
♦ Construct equilateral $\triangle HIJ$ inscribed in circle K. Explain the steps you used to make your construction.

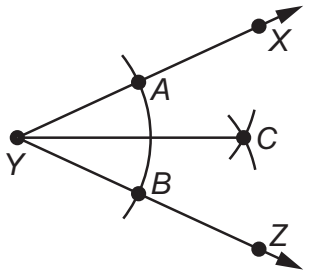
Solution:

(This is an alternate method from the method shown in the Key Ideas.) Use a compass to draw circle K. Draw segment $\overline{FG}$ through the center of circle K. Label the points where $\overline{FG}$ intersects circle K as points I and P. Using the compass setting you used when drawing the circle, place the compass on point P and draw an arc passing through point K. Label the points where the arc intersects circle K as points H and J. Draw $\overline{HJ}$, $\overline{IJ}$, and $\overline{HI}$. Triangle HIJ is an equilateral triangle inscribed in circle K.



SAMPLE ITEMS

1. Consider the construction of the angle bisector shown.



Which could have been the first step in creating this construction?

A. Place the compass point on point A and draw an arc inside $\angle Y$.
B. Place the compass point on point B and draw an arc inside $\angle Y$.
C. Place the compass point on vertex Y and draw an arc that intersects $\overline{YX}$ and $\overline{YZ}$.
D. Place the compass point on vertex Y and draw an arc that intersects point C.

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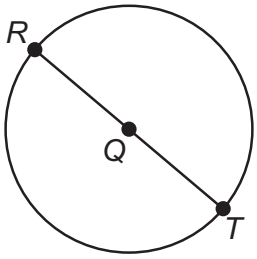
2. Consider the beginning of the construction of a square inscribed in circle *Q*.

Step 1: Label point *R* on circle *Q*.
Step 2: Draw a diameter through *R* and *Q*.
Step 3: Label the point where the diameter intersects the circle as point *T*.



What is the next step in this construction?

A. Draw radius $\overline{SQ}$.
B. Label point *S* on circle *Q*.
C. Construct a line segment parallel to $\overline{RT}$.
D. Construct the perpendicular bisector of $\overline{RT}$.

Answers to Unit 2.5 Sample Items

1. C 2. D

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2.6 Use Coordinates to Prove Simple Geometric Theorems Algebraically

MGSE9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

(Focus on quadrilaterals, right triangles, and circles.)

KEY IDEAS

To prove properties about special parallelograms on a coordinate plane, you can use the midpoint, distance, and slope formulas:

- The ***midpoint formula*** is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. This formula is used to find the coordinates of the midpoint of $\overline{AB}$, given $A(x_1, y_1)$ and $B(x_2, y_2)$.
- The ***distance formula*** is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. This formula is used to find the length of $\overline{AB}$, given $A(x_1, y_1)$ and $B(x_2, y_2)$.
- The ***slope formula*** is $m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$. This formula is used to find the slope of a line or line segment, given any two points on the line or line segment $A(x_1, y_1)$ and $B(x_2, y_2)$.

You can use properties of quadrilaterals to help prove theorems, such as the following:

- To prove a quadrilateral is a parallelogram, show that the opposite sides are parallel using the slope formula.
- To prove a quadrilateral is a rectangle, show that the opposite sides are parallel and the consecutive sides are perpendicular using the slope formula.
- To prove a quadrilateral is a rhombus, show that all four sides are congruent using the distance formula.
- To prove a quadrilateral is a square, show that all four sides are congruent and consecutive sides are perpendicular using the distance and slope formulas.

You can also use diagonals of a quadrilateral to help prove theorems, such as the following:

- To prove a quadrilateral is a parallelogram, show that its diagonals bisect each other using the midpoint formula.
- To prove a quadrilateral is a rectangle, show that its diagonals bisect each other and are congruent using the midpoint and distance formulas.
- To prove a quadrilateral is a rhombus, show that its diagonals bisect each other and are perpendicular using the midpoint and slope formulas.
- To prove a quadrilateral is a square, show that its diagonals bisect each other, are congruent, and are perpendicular using the midpoint, distance, and slope formulas.

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Important Tips

- When using the formulas for midpoint, distance, and slope, the order of the points does not matter. Either point can be (x_1, y_1), but be careful to always subtract in the same order.
- Parallel lines have the same slope. Perpendicular lines have slopes that are the negative reciprocal of each other.

REVIEW EXAMPLE

- Quadrilateral *ABCD* has vertices *A*(–1, 3), *B*(3, 5), *C*(4, 3), and *D*(0, 1). Is *ABCD* a rectangle? Explain how you know.

  Solution:

  First determine whether the figure is a parallelogram. If the figure is a parallelogram, then the diagonals bisect each other. If the diagonals bisect each other, then the midpoints of the diagonals are the same point. Use the midpoint formula to determine the midpoints for each diagonal.

  Midpoint $\overline{AC}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1 + 4}{2}, \frac{3 + 3}{2}\right) = \left(\frac{3}{2}, \frac{6}{2}\right) = (1.5, 3)$

  Midpoint $\overline{BD}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + 0}{2}, \frac{5 + 1}{2}\right) = \left(\frac{3}{2}, \frac{6}{2}\right) = (1.5, 3)$

  The diagonals have the same midpoint; therefore, the diagonals bisect each other and the figure is a parallelogram.

  A parallelogram with congruent diagonals is a rectangle. Determine whether the diagonals are congruent.

  Use the distance formula to find the length of the diagonals.

  $AC = \sqrt{(4 - (-1))^2 + (3 - 3)^2} = \sqrt{(5)^2 + (0)^2} = \sqrt{25 + 0} = \sqrt{25} = 5$

  $BD = \sqrt{(0 - 3)^2 + (1 - 5)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

  The diagonals are congruent because they have the same length.

  The figure is a parallelogram with congruent diagonals, so the figure is a rectangle.

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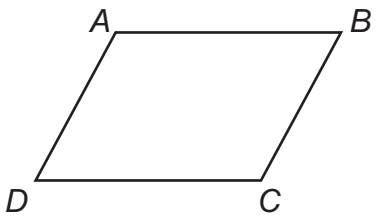
SAMPLE ITEMS

1. Which information is sufficient to show that a parallelogram is a rectangle?

A. The diagonals bisect each other.
B. The diagonals are congruent.
C. The diagonals are congruent and perpendicular.
D. The diagonals bisect each other and are perpendicular.

2. Look at quadrilateral *ABCD*.



Which information is needed to show that quadrilateral *ABCD* is a parallelogram?

A. Use the distance formula to show that diagonals *AC* and *BD* have the same length.
B. Use the slope formula to show that segments *AB* and *CD* are perpendicular and segments *AD* and *BC* are perpendicular.
C. Use the slope formula to show that segments *AB* and *CD* have the same slope and segments *AD* and *BC* have the same slope.
D. Use the distance formula to show that segments *AB* and *AD* have the same length and segments *CD* and *BC* have the same length.

Answers to Unit 2.6 Sample Items

1. B 2. C

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UNIT 3: RIGHT TRIANGLE TRIGONOMETRY

This unit investigates the properties of right triangles. The trigonometric ratios sine, cosine, and tangent along with the Pythagorean Theorem are used to solve right triangles in applied problems. The relationship between the sine and cosine of complementary angles is identified.

3.1 Right Triangle Relationships

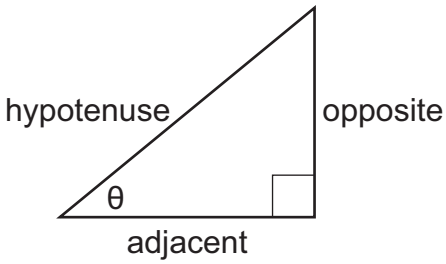
MGSE9-12.G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

MGSE9-12.G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

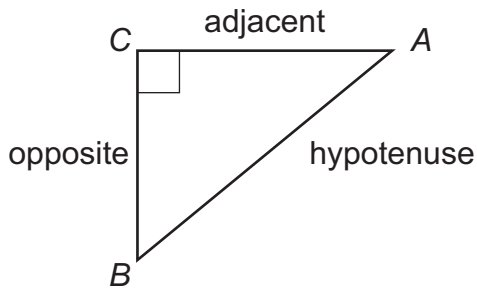
MGSE9-12.G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

KEY IDEAS

The trigonometric ratios ***sine***, ***cosine***, and ***tangent*** are defined as ratios of the lengths of the sides in a right triangle with a given acute angle measure. These terms are usually seen abbreviated as ***sin***, ***cos***, and ***tan***.



- $\sin\theta = \dfrac{\text{length of opposite side}}{\text{length of hypotenuse}}$
- $\cos\theta = \dfrac{\text{length of adjacent side}}{\text{length of hypotenuse}}$
- $\tan\theta = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$



- $\sin A = \dfrac{\text{length of opposite side}}{\text{length of hypotenuse}}$
- $\cos A = \dfrac{\text{length of adjacent side}}{\text{length of hypotenuse}}$
- $\tan A = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$

The two acute angles of any right triangle are complementary. As a result, if angles P and Q are complementary, $\sin P = \cos Q$ and $\sin Q = \cos P$.

When solving problems with right triangles, you can use both trigonometric ratios and the Pythagorean Theorem ($a^2 + b^2 = c^2$). There may be more than one way to solve the problem, so analyze the given information to help decide which method is the most efficient.

Important Tip

✍ The tangent of angle A is also equivalent to $\dfrac{\sin A}{\cos A}$.

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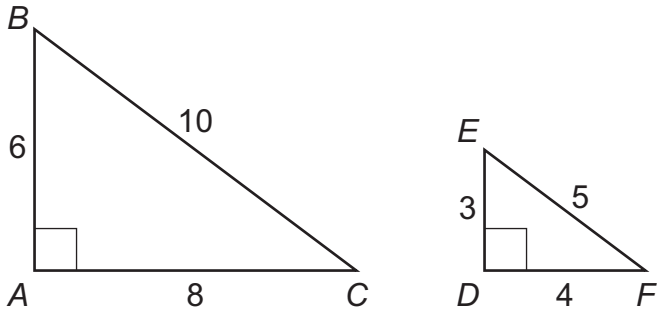
REVIEW EXAMPLES

♦ Triangles *ABC* and *DEF* are similar.



a. Find the ratio of the side opposite angle *B* to the hypotenuse in $\triangle ABC$.
b. What angle in $\triangle DEF$ corresponds to angle *B*?
c. Find the ratio of the side opposite angle *E* to the hypotenuse in $\triangle DEF$.
d. How does the ratio in part (a) compare to the ratio in part (c)?
e. Which trigonometric ratio does this represent?

Solution:

a. $\overline{AC}$ is opposite angle *B*. $\overline{BC}$ is the hypotenuse. The ratio of the side opposite angle *B* to the hypotenuse in $\triangle ABC$ is $\frac{8}{10} = \frac{4}{5}$.
b. Angle *E* in $\triangle DEF$ corresponds to angle *B* in $\triangle ABC$.
c. $\overline{DF}$ is opposite angle *E*. $\overline{EF}$ is the hypotenuse. The ratio of the side opposite angle *E* to the hypotenuse in $\triangle DEF$ is $\frac{4}{5}$.
d. The ratios are the same.
e. This represents sin *B* and sin *E*, because both are the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$.

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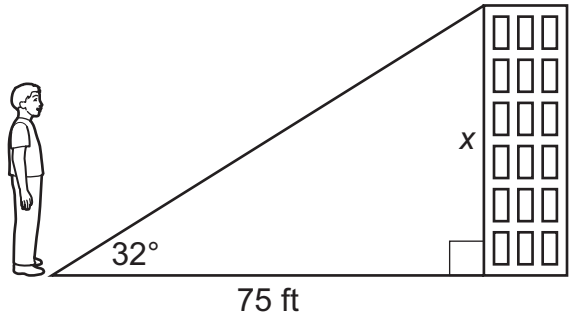
- Ricardo is standing 75 feet away from the base of a building. The ***angle of elevation*** from the ground where Ricardo is standing to the top of the building is 32°.



Note: figure not drawn to scale.

What is x, the height of the building, to the nearest tenth of a foot?

$$\begin{bmatrix} \sin 32° \approx 0.5299 \\ \cos 32° \approx 0.8480 \\ \tan 32° \approx 0.6249 \end{bmatrix}$$

Solution:

When finding the length of the side opposite the 32° angle and given the length of the side adjacent to the 32° angle, use the tangent ratio. Substitute x for the opposite side, 75 for the adjacent side, and 32° for the angle measure. Then solve.

$$\tan 32° = \frac{x}{75}$$

$$75\tan 32° = x$$

$$75 \cdot 0.6249 \approx x$$

$$46.9 \approx x$$

The building is about 46.9 feet tall.

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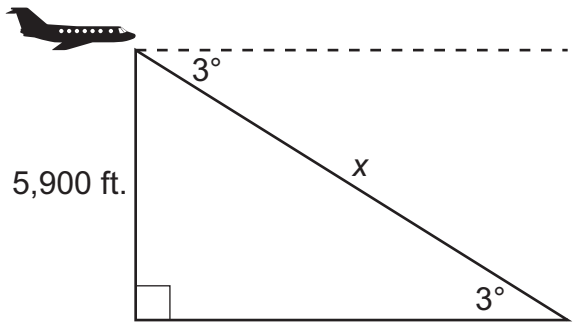
- An airplane is at an altitude of 5,900 feet. The airplane descends at an angle of 3°, called the ***angle of depression***.



Note: figure not drawn to scale.

About how far will the airplane travel in the air until it reaches the ground?

$$\begin{bmatrix} \sin 3° \approx 0.0523 \\ \cos 3° \approx 0.9986 \\ \tan 3° \approx 0.0524 \end{bmatrix}$$

Solution:

Use sin 3° to find the distance the airplane will travel until it reaches the ground, x. Substitute x for the hypotenuse, 5,900 for the opposite side, and 3° for the angle measure. Then solve.

$$\sin 3° = \frac{5{,}900}{x}$$

$$x \sin 3° = 5{,}900$$

$$x = \frac{5{,}900}{\sin 3°}$$

$$x \approx \frac{5{,}900}{0.0523}$$

$$x \approx 112{,}811$$

The airplane will travel about 113,000 feet until it reaches the ground.

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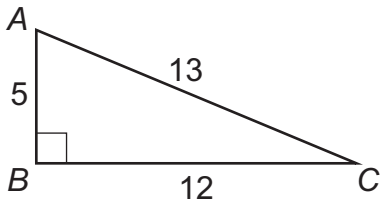
♦ Triangle *ABC* is a right triangle.



What is the best approximation for $m\angle C$?

$$\begin{bmatrix} \sin 67.4° \approx 0.923 \\ \cos 22.6° \approx 0.923 \\ \tan 42.7° \approx 0.923 \end{bmatrix}$$

Solution:

Find the trigonometric ratios for angle *C*.

$\sin C = \frac{5}{13} \approx 0.385$

$\cos C = \frac{12}{13} \approx 0.923$

$\tan C = \frac{5}{12} \approx 0.417$

Using the table, $\cos 22.6° \approx 0.923$, so $m\angle C \approx 22.6°$, or using trigonometric inverses, $\sin^{-1} \frac{5}{13} = 22.6$, $\cos^{-1} \frac{12}{13} = 22.6$, or $\tan^{-1} \frac{5}{12} = 22.6$.

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SAMPLE ITEMS

1. In right triangle *ABC*, angle *A* and angle *B* are complementary angles. The value of cos *A* is $\frac{5}{13}$. What is the value of sin *B*?

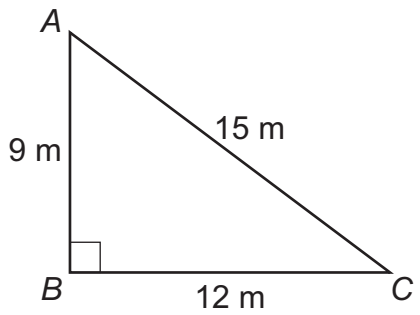
A. $\frac{5}{13}$

B. $\frac{12}{13}$

C. $\frac{13}{12}$

D. $\frac{13}{5}$

2. Triangle *ABC* is given below.



What is the value of cos *A*?

A. $\frac{3}{5}$

B. $\frac{3}{4}$

C. $\frac{4}{5}$

D. $\frac{5}{3}$

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3. In right triangle *HJK*, $\angle J$ is a right angle and tan $\angle H = 1$. Which statement about triangle *HJK* must be true?

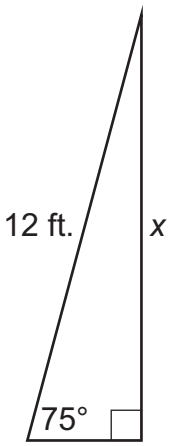
A. $\sin \angle H = \frac{1}{2}$

B. $\sin \angle H = 1$

C. $\sin \angle H = \cos \angle H$

D. $\sin \angle H = \frac{1}{\cos \angle H}$

4. A 12-foot ladder is leaning against a building at a 75° angle to the ground.



Which equation can be used to find how high the ladder reaches up the side of the building?

A. $\sin 75° = \frac{12}{x}$

B. $\tan 75° = \frac{12}{x}$

C. $\cos 75° = \frac{x}{12}$

D. $\sin 75° = \frac{x}{12}$

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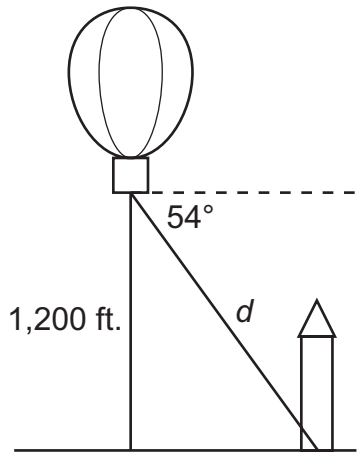
5. A hot-air balloon is 1,200 feet above the ground. The angle of depression from the basket of the hot-air balloon to the base of a monument is 54°.



Note: figure not drawn to scale.

Which equation can be used to find the distance, *d*, in feet, from the basket of the hot air balloon to the base of the monument?

A. $\sin 54° = \frac{d}{1200}$

B. $\sin 54° = \frac{1200}{d}$

C. $\cos 54° = \frac{d}{1200}$

D. $\cos 54° = \frac{1200}{d}$

Answers to Unit 3.1 Sample Items

1. A 2. A 3. C 4. D 5. B

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UNIT 4: CIRCLES AND VOLUME

This unit investigates the properties of circles and addresses finding the volume of solids. Properties of circles are used to solve problems involving arcs, angles, sectors, chords, tangents, and secants. Volume formulas are derived and used to calculate the volumes of cylinders, pyramids, cones, and spheres.

4.1 Understand and Apply Theorems about Circles

MGSE9-12.G.C.1 Understand that all circles are similar.

MGSE9-12.G.C.2 Identify and describe relationships among inscribed angles, radii, chords, tangents, and secants. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

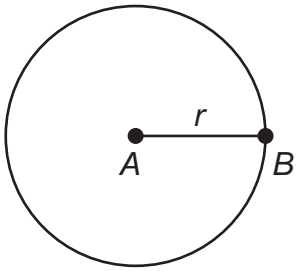
MGSE9-12.G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

MGSE9-12.G.C.4 Construct a tangent line from a point outside a given circle to the circle.

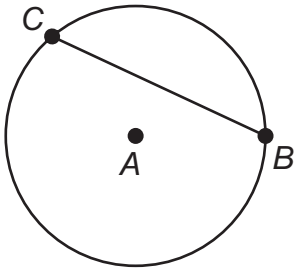
KEY IDEAS

A ***circle*** is the set of points in a plane equidistant from a given point, which is the center of the circle. All circles are similar.

A ***radius*** is a line segment from the center of a circle to any point on the circle. The word radius is also used to describe the length, *r*, of the segment. $\overline{AB}$ is a radius of circle *A*.



A ***chord*** is a line segment whose endpoints are on a circle. $\overline{BC}$ is a chord of circle *A*.



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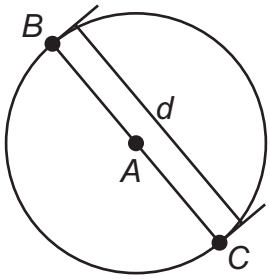
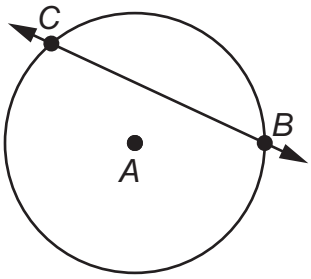
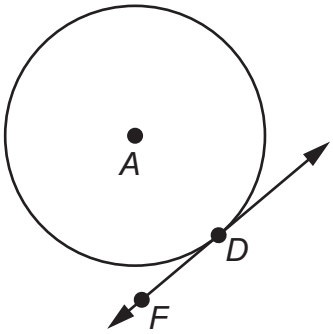
A ***diameter*** is a chord that passes through the center of a circle. The word diameter is also used to describe the length, *d,* of the segment. $\overline{BC}$ is a diameter of circle *A*.



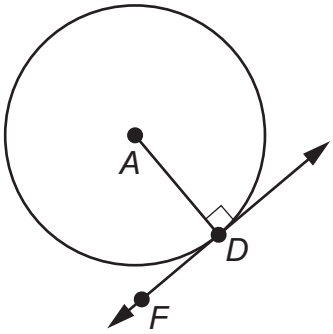
A ***secant line*** is a line that is in the plane of a circle and intersects the circle at exactly two points. Every chord lies on a secant line. $\overleftrightarrow{BC}$ is a secant line of circle *A*.



A ***tangent line*** is a line that is in the plane of a circle and intersects the circle at only one point, the ***point of tangency***. $\overleftrightarrow{DF}$ is tangent to circle *A* at the point of tangency, point *D.*



If a line is tangent to a circle, the line is perpendicular to the radius drawn to the point of tangency. $\overleftrightarrow{DF}$ is tangent to circle *A* at point *D*, so $\overline{AD} \perp \overline{DF}$.



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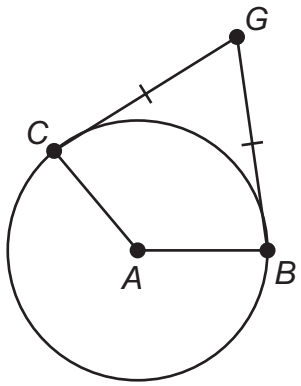
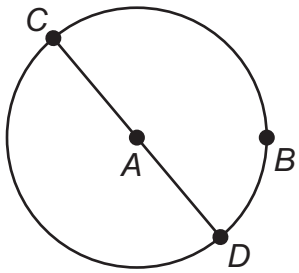
Tangent segments drawn from the same point are congruent. In circle A, $\overline{CG} \cong \overline{BG}$.

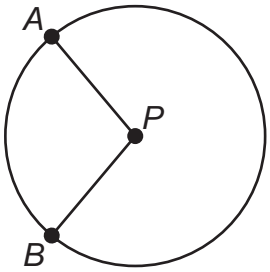


Circumference is the distance around a circle. The formula for the circumference, C, of a circle is $C = \pi d$, where d is the diameter of the circle. The formula is also written as $C = 2\pi r$, where r is the length of the radius of the circle. π is the ratio of circumference to diameter of any circle.

An ***arc*** is a part of the circumference of a circle. A ***minor arc*** has a measure less than 180°. Minor arcs are written using two points on a circle. A ***semicircle*** is an arc that measures exactly 180°. Semicircles are written using three points on a circle. This is done to show which half of the circle is being described. A ***major arc*** has a measure greater than 180°. Major arcs are written with three points to distinguish them from the corresponding minor arc. In circle A, $\overset{\frown}{CB}$ is a minor arc, $\overset{\frown}{CBD}$ is a semicircle, and $\overset{\frown}{CDB}$ is a major arc.



A ***central angle*** is an angle whose vertex is at the center of a circle and whose sides are radii of the circle. The measure of a central angle of a circle is equal to the measure of the intercepted arc. $\angle APB$ is a central angle for circle P, and $\overset{\frown}{AB}$ is the intercepted arc.



$$m\angle APB = m\overset{\frown}{AB}$$

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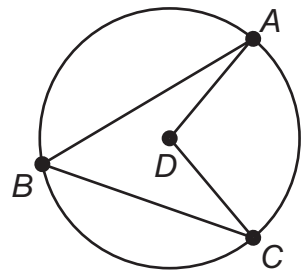
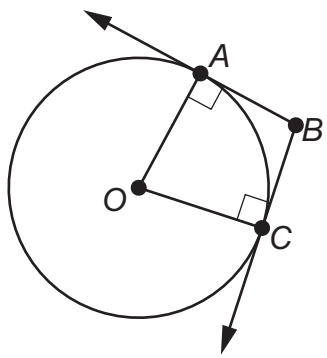
An ***inscribed angle*** is an angle whose vertex is on a circle and whose sides are chords of the circle. The measure of an angle inscribed in a circle is half the measure of the intercepted arc. For circle D, $\angle ABC$ is an inscribed angle, and $\widehat{AC}$ is the intercepted arc.



$$m\angle ABC = \frac{1}{2}\left(m\widehat{AC}\right) = \frac{1}{2}(m\angle ADC)$$

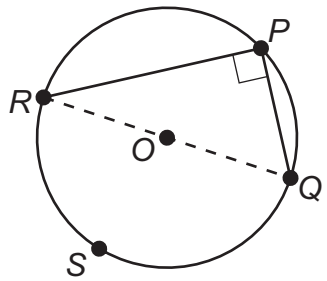
$$m\angle ADC = m\widehat{AC} = 2(m\angle ABC)$$

A ***circumscribed angle*** is an angle formed by two rays that are each tangent to a circle. These rays are perpendicular to radii of the circle. In circle O, the measure of the circumscribed angle is equal to 180° minus the measure of the central angle that forms the intercepted arc. The measure of the circumscribed angle can also be found by using the measures of two intercepted arcs.



$$m\angle ABC = 180° - m\angle AOC$$

When an inscribed angle intercepts a semicircle, the inscribed angle has a measure of 90°. For circle O, $\angle RPQ$ intercepts semicircle $\widehat{RSQ}$ as shown.



$$m\angle RPQ = \frac{1}{2}\left(m\widehat{RSQ}\right) = \frac{1}{2}(180°) = 90°$$

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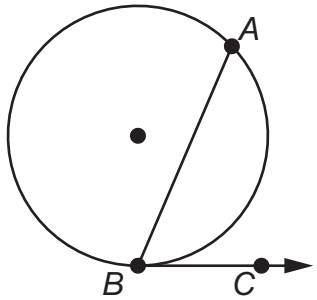
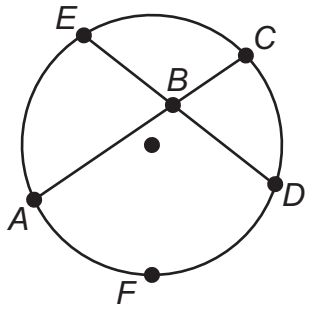
The measure of an angle formed by a tangent and a chord with its vertex on the circle is half the measure of the intercepted arc. $\overline{AB}$ is a chord for the circle, and $\overrightarrow{BC}$ is tangent to the circle at point B. So $\angle ABC$ is formed by a tangent and a chord.



$$m\angle ABC = \frac{1}{2}\left(m\widehat{AB}\right)$$

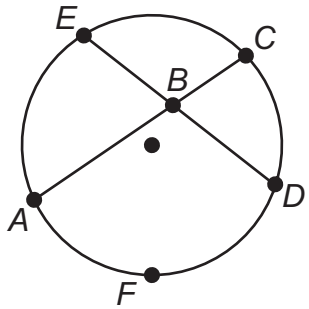
When two chords intersect inside a circle, two pairs of vertical angles are formed. The measure of any one of the angles is half the sum of the measures of the arcs intercepted by the pair of vertical angles.



$$m\angle ABE = \frac{1}{2}\left(m\widehat{AE} + m\widehat{CD}\right)$$

$$m\angle ABD = \frac{1}{2}\left(m\widehat{AFD} + m\widehat{EC}\right)$$

When two chords intersect inside a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$$AB \bullet BC = EB \bullet BD$$

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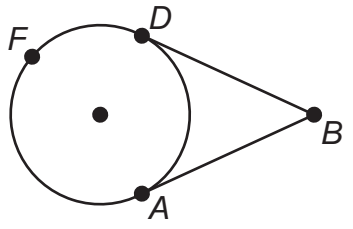
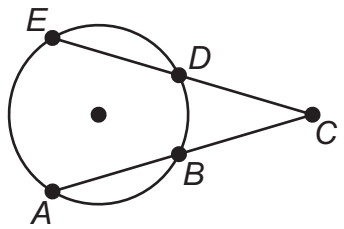
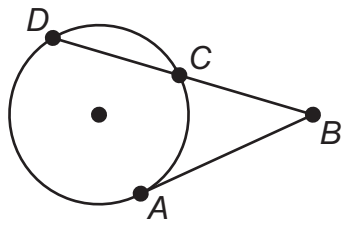
Angles outside a circle can be formed by the intersection of two tangents (circumscribed angle), two secants, or a secant and a tangent. For all three situations, the measure of the angle is half the difference of the measure of the larger intercepted arc and the measure of the smaller intercepted arc.



$$m\angle ABD = \frac{1}{2}\left(m\widehat{AFD} - m\widehat{AD}\right)$$

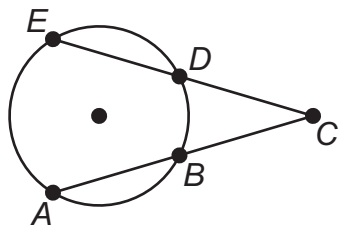


$$m\angle ACE = \frac{1}{2}\left(m\widehat{AE} - m\widehat{BD}\right)$$



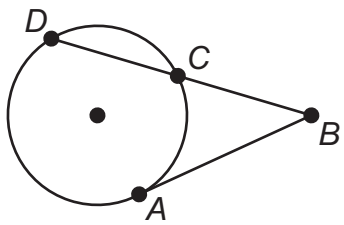
$$m\angle ABD = \frac{1}{2}\left(m\widehat{AD} - m\widehat{AC}\right)$$

When two ***secant segments*** intersect outside a circle, part of each secant segment is a segment formed outside the circle. The product of the length of one secant segment and the length of the segment formed outside the circle is equal to the product of the length of the other secant segment and the length of the segment formed outside the circle.



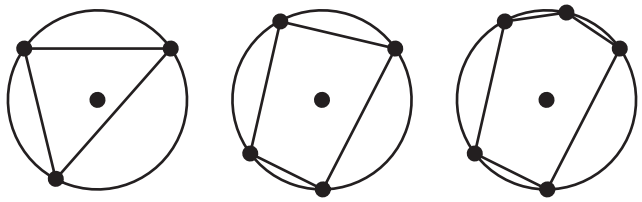
$$EC \bullet DC = AC \bullet BC$$

When a secant segment and a tangent segment intersect outside a circle, the product of the length of the secant segment and the length of the segment formed outside the circle is equal to the square of the length of the tangent segment.



$$DB \bullet CB = AB^2$$

An ***inscribed polygon*** is a polygon whose vertices all lie on a circle. This diagram shows a triangle, a quadrilateral, and a pentagon each inscribed in a circle.



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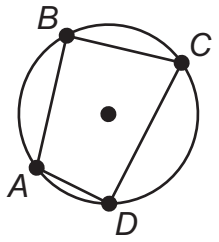
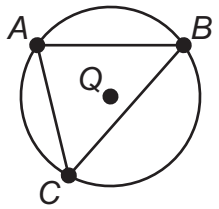
In a quadrilateral inscribed in a circle, the opposite angles are supplementary.



$$m\angle ABC + m\angle ADC = 180°$$

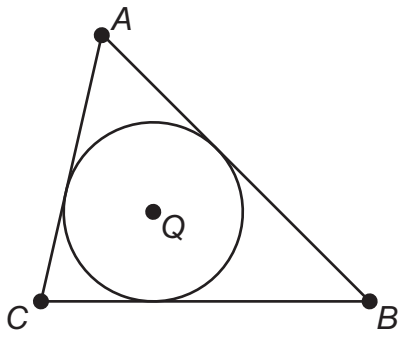
$$m\angle BCD + m\angle BAD = 180°$$

When a triangle is inscribed in a circle, the center of the circle is the ***circumcenter*** of the triangle. The circumcenter is equidistant from the vertices of the triangle. Triangle *ABC* is inscribed in circle *Q*, and point *Q* is the circumcenter of the triangle.



$$AQ = BQ = CQ$$

An ***inscribed circle*** is a circle enclosed in a polygon, where every side of the polygon is tangent to the circle. Specifically, when a circle is inscribed in a triangle, the center of the circle is the ***incenter*** of the triangle. The incenter is equidistant from the sides of the triangle. Circle *Q* is inscribed in triangle *ABC*, and point *Q* is the incenter of the triangle. Notice also that the sides of the triangle form circumscribed angles with the circle.



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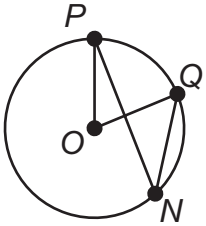
REVIEW EXAMPLES

♦ $\angle PNQ$ is inscribed in circle O, and $m\widehat{PQ} = 70°$.



a. What is the measure of $\angle POQ$?
b. What is the relationship between $\angle POQ$ and $\angle PNQ$?
c. What is the measure of $\angle PNQ$?

Solution:

a. The measure of a central angle is equal to the measure of the intercepted arc.
$m\angle POQ = m\widehat{PQ} = 70°$.

b. $\angle POQ$ is a central angle that intercepts $\widehat{PQ}$. $\angle PNQ$ is an inscribed angle that intercepts $\widehat{PQ}$. The measure of the central angle is equal to the measure of the intercepted arc. The measure of the inscribed angle is equal to one-half the measure of the intercepted arc. So $m\angle POQ = m\widehat{PQ}$, $m\angle PNQ = \frac{1}{2}\left(m\widehat{PQ}\right)$, and $m\angle POQ = 2(m\angle PNQ)$.

c. From part (b), $m\angle POQ = 2(m\angle PNQ)$
Substitute: $70° = 2(m\angle PNQ)$
Divide: $35° = m\angle PNQ$

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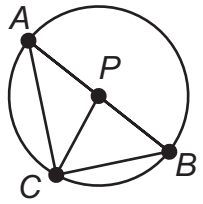
♦ In circle P below, $\overline{AB}$ is a diameter.



If $m\angle APC = 100°$, find the following:

a. $m\angle BPC$
b. $m\angle BAC$
c. $m\widehat{BC}$
d. $m\widehat{AC}$

Solution:

a. $\angle APC$ and $\angle BPC$ are supplementary, so $m\angle BPC = 180° - m\angle APC = 180° - 100° = 80°$.
b. $\angle BAC$ is an angle in $\triangle APC$. The sum of the measures of the angles of a triangle is 180°.

For $\triangle APC$: $m\angle APC + m\angle BAC + m\angle ACP = 180°$

You are given that $m\angle APC = 100°$.

Substitute: $100° + m\angle BAC + m\angle ACP = 180°$

Subtract 100° from both sides: $m\angle BAC + m\angle ACP = 80°$

Because two sides of $\triangle APC$ are radii of the circle, $\triangle APC$ is an isosceles triangle. This means that the two base angles are congruent, so $m\angle BAC = m\angle ACP$.

Substitute: $m\angle BAC$ for $m\angle ACP$: $m\angle BAC + m\angle BAC = 80°$

Add: $2(m\angle BAC) = 80°$

Divide: $m\angle BAC = 40°$

You could also use the answer from part (a) to solve for $m\angle BAC$. Part (a) shows $m\angle BPC = 80°$.

Because the central angle measure is equal to the measure of the intercepted arc, $m\angle BPC = m\widehat{BC} = 80°$.

Because an inscribed angle is equal to one-half the measure of the intercepted arc, $m\angle BAC = \frac{1}{2}(m\widehat{BC})$.

Substitute: $m\angle BAC = \frac{1}{2}(80°)$

Therefore, $m\angle BAC = 40°$.

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c. $\angle BAC$ is an inscribed angle intercepting $\overset{\frown}{BC}$. The intercepted arc is twice the measure of the inscribed angle.

$m\overset{\frown}{BC} = 2(m\angle BAC)$

From part (b), $m\angle BAC = 40°$.

Substitute: $m\overset{\frown}{BC} = 2 \bullet 40°$

$m\overset{\frown}{BC} = 80°$

You could also use the answer from part (a) to solve. Part (a) shows $m\angle BPC = 80°$. Because $\angle BPC$ is a central angle that intercepts $\overset{\frown}{BC}$, $m\angle BPC = m\overset{\frown}{BC} = 80°$.

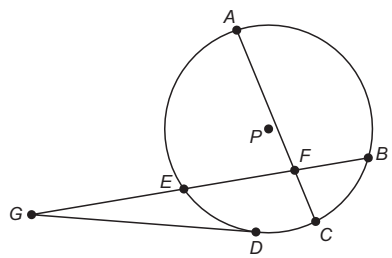
d. $\angle APC$ is a central angle intercepting $\overset{\frown}{AC}$. The measure of the intercepted arc is equal to the measure of the central angle.

$m\overset{\frown}{AC} = m\angle APC$

You are given $m\angle APC = 100°$.

Substitute: $m\overset{\frown}{AC} = 100°$

♦ In circle P below, $\overline{DG}$ is a tangent, $AF = 8$, $EF = 6$, $BF = 4$, and $EG = 8$.



Find CF and DG.

Solution:

First, find CF. Use the fact that $\overline{CF}$ is part of a pair of intersecting chords.

$$AF \bullet CF = EF \bullet BF$$
$$8 \bullet CF = 6 \bullet 4$$
$$8 \bullet CF = 24$$
$$CF = 3$$

Next, find DG. Use the fact that $\overline{DG}$ is tangent to the circle.

$$EG \bullet BG = DG^2$$
$$8 \bullet (8 + 6 + 4) = DG^2$$
$$8 \bullet 18 = DG^2$$
$$144 = DG^2$$
$$\pm 12 = DG$$
$$12 = DG \text{ (since length cannot be negative)}$$

$CF = 3$ and $DG = 12$

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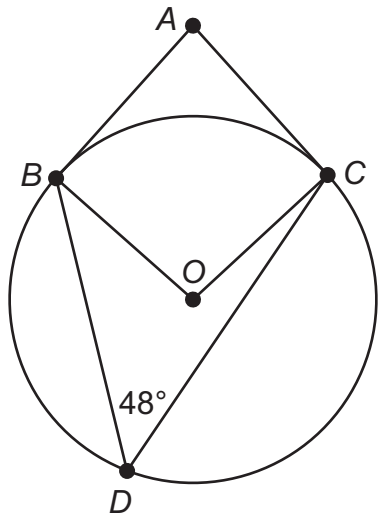
♦ In this circle, $\overline{AB}$ is tangent to the circle at point B, $\overline{AC}$ is tangent to the circle at point C, and point D lies on the circle. What is $m\angle BAC$?



Solution:

Method 1

First, find the measure of angle *BOC*. Angle *BDC* is an inscribed angle, and angle *BOC* is a central angle.

$$\begin{aligned} m\angle BOC &= 2(m\angle BDC) \\ &= 2 \bullet 48° \\ &= 96° \end{aligned}$$

Angle *BAC* is a circumscribed angle. Use the measure of angle *BOC* to find the measure of angle *BAC*.

$$\begin{aligned} m\angle BAC &= 180° - m\angle BOC \\ &= 180° - 96° \\ &= 84° \end{aligned}$$

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Method 2

Angle *BDC* is an inscribed angle. First, find the measures of $\widehat{BC}$ and $\widehat{BDC}$.

$$m\angle BDC = \frac{1}{2}\left(m\widehat{BC}\right)$$
$$48° = \frac{1}{2}\left(m\widehat{BC}\right)$$
$$2 \bullet 48° = m\widehat{BC}$$
$$96° = m\widehat{BC}$$

$$m\widehat{BDC} = 360° - m\widehat{BC}$$
$$= 360° - 96°$$
$$= 264°$$

Angle *BAC* is a circumscribed angle. Use the measures of $\widehat{BC}$ and $\widehat{BDC}$ to find the measure of angle *BAC*.

$$m\angle BAC = \frac{1}{2}\left(m\widehat{BDC} - m\widehat{BC}\right)$$
$$= \frac{1}{2}(264° - 96°)$$
$$= \frac{1}{2}(168°)$$
$$= 84°$$

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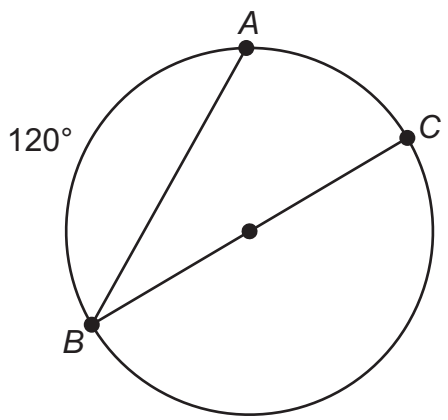
SAMPLE ITEMS

1. Circle *P* is dilated to form circle *P′*. Which statement is ALWAYS true?

A. The radius of circle *P* is equal to the radius of circle *P′*.
B. The length of any chord in circle *P* is greater than the length of any chord in circle *P′*.
C. The diameter of circle *P* is greater than the diameter of circle *P′*.
D. The ratio of the diameter to the circumference is the same for both circles.

2. In the circle shown, $\overline{BC}$ is a diameter and $m\widehat{AB} = 120°$.



What is the measure of $\angle ABC$?

A. 15°
B. 30°
C. 60°
D. 120°

Answers to Unit 4.1 Sample Items

1. D 2. B

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4.2 Find Arc Lengths and Areas of Sectors of Circles

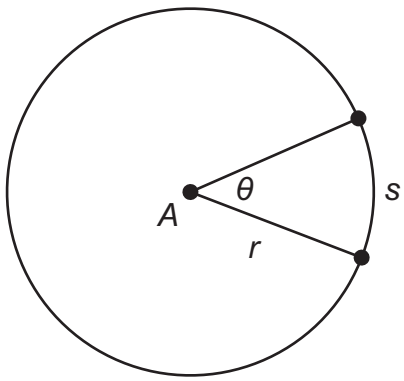
MGSE9-12.G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

KEY IDEAS

Circumference is the distance around a circle. The formula for the circumference, C, of a circle is $C = 2\pi r$, where r is the length of the radius of the circle.

Area is a measure of the amount of space a circle covers. The formula for the area, A, of a circle is $A = \pi r^2$, where r is the length of the radius of the circle.

Arc length is a portion of the circumference of a circle. To find the length of an arc, divide the number of degrees in the central angle of the arc by 360 and then multiply that amount by the circumference of the circle. The formula for the arc length, s, is $s = \frac{2\pi r\theta}{360}$, where θ is the degree measure of the central angle and r is the radius of the circle.



Important Tip

✍ Do not confuse arc length with the ***arc measure*** in degrees. Arc length depends on the size of the circle because it is part of the circumference of the circle. The measure of the arc is independent of (does not depend on) the size of the circle.

The following shows one way to remember the formula for arc length:

arc length = fraction of the circle × circumference = $s = \frac{2\pi r\theta}{360}$.

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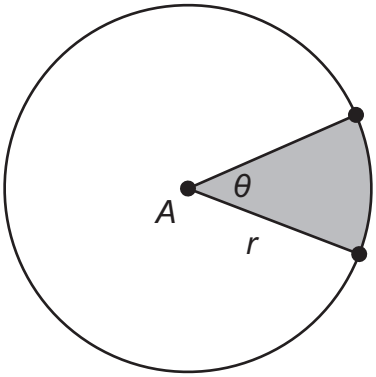
A ***sector*** of a circle is the region bounded by two radii of a circle and the resulting arc between them. To find the area of a sector, divide the number of degrees in the central angle of the arc by 360 and then multiply that amount by the area of the circle. The formula for the area of a sector is $\frac{\pi r^2 \theta}{360}$, where θ is the degree measure of the central angle and r is the radius of the circle.



Important Tip

✍ The following shows one way to remember the formula for the area of a sector:

area of a sector = fraction of the circle × area = $\frac{\theta}{360}(\pi r^2)$.

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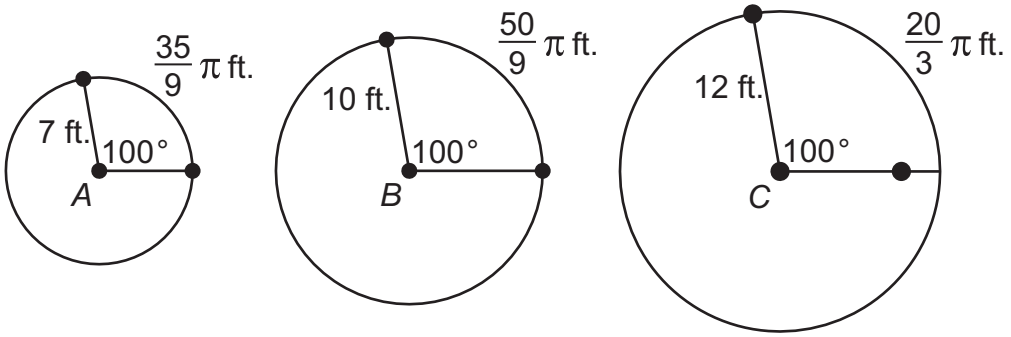
REVIEW EXAMPLES

♦ Circles *A*, *B*, and *C* have a central angle measuring 100°. The length of each radius and the length of each intercepted arc are shown.



a. What is the ratio of the radius of circle *B* to the radius of circle *A*?

b. What is the ratio of the length of the intercepted arc of circle *B* to the length of the intercepted arc of circle *A*?

c. Compare the ratios in parts (a) and (b).

d. What is the ratio of the radius of circle *C* to the radius of circle *B*?

e. What is the ratio of the length of the intercepted arc of circle *C* to the length of the intercepted arc of circle *B*?

f. Compare the ratios in parts (d) and (e).

g. Based on your observations of circles *A*, *B*, and *C*, what conjecture can you make about the length of the arc intercepted by a central angle and the radius?

h. What is the ratio of arc length to radius for each circle?

Solution:

a. Divide the radius of circle *B* by the radius of circle *A*: $\frac{\text{circle B}}{\text{circle A}} = \frac{10}{7}$

b. Divide the length of the intercepted arc of circle *B* by the length of the intercepted arc of circle *A*:

$$\frac{\frac{50}{9}\pi}{\frac{35}{9}\pi} = \frac{50\pi}{9} \bullet \frac{9}{35\pi} = \frac{10}{7}$$

c. The ratios are the same.

d. Divide the radius of circle *C* by the radius of circle *B*: $\frac{\text{circle C}}{\text{circle B}} = \frac{12}{10} = \frac{6}{5}$

e. Divide the length of the intercepted arc of circle *C* by the length of the intercepted arc of circle *B*:

$$\frac{\frac{20}{3}\pi}{\frac{50}{9}\pi} = \frac{20\pi}{3} \bullet \frac{9}{50\pi} = \frac{6}{5}$$

f. The ratios are the same.

g. When circles, such as circles *A*, *B*, and *C*, have the same central angle measure, the ratio of the lengths of the intercepted arcs is the same as the ratio of the radii.

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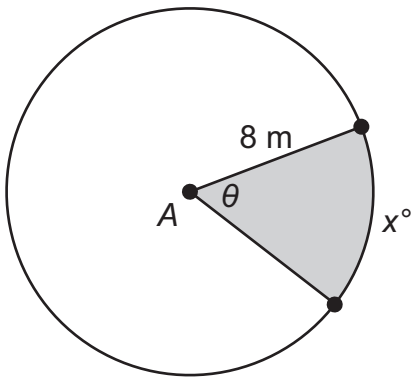
h. Circle A: $\dfrac{\frac{35}{9}\pi}{7} = \dfrac{35}{63}\pi = \dfrac{5}{9}\pi$

Circle B: $\dfrac{\frac{50}{9}\pi}{10} = \dfrac{50}{90}\pi = \dfrac{5}{9}\pi$

Circle C: $\dfrac{\frac{20}{3}\pi}{12} = \dfrac{20}{36}\pi = \dfrac{5}{9}\pi$

♦ Circle A is shown.



If $x = 50$, what is the area of the shaded sector of circle A?

Solution:

To find the area of the sector, divide the measure of the central angle of the arc in degrees by 360 and then multiply that amount by the area of the circle. The arc measure, x, is equal to the measure of the central angle, θ. The formula for the area of a circle is $A = \pi r^2$.

$A_{\text{sector}} = \dfrac{\pi r^2 \theta}{360}$ Area of sector of a circle with radius r and central angle θ in degrees

$A_{\text{sector}} = \dfrac{50\pi(8)^2}{360}$ Substitute 50 for θ and 8 for r.

$A_{\text{sector}} = \dfrac{5\pi(64)}{36}$ Rewrite the fraction and the power.

$A_{\text{sector}} = \dfrac{320\pi}{36}$ Multiply.

$A_{\text{sector}} = \dfrac{80\pi}{9}$ Rewrite.

The area of the sector is $\dfrac{80}{9}\pi$ square meters.

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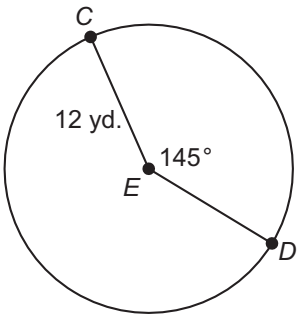
SAMPLE ITEMS

1. Circle *E* is shown.



What is the length of $\overset{\frown}{CD}$?

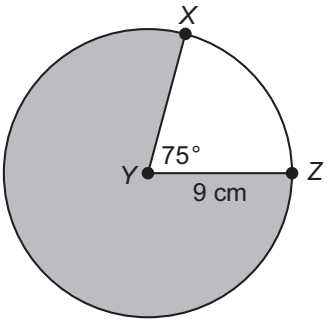
A. $\frac{29}{72}\pi$ yd.

B. $\frac{29}{6}\pi$ yd.

C. $\frac{29}{3}\pi$ yd.

D. $\frac{29}{2}\pi$ yd.

2. Circle *Y* is shown.



What is the area of the shaded part of the circle?

A. $\frac{57}{4}\pi$ cm^2

B. $\frac{135}{8}\pi$ cm^2

C. $\frac{405}{8}\pi$ cm^2

D. $\frac{513}{8}\pi$ cm^2

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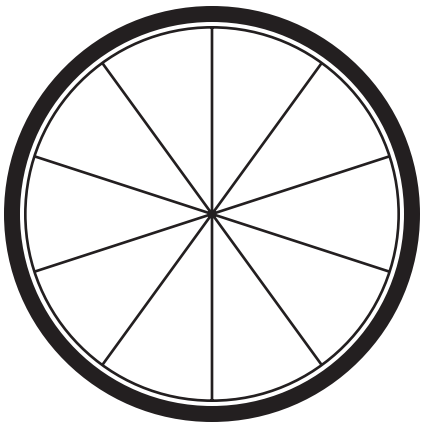
3. The spokes of a bicycle wheel form 10 congruent central angles. The diameter of the circle formed by the outer edge of the wheel is 18 inches.



What is the length, to the nearest tenth inch, of the outer edge of the wheel between two consecutive spokes?

A. 1.8 inches
B. 5.7 inches
C. 11.3 inches
D. 25.4 inches

Answers to Unit 4.2 Sample Items

1. C 2. D 3. B

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4.3 Explain Volume Formulas and Use Them to Solve Problems

MGSE9-12.G.GMD.1 Give informal arguments for geometric formulas.

a. Give informal arguments for the formulas of the circumference of a circle and area of a circle using dissection arguments and informal limit arguments.

b. Give informal arguments for the formula of the volume of a cylinder, pyramid, and cone using Cavalieri's principle.

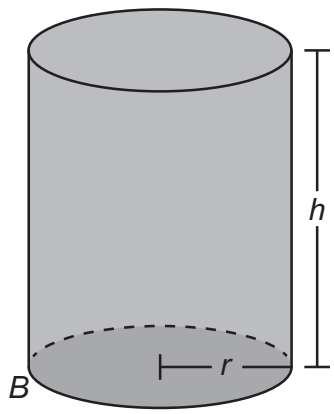
MGSE9-12.G.GMD.2 Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

MGSE9-12.G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

KEY IDEAS

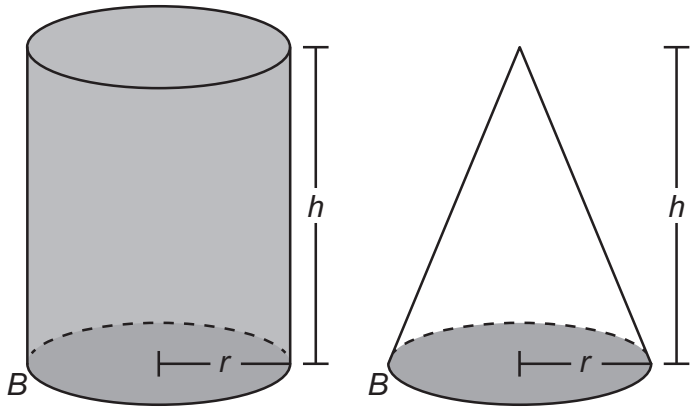
The ***volume*** of a figure is a measure of how much space it takes up. Volume is a measure of capacity.

The formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height. The volume formula can also be given as $V = Bh$, where B is the area of the base. In a cylinder, the base is a circle and the area of a circle is given by $A = \pi r^2$. Therefore, $V = Bh = \pi r^2 h$.



When a cylinder and a cone have congruent bases and equal heights, the volume of exactly three cones will fit into the cylinder. So, for a cone and cylinder that have the same radius r and height h, the volume of the cone is one-third of the volume of the cylinder.

The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height.



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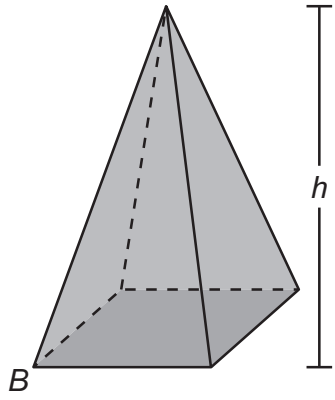
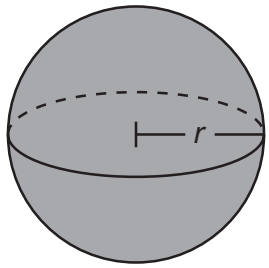
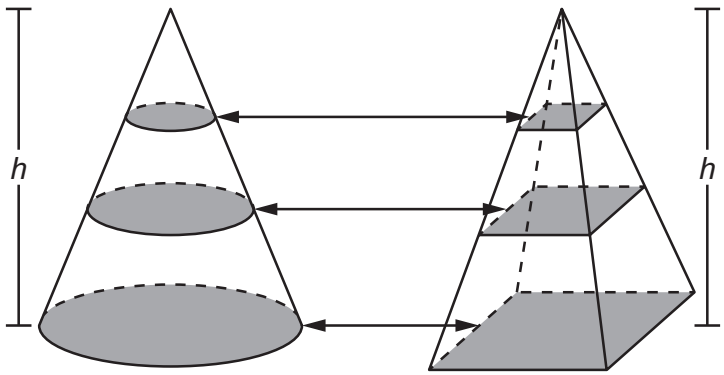
The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.



The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius.



Cavalieri's principle states that if two solids are between parallel planes and all cross sections at equal distances from their bases have equal areas, the solids have equal volumes. For example, this cone and this pyramid have the same height and the cross sections have the same area, so they have equal volumes.



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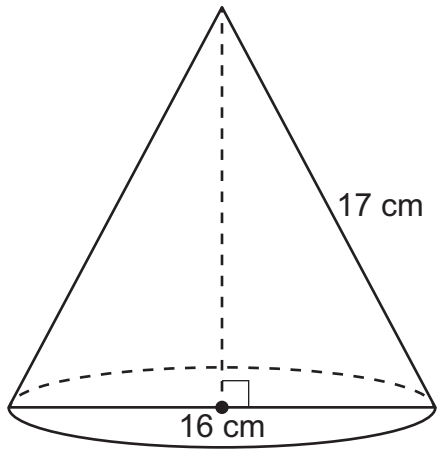
REVIEW EXAMPLES

- What is the volume of the cone shown below?



Solution:

The diameter of the cone is 16 cm. So the radius is 16 cm ÷ 2 = 8 cm. Use the Pythagorean Theorem, $a^2 + b^2 = c^2$, to find the height of the cone. Substitute 8 for b and 17 for c and solve for a:

$$a^2 + 8^2 = 17^2$$
$$a^2 + 64 = 289$$
$$a^2 = 225$$
$$a = 15$$

The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Substitute 8 for r and 15 for h:

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8)^2(15)$$

The volume is 320π cm^3.

- A sphere has a radius of 3 feet. What is the volume of the sphere?

Solution:

The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Substitute 3 for r and solve.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(3)^3$$
$$V = \frac{4}{3}\pi(27)$$
$$V = 36\pi \text{ ft}^3$$

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- A cylinder has a radius of 10 cm and a height of 9 cm. A cone has a radius of 10 cm and a height of 9 cm. Show that the volume of the cylinder is three times the volume of the cone.

  Solution:

  The formula for the volume of a cylinder is $V = \pi r^2 h$. Substitute 10 for r and 9 for h:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(10)^2(9) \\ &= \pi(100)(9) \\ &= 900\pi \text{ cm}^3 \end{aligned}$$

  The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Substitute 10 for r and 9 for h:

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(10)^2(9) \\ &= \frac{1}{3}\pi(100)(9) \\ &= 300\pi \text{ cm}^3 \end{aligned}$$

  Divide: $900\pi \div 300\pi = 3$

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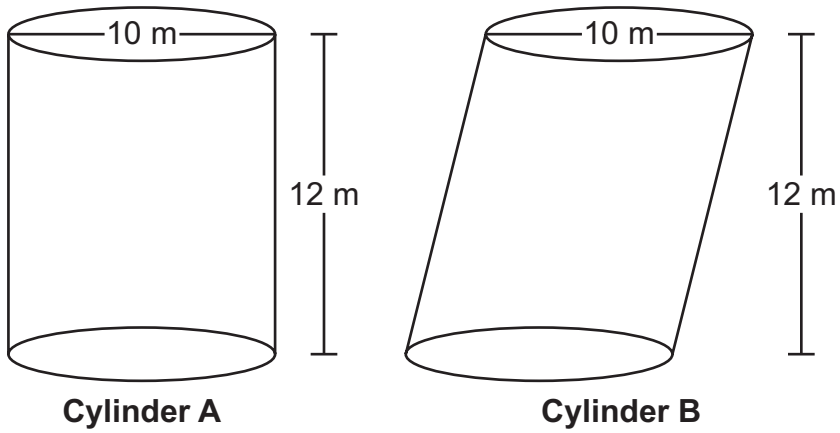
♦ Cylinder A and Cylinder B are shown below. What is the volume of each cylinder?

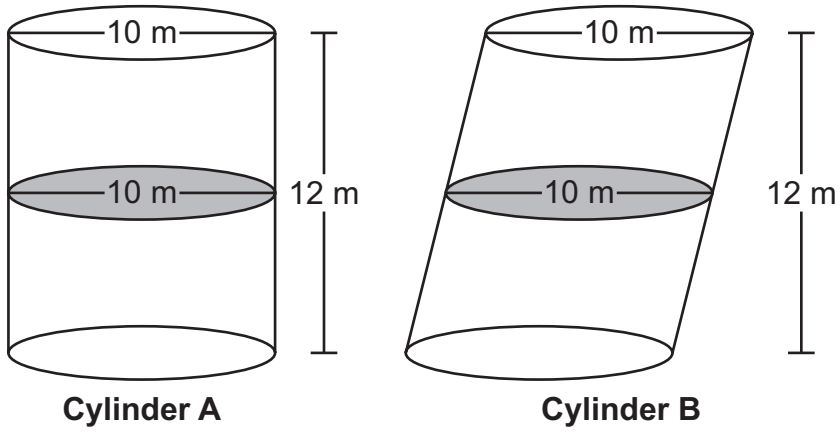


Solution**:**

To find the volume of Cylinder A, use the formula for the volume of a cylinder, which is $V = \pi r^2 h$. Divide the diameter by 2 to find the radius: $10 \div 2 = 5$. Substitute 5 for r and 12 for h:

$$\begin{aligned} V_{\text{Cylinder A}} &= \pi r^2 h \\ &= \pi(5)^2(12) \\ &= \pi(25)(12) \\ &= 300\pi \text{ m}^3 \\ &\approx 942 \text{ m}^3 \end{aligned}$$

Notice that Cylinder B has the same height and the same radius as Cylinder A. The only difference is that Cylinder B is slanted. For both cylinders, the cross section at every plane parallel to the bases is a circle with the same area. By Cavalieri's principle, the cylinders have the same volume; therefore, the volume of Cylinder B is 300π m^3, or about 942 m^3.



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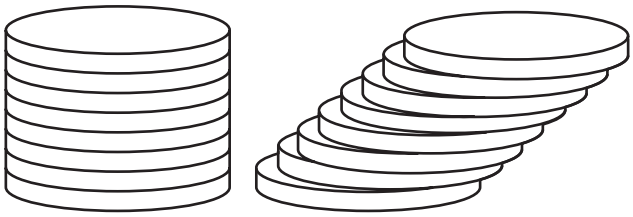
SAMPLE ITEMS

1. Jason constructed two cylinders using solid metal washers. The cylinders have the same height, but one of the cylinders is slanted as shown.



Which statement is true about Jason's cylinders?

A. The cylinders have different volumes because they have different radii.
B. The cylinders have different volumes because they have different surface areas.
C. The cylinders have the same volume because the washers are solid.
D. The cylinders have the same volume because they have the same cross-sectional area at every plane parallel to the bases.

2. What is the volume of a cylinder with a radius of 3 in. and a height of $\frac{9}{2}$ in.?

A. $\frac{81}{2}\pi$ in.^{3}
B. $\frac{27}{4}\pi$ in.^{3}
C. $\frac{27}{8}\pi$ in.^{3}
D. $\frac{9}{4}\pi$ in.^{3}

Answers to Unit 4.3 Sample Items

1. D 2. A

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UNIT 5: GEOMETRIC AND ALGEBRAIC CONNECTIONS

This unit investigates coordinate geometry. Students look at equations for circles and use given information to derive equations for representations of these figures on a coordinate plane. Students also use coordinates to prove simple geometric theorems using the properties of distance, slope, and midpoints. Students will verify whether a figure is a special quadrilateral by showing that sides of figures are parallel or perpendicular.

5.1 Apply Geometric Concepts in Modeling Situations

MGSE9-12.G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

MGSE9-12.G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

MGSE9-12.G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

KEY IDEAS

Modeling can be applied to describe real-life objects with geometric shapes.

Density is the mass of an object divided by its volume.

Population density can be determined by calculating the quotient of the number of people in an area and the area itself.

Apply constraints to maximize or minimize the cost of a cardboard box used to package a product that represents a geometric figure. Apply volume relationships of cylinders, pyramids, cones, and spheres.

REVIEW EXAMPLE

- A city has a population of 6,688 people. The area of the city is approximately 7.267 square miles.

  How many people per square mile live in the city?

  Solution:

  Find the quotient of 6,688 and 7.267 to find the number of people per square mile in the city.

  People per square mile $= \frac{6{,}688}{7.267} \approx 920$ people per square mile

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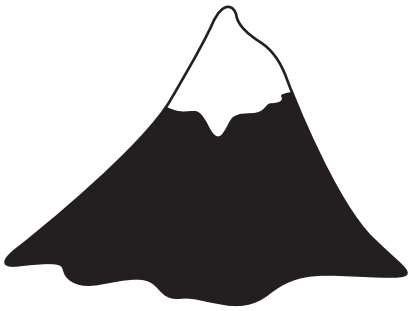
- This is a hand drawing of a mountain.



Explain which geometric shape could be used to estimate the total amount of earth the mountain is made of.

Solution:

The most accurate shape that could be used to model the mountain is a cone because, to determine the total amount of earth the mountain is made from, a three-dimensional shape is needed, which is why a triangle is not as accurate as a cone.

- A construction company is preparing 10 acres of land for a new housing community. The land contains large rocks that need to be removed. A machine removes 10 rocks from 360 square feet of land.

1 acre = 43,560 square feet

About how many rocks will need to be removed from the 10 acres of land?

Solution:

If there are 10 rocks in 360 square feet, then we can predict that there will be about 10 rocks every 360 square feet of land.

We will need to determine how many 360 square feet are in 10 acres.

10(43,560) = 435,600, so 435,600 square feet are in 10 acres.

$\frac{435{,}600}{360}$ = 1,210, so 1,210 parcels of 360 square feet are on the 10 acres.

(1,210)(10) = 12,100, so there should be about 12,100 rocks on the 10 acres of land.

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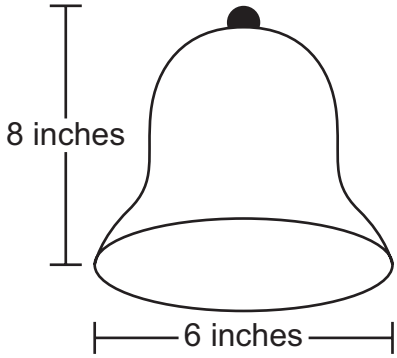
- A company needs to package this bell in a rectangular box.



What are the smallest dimensions (length, width, and height) the rectangular box can have so that the lid of the box can also close?

Solution:

Since the diameter of the base of the bell is 6 inches, the width and length of the box cannot be smaller than 6 inches. Since the height of the bell is 8 inches, then the height of the box cannot be smaller than 8 inches.

This gives us a rectangular box with these dimensions:

Length = 6 inches
Width = 6 inches
Height = 8 inches

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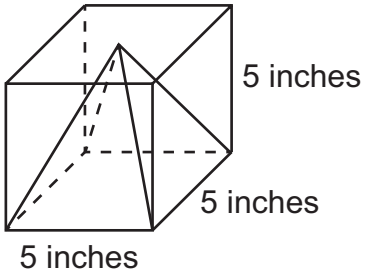
SAMPLE ITEMS

1. Joe counts 250 peach trees on 25% of the land he owns. He determines that there are 10 trees for every 1,000 square feet of land. About how many acres of land does Joe own?

1 acre = 43,560 square feet

A. 2.3
B. 10
C. 43.56
D. 2,500

2. A square pyramid is packaged inside a box.



The space inside the box around the pyramid is then filled with protective foam. About how many cubic inches of foam is needed to fill the space around the pyramid?

A. 8
B. 41
C. 83
D. 125

Answers to Unit 5.1 Sample Items

1. A 2. C

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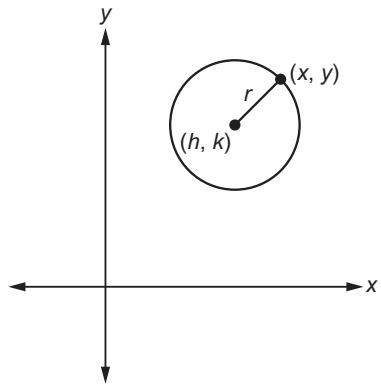
5.2 Translate between the Geometric Description and the Equation for a Conic Section

MGSE9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

KEY IDEAS

A ***circle*** is the set of points in a plane equidistant from a given point, or center, of the circle.

The standard form of the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the ***center*** of the circle and r is the ***radius*** of the circle.

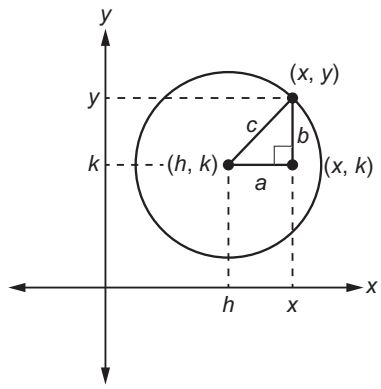


The equation of a circle can be derived from the ***Pythagorean Theorem***; $a^2 + b^2 = c^2$.

Example: Given a circle with a center at (h, k) and a point (x, y) on the circle, draw a horizontal line segment from (h, k) to (x, k). Label this line segment a. Draw a vertical line segment from (x, y) to (x, k). Label this line segment b. Label the radius c. A right triangle is formed.

The length of line segment a is given by $(x - h)$.

The length of line segment b is given by $(y - k)$.



Using the Pythagorean Theorem, substitute $(x - h)$ for a, $(y - k)$ for b, and r for c in the equation.

$a^2 + b^2 = c^2$	Use the Pythagorean Theorem.
$(x - h)^2 + (y - k)^2 = r^2$	Substitution

The equation for a circle with a center at (h, k) and a radius r is $(x - h)^2 + (y - k)^2 = r^2$.

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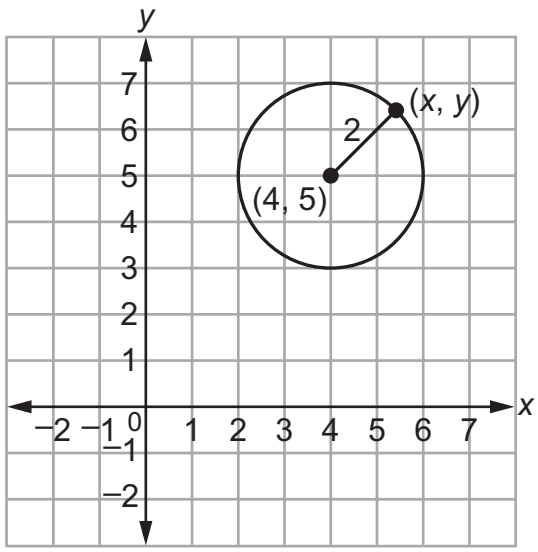
REVIEW EXAMPLES

- What is the equation of the circle with a center at (4, 5) and a radius of 2?



Solution:

Use the standard form for the equation for a circle: $(x - h)^2 + (y - k)^2 = r^2$. Substitute the values into the equation, with $h = 4$, $k = 5$, and $r = 2$.

$(x - h)^2 + (y - k)^2 = r^2$	Equation for a circle
$(x - (4))^2 + (y - (5))^2 = (2)^2$	Substitute the values in the equation of a circle.
$(x - 4)^2 + (y - 5)^2 = 4$	Evaluate.

The equation of a circle with a center at (4, 5) and a radius of 2 is $(x - 4)^2 + (y - 5)^2 = 4$, or $x^2 + y^2 - 8x - 10y + 37 = 0$ when expanded.

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♦ What is the center and radius of the circle given by $8x^2 + 8y^2 - 16x - 32y + 24 = 0$?

Solution:

Write the equation in standard form to identify the center and radius of the circle. First, write the equation so the x-terms are next to each other and the y-terms are next to each other, both on the left side of the equation, and the constant term is on the right side of the equation.

$8x^2 + 8y^2 - 16x - 32y + 24 = 0$	Original equation
$8x^2 + 8y^2 - 16x - 32y = -24$	Subtract 24 from both sides.
$8x^2 - 16x + 8y^2 - 32y = -24$	Commutative Property
$x^2 - 2x + y^2 - 4y = -3$	Divide both sides by 8.
$(x^2 - 2x) + (y^2 - 4y) = -3$	Associative Property

Next, to write the equation in standard form, complete the square for the x-terms and the y-terms.

Using $ax^2 + bx + c = 0$, find $\left(\frac{b}{2a}\right)^2$ for the x- and y-terms.

x-term: $\left(\frac{b}{2a}\right)^2 = \left(\frac{-2}{2(1)}\right)^2 = (-1)^2 = 1$

y-term: $\left(\frac{b}{2a}\right)^2 = \left(\frac{-4}{2(1)}\right)^2 = (-2)^2 = 4$

$(x^2 - 2x + 1) + (y^2 - 4y + 4) = -3 + 1 + 4$	Add 1 and 4 to each side of the equation.
$(x - 1)^2 + (y - 2)^2 = 2$	Write the trinomials as squares of binomials.

This equation for the circle is written in standard form, where $h = 1$, $k = 2$, and $r^2 = 2$. The center of the circle is (1, 2), and the radius is $\sqrt{2}$.

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SAMPLE ITEMS

1. Which is an equation for the circle with a center at (–2, 3) and a radius of 3?

A. $x^2 + y^2 + 4x - 6y + 22 = 0$
B. $2x^2 + 2y^2 + 3x - 3y + 4 = 0$
C. $x^2 + y^2 + 4x - 6y + 4 = 0$
D. $3x^2 + 3y^2 + 4x - 6y + 4 = 0$

2. What is the center of the circle given by the equation $x^2 + y^2 - 10x - 11 = 0$?

A. (5, 0)
B. (0, 5)
C. (–5, 0)
D. (0, –5)

Answers to Unit 5.2 Sample Items

1. C 2. A

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5.3 Use Coordinates to Prove Simple Geometric Theorems Algebraically

MGSE9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*

MGSE9-12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

MGSE9-12.G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

MGSE9-12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

KEY IDEAS

Given the equation of a circle, you can verify whether a point lies on the circle by substituting the coordinates of the point into the equation. If the resulting equation is true, then the point lies on the figure. If the resulting equation is not true, then the point does not lie on the figure.

Given the center and radius of a circle, you can verify whether a point lies on the circle by determining whether the distance between the given point and the center is equal to the radius.

To prove properties about special parallelograms on a coordinate plane, you can use the partitioning of a segment, distance, and slope formulas:

- The ***partitioning of a segment formula*** is $(x, y) = \left(\frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a}\right)$ or $(x, y) = (x_1 + \frac{a}{a + b}(x_2 - x_1), y_1 + \frac{a}{a + b}(y_2 - y_1)$. This formula is used to find the coordinates of a point that partitions a directed line segment AB at the ratio of a:b from $A(x_1, y_1)$ to $B(x_2, y_2)$. This formula can also be used to derive the midpoint formula.

  The ***midpoint formula*** is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. This formula is used to find the coordinates of the midpoint of $\overline{AB}$, given $A(x_1, y_1)$ and $B(x_2, y_2)$.

- The ***distance formula*** is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. This formula is used to find the length of $\overline{AB}$, given $A(x_1, y_1)$ and $B(x_2, y_2)$.

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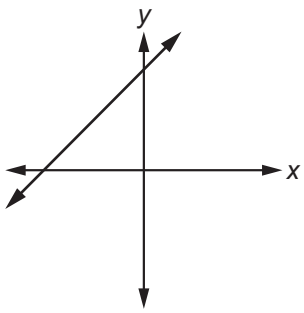
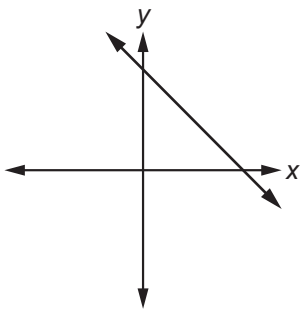
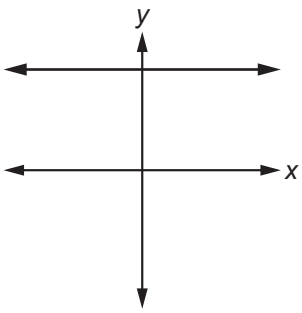
- The ***slope formula*** is $m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$. This formula is used to find the slope of a line or line segment, given any two points on the line or line segment $A(x_1, y_1)$ and $B(x_2, y_2)$. Slopes can be positive, negative, 0, or undefined.



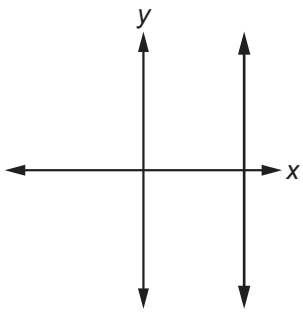
A line with a ***positive slope*** slants up to the right.



A line with a ***negative slope*** slants down to the right.



A line with a ***slope of 0*** is horizontal.



A line with an ***undefined slope*** is vertical.

To prove a triangle is isosceles, you can use the distance formula to show that at least two sides are congruent.

You can use properties of quadrilaterals to help prove theorems:

- To prove a quadrilateral is a parallelogram, show that the opposite sides are parallel using slope.
- To prove a quadrilateral is a rectangle, show that the opposite sides are parallel and the consecutive sides are perpendicular using slope.
- To prove a quadrilateral is a rhombus, show that all four sides are congruent using the distance formula.
- To prove a quadrilateral is a square, show that all four sides are congruent and consecutive sides are perpendicular using slope and the distance formula.

You can also use diagonals of a quadrilateral to help prove theorems:

- To prove a quadrilateral is a parallelogram, show that its diagonals bisect each other using the midpoint formula.
- To prove a quadrilateral is a rectangle, show that its diagonals bisect each other and are congruent using the midpoint and distance formulas.
- To prove a quadrilateral is a rhombus, show that its diagonals bisect each other and are perpendicular using the midpoint and slope formulas.
- To prove a quadrilateral is a square, show that its diagonals bisect each other, are congruent, and are perpendicular using the midpoint, distance, and slope formulas.

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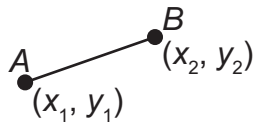
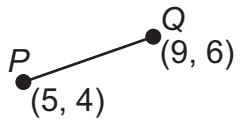
A ***directed line segment*** is a line segment from one point to another point in the coordinate plane.



Example: Notice on $\overline{PQ}$ we can subtract to find the difference of the *x*- and *y*-values of *Q* and *P*:

(9 – 5, 6 – 4) = (4, 2). They tell you that a "route" from *P* to *Q* is 4 units right and 2 units up. Note that these are used in the slope: $\frac{6-4}{9-5} = \frac{2}{4} = \frac{1}{2}$.



Important Tips

- When using the formulas for partitioning of a segment, distance, and slope, the order of the points does not matter. You can use either point as (x_1, y_1) and (x_2, y_2), but be careful to always subtract in the same order.
- Parallel lines have the same slope. Perpendicular lines have slopes that are the negative reciprocal of each other.
- When using directed line segments, pay close attention to the beginning and end points of the line. For example, the directed line segments $\overline{PQ}$ and $\overline{QP}$ have the same length but different directions.

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REVIEW EXAMPLES

- Quadrilateral *ABCD* has vertices *A*(–1, 3), *B*(3, 5), *C*(4, 3), and *D*(0, 1). Is *ABCD* a rectangle? Explain how you know.

  Solution:

  First determine whether the figure is a parallelogram. If the figure is a parallelogram, then the diagonals bisect each other. If the diagonals bisect each other, then the midpoints of the diagonals are the same point. Use the midpoint formula to determine the midpoints for each diagonal.

  Midpoint $\overline{AC}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1 + 4}{2}, \frac{3 + 3}{2}\right) = \left(\frac{3}{2}, \frac{6}{2}\right) = (1.5, 3)$

  Midpoint $\overline{BD}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + 0}{2}, \frac{5 + 1}{2}\right) = \left(\frac{3}{2}, \frac{6}{2}\right) = (1.5, 3)$

  The diagonals have the same midpoint; therefore, the diagonals bisect each other and the figure is a parallelogram.

  A parallelogram with congruent diagonals is a rectangle. Determine whether the diagonals are congruent.

  Use the distance formula to find the length of the diagonals:

  $AC = \sqrt{(4 - (-1))^2 + (3 - 3)^2} = \sqrt{(5)^2 + (0)^2} = \sqrt{25 + 0} = \sqrt{25} = 5$

  $BD = \sqrt{(0 - 3)^2 + (1 - 5)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

  The diagonals are congruent because they have the same length.

  The figure is a parallelogram with congruent diagonals, so the figure is a rectangle.

- Circle *C* has a center of (–2, 3) and a radius of 4. Does point (–4, 6) lie on circle *C*?

  Solution:

  The distance from any point on the circle to the center of the circle is equal to the radius. Use the distance formula to find the distance from (–4, 6) to the center (–2, 3). Then see if it is equal to the radius, 4.

$\sqrt{(-4 - (-2))^2 + (6 - 3)^2}$	Substitute the coordinates of the points in the distance formula.
$\sqrt{(-2)^2 + (3)^2}$	Evaluate within parentheses.
$\sqrt{4 + 9}$	Evaluate the exponents.
$\sqrt{13}$	Add.

  The distance from (–4, 6) to (–2, 3) is not equal to the radius, so (–4, 6) does not lie on the circle. (In fact, since $\sqrt{13} < 4$, the distance is less than the radius, so the point lies inside the circle.)

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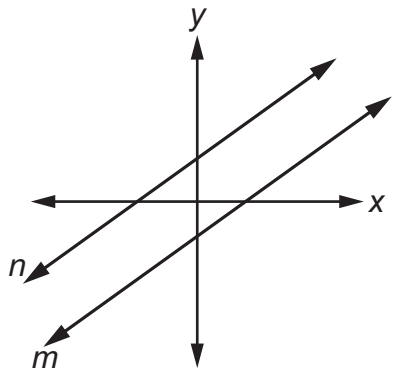
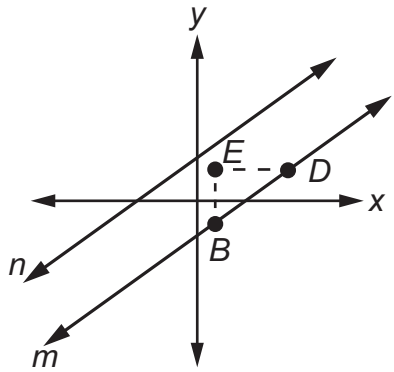
♦ Follow the steps below to prove that if two nonvertical lines are parallel, then they have equal slopes.

a. Let the straight lines *n* and *m* be parallel. Sketch these on a coordinate grid.

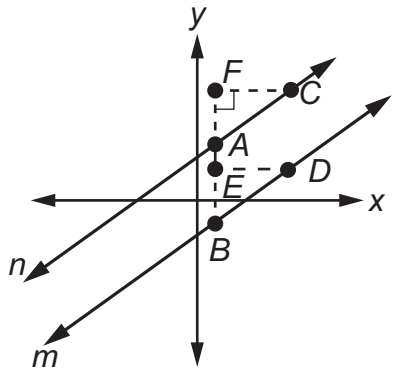


b. Plot any points *B* and *D* on line *m* and the point *E* so that segment *BE* is the rise and segment *ED* is the run of the slope of line *m*. (A straight line can have only one slope.)



That is, the slope of line *m* is $\frac{BE}{ED}$.

c. Draw the straight line *BE* so that it intersects line *n* at point *A* and extends to include point *F* such that segment *FC* is perpendicular to *BE*.



d. Look at the figure above. What is the slope of line *n*?

Solution:

The slope is $\frac{AF}{FC}$.

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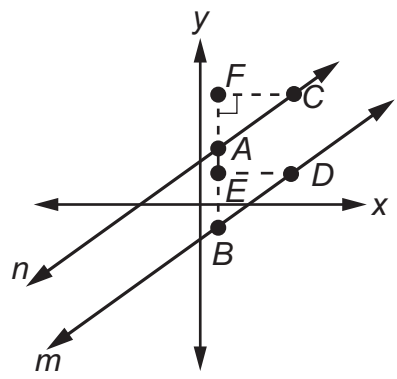


e. Line *BF* is the _____ of lines *m* and *n*, so $\angle EBD$ and $\angle FAC$ are _____ angles, which means $\angle EBD$ ____ $\angle FAC$.

Solution:

transversal, corresponding, $\cong$

f. Why is it true that $\angle DEB \cong \angle AFC$?

Solution:

The angles are right angles.

g. Now, $\triangle DEB$ and $\triangle CFA$ are similar, so the ratio of their sides is proportional. Write the proportion that relates the vertical leg to the horizontal leg of the triangles.

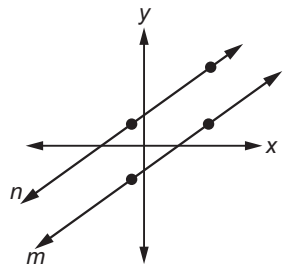
Solution:

$$\frac{BE}{ED} \cong \frac{AF}{FC}$$

h. Note that this proportion shows the slope of line *m* is the same as the slope of line *n*. Therefore, parallel lines have the same slope.

♦ Prove that if two nonvertical lines have equal slopes, then they are parallel.

Solution:



Use a proof by contradiction. Assume that the lines have equal slopes but are not parallel—that is, assume the lines intersect. If you can show this is not true, it is equivalent to proving the original statement.

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Write the equations for both lines. The slopes are the same, so use m for the slope of each line. The two lines are different, so $b_1 \neq b_2$.

Equation for line n: $y = mx + b_1$

Equation for line m: $y = mx + b_2$

Solve a system of equations to find the point of intersection. Both equations are solved for y, so use substitution.

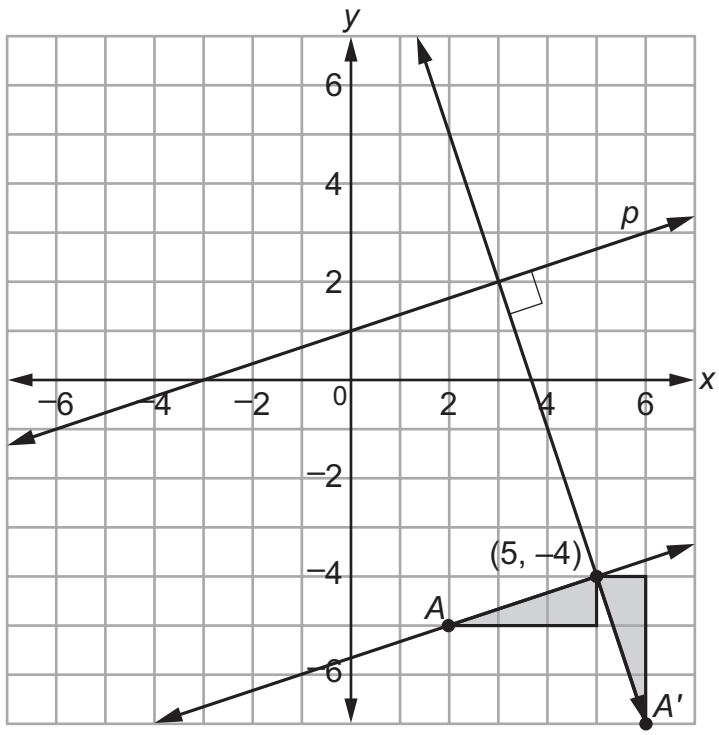
$$mx + b_1 = mx + b_2$$
$$b_1 = b_2$$

Because it was assumed that $b_1 \neq b_2$, this is a contradiction. So the original statement is true—if two nonvertical lines have equal slopes, then they are parallel.

♦ The line p is represented by the equation $y = \frac{1}{3}x + 1$. What is the equation of the line that is perpendicular to line p and passes through the point (5, –4)?

Solution:

On a coordinate grid, graph the line $y = \frac{1}{3}x + 1$ and the line parallel to $y = \frac{1}{3}x + 1$ that passes through the point (5, –4), which is $y = \frac{1}{3}x - \frac{17}{3}$. Next, choose any point on the parallel line, (2, –5) for example, and name the point A. Form a right triangle with the point (5, –4) and point A. Rotate the triangle 90° around the point (5, –4). Use the hypotenuse of the new triangle to draw the perpendicular line. The new line has a slope of –3 and passes through the point (5, –4).



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The slope-intercept form of the equation of a line is $y = mx + b$. Substitute –3 for m. The line that is perpendicular to line p passes through the point (5, –4). So substitute 5 in for x and –4 for y. Solve for b.

$$-4 = -3(5) + b$$
$$-4 = -15 + b$$
$$11 = b$$

The equation of the line that is perpendicular to line p and passes through the point (5, –4) is $y = -3x + 11$.

♦ For what value of n are the lines $7x + 3y = 8$ and $nx + 3y = 8$ perpendicular?

Solution:

The two lines will be perpendicular when the slopes are opposite reciprocals.

First, find the slope of the line $7x + 3y = 8$.

$$7x + 3y = 8$$
$$3y = -7x + 8$$
$$y = -\frac{7}{3}x + \frac{8}{3}$$

The slope is $-\frac{7}{3}$.

Next, find the slope of the line $nx + 3y = 8$, in terms of n.

$$nx + 3y = 8$$
$$3y = -nx + 8$$
$$y = -\frac{n}{3}x + \frac{8}{3}$$

The slope is $-\frac{n}{3}$.

The opposite reciprocal of $-\frac{7}{3}$ is $\frac{3}{7}$. Find the value of n that makes the slope of the second line $\frac{3}{7}$.

$$-\frac{n}{3} = \frac{3}{7}$$
$$-n = \frac{9}{7}$$
$$n = -\frac{9}{7}$$

When $n = -\frac{9}{7}$, the two lines are perpendicular.

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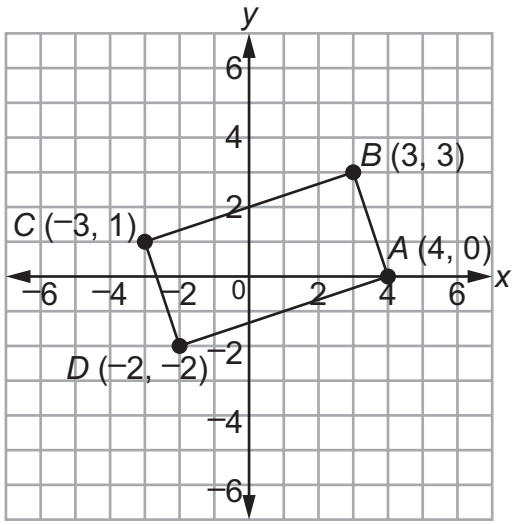
♦ Quadrilateral *ABCD* has vertices *A*(4, 0), *B*(3, 3), *C*(–3, 1), and *D*(–2, –2). Prove that *ABCD* is a rectangle.

Solution:

The slopes of the sides are:

$\overline{AB}$: $\frac{3-0}{3-4} = \frac{3}{-1} = -3$ $\overline{BC}$: $\frac{1-3}{-3-3} = \frac{-2}{-6} = \frac{1}{3}$

$\overline{CD}$: $\frac{-2-1}{-2+3} = \frac{-3}{1} = -3$ $\overline{DA}$: $\frac{0+2}{4+2} = \frac{2}{6} = \frac{1}{3}$



$\overline{AB} \parallel \overline{CD}$ because they have equal slopes.

$\overline{BC} \parallel \overline{DA}$ because they have equal slopes.

So *ABCD* is a parallelogram because both pairs of opposite sides are parallel.

$\overline{AB} \perp \overline{BC}$ because the product of their slopes is –1: $-3 \bullet \frac{1}{3} = -1$.

Therefore, *ABCD* is a rectangle because it is a parallelogram with a right angle.

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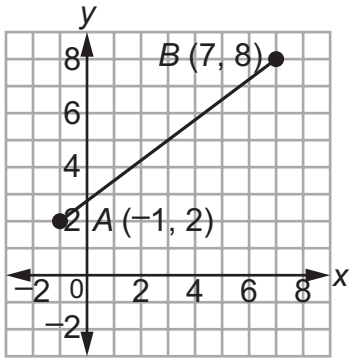
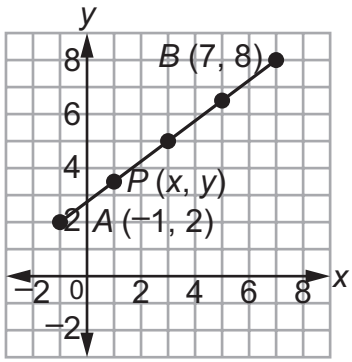
♦ Given the points $A(-1, 2)$ and $B(7, 8)$, find the coordinates of the point P on directed line segment $\overline{AB}$ that partitions $\overline{AB}$ in the ratio 1:3.



Solution:

Point P partitions $\overline{AB}$ in the ratio 1:3 if P is on $\overline{AB}$. This means that you need to divide $\overline{AB}$ into 4 equal parts, since $\overline{AP}$ is one-fourth of $\overline{AB}$.



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Let $P(x, y)$ be on $\overline{AB}$. Solve two equations to find x and y, where (x_1, y_1) is the starting point, (x_2, y_2) is the ending point, and $\frac{a}{(a+b)} = \frac{1}{4}$.

$$(x, y) = \left(x_1 + \frac{a}{a+b}(x_2 - x_1), y_1 + \frac{a}{a+b}(y_2 - y_1)\right)$$

$$(x, y) = \left(-1 + \frac{1}{4}(7 - (-1)), 2 + \frac{1}{4}(8 - 2)\right)$$

$$x = -1 + \frac{1}{4}(7 - (-1)) \qquad y = 2 + \frac{1}{4}(8 - 2)$$

$$x = -1 + \frac{1}{4}(8) \qquad y = 2 + \frac{1}{4}(6)$$

$$x = -1 + 2 \qquad y = 2 + \frac{3}{2}$$

$$x = 1 \qquad y = \frac{7}{2}$$

The coordinates of P are $\left(1, \frac{7}{2}\right)$.

Here is another method for partitioning segment AB in the ratio 1:3 from $A(x_1, y_1)$ to $B(x_2, y_2)$.

$$\begin{aligned}(x, y) &= \frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a} \\ &= \frac{3(-1) + 1(7)}{3 + 1}, \frac{3(2) + 1(8)}{3 + 1} \\ &= \frac{-3 + 7}{4}, \frac{6 + 8}{4} \\ &= \frac{4}{4}, \frac{14}{4} \\ &= (1, \frac{7}{2})\end{aligned}$$

Important Tip

✍ Be careful when using directed line segments. If point P partitions $\overline{AB}$ in the ratio 1:3, then point P partitions $\overline{BA}$ in the ratio 3:1.

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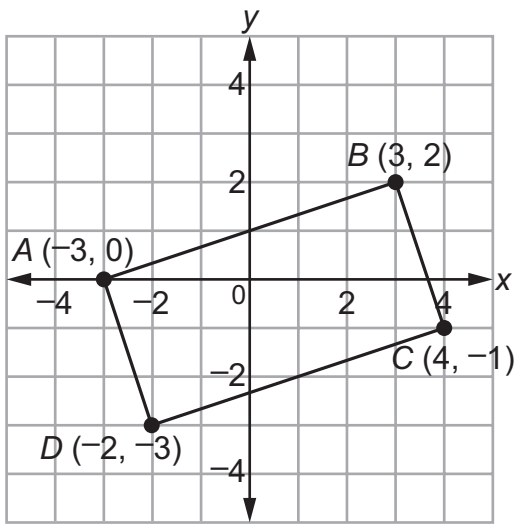
♦ Find the area of rectangle *ABCD* with vertices *A*(–3, 0), *B*(3, 2), *C*(4, –1), and *D*(–2, –3).

Solution:



One strategy is to use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, to find the length and width of the rectangle.

$$AB = \sqrt{(3 - (-3))^2 + (2 - 0)^2} = \sqrt{36 + 4} = \sqrt{40}$$

$$BC = \sqrt{(4 - 3)^2 + (-1 - 2)^2} = \sqrt{1 + 9} = \sqrt{10}$$

The length of the rectangle is usually considered to be the longer side. Therefore, the length of the rectangle is $\sqrt{40}$ and the width is $\sqrt{10}$.

NOTE: Other strategies are possible to find the length of *AB* and *BC*, such as using the Pythagorean Theorem.

Use the area formula.

$$A = lw$$

$$A = (\sqrt{40})(\sqrt{10})$$

$$A = \sqrt{400}$$

$$A = 20$$

The area of the rectangle is 20 square units.

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SAMPLE ITEMS

1. Which information is needed to show that a parallelogram is a rectangle?

A. The diagonals bisect each other.
B. The diagonals are congruent.
C. The diagonals are congruent and perpendicular.
D. The diagonals bisect each other and are perpendicular.

2. Which point is on a circle with a center of (3, –9) and a radius of 5?

A. (–6, 5)
B. (–1, 6)
C. (1, 6)
D. (6, –5)

3. Given the points $P(2, -1)$ and $Q(-9, -6)$, what are the coordinates of the point on directed line segment $\overline{PQ}$ that partitions $\overline{PQ}$ in the ratio $\frac{3}{2}$?

A. $\left(-\frac{23}{5}, -4\right)$

B. $\left(-\frac{12}{5}, -3\right)$

C. $\left(\frac{5}{3}, \frac{8}{3}\right)$

D. $\left(-\frac{5}{3}, -\frac{8}{3}\right)$

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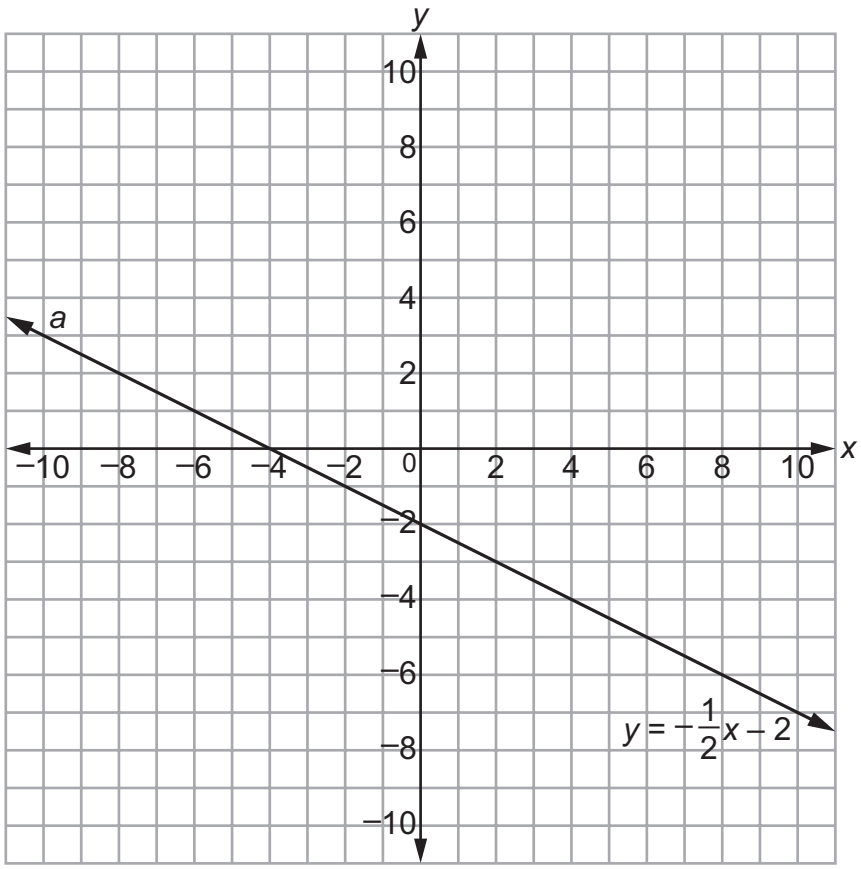
4. An equation of line *a* is $y = -\frac{1}{2}x - 2$.



Which equation is an equation of the line that is perpendicular to line *a* and passes through the point (–4, 0)?

A. $y = -\frac{1}{2}x + 2$

B. $y = -\frac{1}{2}x + 8$

C. $y = 2x - 2$

D. $y = 2x + 8$

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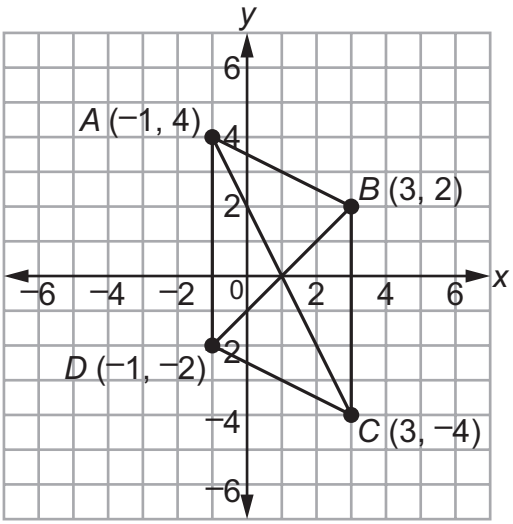
5. Parallelogram *ABCD* has vertices as shown.



Which equation would be used in proving that the diagonals of parallelogram *ABCD* bisect each other?

A. $\sqrt{(3-1)^2+(2-0)^2}=\sqrt{(1-3)^2+(0+4)^2}$

B. $\sqrt{(3+1)^2+(2+0)^2}=\sqrt{(1+3)^2+(0-4)^2}$

C. $\sqrt{(-1-1)^2+(4-0)^2}=\sqrt{(1-3)^2+(0+4)^2}$

D. $\sqrt{(-1+1)^2+(4+0)^2}=\sqrt{(1+3)^2+(0-4)^2}$

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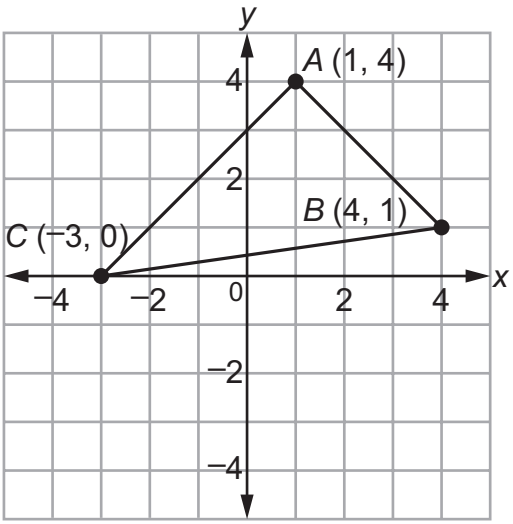
6. Triangle *ABC* has vertices as shown.



What is the area of the triangle?

A. $\sqrt{72}$ square units

B. 12 square units

C. $\sqrt{288}$ square units

D. 24 square units

Answers to Unit 5.3 Sample Items

1. B 2. D 3. A 4. D 5. C 6. B

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UNIT 6: APPLICATIONS OF PROBABILITY

This unit investigates the concept of probability. Students look at sample spaces and identify unions, intersections, and complements. They identify ways to tell whether events are independent. The concept of conditional probability is related to independence, and students use the concepts to solve real-world problems, including those that are presented in two-way frequency tables. Students find probabilities of compound events using the rules of probability.

6.1 Understand Independence and Conditional Probability and Use Them to Interpret Data

MGSE9-12.S.CP.1 Describe categories of events as subsets of a sample space using unions, intersections, or complements of other events (*or, and, not*).

MGSE9-12.S.CP.2 Understand that if two events A and B are independent, the probability of A and B occurring together is the product of their probabilities, and that if the probability of two events A and B occurring together is the product of their probabilities, the two events are independent.

MGSE9-12.S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B). Interpret independence of A and B in terms of conditional probability; that is, the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

MGSE9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, use collected data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

MGSE9-12.S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

KEY IDEAS

In probability, a ***sample space*** is the set of all possible outcomes. Any subset from the sample space is an ***event***.

If the outcome of one event does not change the possible outcomes of the other event, the events are ***independent***. If the outcome of one event does change the possible outcomes of the other event, the events are ***dependent***.

The ***intersection*** of two or more events is all of the outcomes shared by both events. The intersection is denoted with the word “and” or with the $\cap$ symbol. For example, the intersection of A and B is shown as $A \cap B$.

The ***union*** of two or more events is all of the outcomes for either event. The union is denoted with the word “or” or with the $\cup$ symbol. For example, the union of A and B is shown as $A \cup B$. The probability of the union of two events that have no outcomes in common is the sum of each individual probability.

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The ***complement*** of an event is the set of outcomes in the same sample space that are not included in the outcomes of the event. The complement is denoted with the word "not" or with the $'$ symbol. For example, the complement of A is shown as A'. The set of outcomes and its complement make up the entire sample space.

Conditional probabilities are found when one event has already occurred and a second event is being analyzed. Conditional probability is denoted $P(A \mid B)$ and is read as "the probability of A given B."

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Two events—A and B—are independent if the probability of the intersection is the same as the product of each individual probability. That is, $P(A \text{ and } B) = P(A) \bullet P(B)$. This is called the ***Multiplication Rule for Independent Events***.

If two events are independent, then $P(A \mid B) = P(A)$ *and* $P(B \mid A) = P(B)$.

Two-way frequency tables summarize data in two categories. These tables can be used to show whether the two events are independent and to approximate conditional probabilities.

Example: A random survey was taken to gather information about grade level and car ownership status of students at a school. This table shows the results of the survey.

Car Ownership by Grade

	Owns a Car	**Does Not Own a Car**	**Total**
Junior	6	10	16
Senior	12	8	20
Total	18	18	36

Estimate the probability that a randomly selected student will be a junior given that the student owns a car.

Let $P(J)$ be the probability that the student is a junior. Let $P(C)$ be the probability that the student owns a car.

$$P(J \mid C) = \frac{P(J \text{ and } C)}{P(C)} = \frac{\frac{6}{36}}{\frac{18}{36}} = \frac{6}{18} = \frac{1}{3}$$

The probability that a randomly selected student will be a junior given that the student owns a car is $\frac{1}{3}$.

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REVIEW EXAMPLES

- This chart shows the names of students in Mr. Leary's class sorted by bicycle and skateboard ownership.

Bicycle and Skateboard Ownership

Owns a Bicycle	Owns a Skateboard	Owns a Bicycle AND a Skateboard	Does NOT Own a Bicycle OR a Skateboard
Ryan	Brett	Joe	Amy
Sarah	Juan	Mike	Gabe
Mariko	Tobi	Linda	Abi
Nina		Rose	
Dion			

Let set *A* be the names of students who own bicycles, and let set *B* be the names of students who own skateboards.

a. Find *A* and *B.* What does the set represent?

b. Find *A* or *B.* What does the set represent?

c. Find (*A* or *B*)′. What does the set represent?

Solution:

a. The intersection is the set of elements that are common to both set *A* and set *B*, so *A* and *B* is {Joe, Mike, Linda, Rose}. This set represents the students who own both a bicycle and a skateboard.

b. The union is the set of elements that are in set *A* or set *B* or in both set *A* and set *B*. You need to list the names in the intersection only one time, so *A* or *B* is {Ryan, Sarah, Mariko, Nina, Dion, Brett, Juan, Tobi, Joe, Mike, Linda, Rose}. This set represents the students who own a bicycle, a skateboard, or both.

c. The complement of *A* or *B* is the set of names that are not in *A* or *B*. So the complement of *A* or *B* is {Amy, Gabe, Abi}. This set represents the students who own neither a bicycle nor a skateboard.

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- In a certain town, the probability that a person plays sports is 65%. The probability that a person is between the ages of 12 and 18 is 40%. The probability that a person plays sports and is between the ages of 12 and 18 is 25%. Are the events independent? How do you know?

  Solution:

  Let $P(S)$ be the probability that a person plays sports.

  Let $P(A)$ be the probability that a person is between the ages of 12 and 18.

  If the two events are independent, then $P(S \text{ and } A) = P(S) \bullet P(A)$.

  Because $P(S \text{ and } A)$ is given as 25%, find $P(S) \bullet P(A)$ and then compare.

  $$\begin{aligned} P(S) \bullet P(A) &= 0.65 \bullet 0.4 \\ &= 0.26 \end{aligned}$$

  Because $0.26 \neq 0.25$, the events are not independent.

- A random survey was conducted to gather information about employment status and age. This table shows the data that were collected.

Employment Survey Results

		Age (years)		
		Younger Than 18	**18 or Older**	**Total**
Status	**Has Job**	20	587	607
	Does Not Have Job	245	92	337
	Total	265	679	944

a. What is the probability that a randomly selected person surveyed has a job given that the person is younger than 18 years old?

b. What is the probability that a randomly selected person surveyed has a job given that the person is 18 years old or older?

c. Are having a job (A) and being 18 years old or older (B) independent events? Explain.
   - $P(A)$ = has a job
   - $P(A')$ = does not have a job
   - $P(B)$ = 18 years old or older
   - $P(B')$ = younger than 18 years old

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Solution:

a. Find the total number of people surveyed younger than 18 years old: 265. Divide the number of people who have a job and are younger than 18 years old, 20, by the number of people younger than 18 years old, 265: $\frac{20}{265} \approx 0.08$. The probability that a person surveyed has a job given that the person is younger than 18 years old is about 0.08.

$$P(A \mid B') = \frac{P(A \text{ and } B')}{P(B')} = \frac{\frac{20}{944}}{\frac{265}{944}} = \frac{20}{265} = 0.08$$

b. Find the total number of people surveyed 18 years old or older: 679. Divide the number of people who have a job and are 18 years old or older, 587, by the number of people 18 years old or older, 679: $\frac{587}{679} \approx 0.86$. The probability that a person surveyed has a job given that the person is 18 years old or older, is about 0.86.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{\frac{587}{944}}{\frac{679}{944}} = \frac{587}{679} = 0.86$$

c. The events are independent if $P(A \mid B) = P(A)$ *and* $P(B \mid A) = P(B)$.
From part (b), $P(A \mid B) \approx 0.86$.

$P(A) = \frac{607}{944} \approx 0.64$

$P(A \mid B) \neq P(A)$, so the events are not independent.

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SAMPLE ITEMS

1. In a particular state, the first character on a license plate is always a letter. The last character is always a digit from 0 to 9.

If *V* represents the set of all license plates beginning with a vowel and *O* represents the set of all license plates that end with an odd number, which license plate belongs to the set *V* and O′?

A. E23 PC8

B. MG4 3F5

C. AR8 8X9

D. P7M Z56

2. For which set of probabilities would events *A* and *B* be independent?

A. $P(A) = 0.25$; $P(B) = 0.25$; $P(A \text{ and } B) = 0.5$
B. $P(A) = 0.08$; $P(B) = 0.4$; $P(A \text{ and } B) = 0.12$
C. $P(A) = 0.16$; $P(B) = 0.24$; $P(A \text{ and } B) = 0.32$
D. $P(A) = 0.3$; $P(B) = 0.15$; $P(A \text{ and } B) = 0.045$

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3. Assume that the following events are independent:

- The probability that a high school senior will go to college is 0.72.
- The probability that a high school senior will go to college and live on campus is 0.46.

What is the probability that a high school senior will live on campus given that the high school senior will go to college?

A. 0.26
B. 0.33
C. 0.57
D. 0.64

4. A random survey was conducted about gender and hair color. This table records the data.

Hair Color

	Brown	Blond	Red	Total
Male	548	876	82	1,506
Female	612	716	66	1,394
Total	1,160	1,592	148	2,900

What is the probability that a randomly selected person has blond hair given that the person selected is male?

A. 0.51
B. 0.55
C. 0.58
D. 0.63

Answers to Unit 6.1 Sample Items

1. A 2. D 3. D 4. C

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6.2 Use the Rules of Probability to Compute Probabilities of Compound Events in a Uniform Probability Model

MGSE9-12.S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in context.

MGSE9-12.S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answers in context.

KEY IDEAS

Two events are ***mutually exclusive*** if the events cannot occur at the same time.

When two events A and B are mutually exclusive, the probability that event A or event B will occur is the sum of the probabilities of each event: $P(A \text{ or } B) = P(A) + P(B)$.

When two events A and B are not mutually exclusive, the probability that event A or B will occur is the sum of the probability of each event minus the intersection of the two events. That is, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. This is called the ***Addition Rule***.

You can find the conditional probability, $P(A \mid B)$, by finding the fraction of B's outcomes that also belong to A.

Example: Event A is choosing a heart card from a standard deck of cards.

Event B is choosing a face card from a standard deck of cards.

$P(A \mid B)$ is the probability that a card is a heart given that the card is a face card. You can look at B's outcomes and determine what fraction belongs to A; there are 12 face cards, 3 of which are also hearts:

$$P(A \mid B) = \frac{3}{12} = \frac{1}{4}$$

	Heart	**Not a Heart**	**Total**
Face Card	3	9	12
Not a Face Card	10	30	40
Total	13	39	52

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REVIEW EXAMPLES

- In Mr. Mabry's class, there are 12 boys and 16 girls. On Monday, 4 boys and 5 girls were wearing white shirts.

  a. If a student is chosen at random from Mr. Mabry's class, what is the probability of choosing a boy or a student wearing a white shirt?

  b. If a student is chosen at random from Mr. Mabry's class, what is the probability of choosing a girl or a student not wearing a white shirt?

  Solution:

  a. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, where A is the set of boys and B is the set of students wearing a white shirt.

     (A and B) is the set of boys wearing a white shirt. There are 12 + 16 = 28 students in Mr. Mabry's class.

     So $P(A) = \frac{12}{28}$, $P(B) = \frac{4+5}{28} = \frac{9}{28}$, and $P(A \text{ and } B) = \frac{4}{28}$.

     $P(\text{a boy or a student wearing a white shirt}) = \frac{12}{28} + \frac{9}{28} - \frac{4}{28} = \frac{17}{28}$

  b. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, where A is the set of girls and B is the set of students not wearing a white shirt.

     (A and B) is the set of girls not wearing a white shirt. There are 12 + 16 = 28 students in Mr. Mabry's class.

     So $P(A) = \frac{16}{28}$, $P(B) = \frac{8+11}{28} = \frac{19}{28}$, and $P(A \text{ and } B) = \frac{11}{28}$.

     $P(\text{a girl or a student not wearing a white shirt}) = \frac{16}{28} + \frac{19}{28} - \frac{11}{28} = \frac{24}{28} = \frac{6}{7}$

- Terry has a number cube with sides labeled 1 through 6. He rolls the number cube twice.

  a. What is the probability that the sum of the two rolls is a prime number given that at least one of the rolls is a 3?

  b. What is the probability that the sum of the two rolls is a prime number or at least one of the rolls is a 3?

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Solution:

a. This is an example of a mutually exclusive event. Make a list of the combinations where at least one of the rolls is a 3. There are 11 such pairs.

1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
3, 1	3, 2	3, 4	3, 5	3, 6	

Then identify the pairs that have a prime sum.

2, 3	3, 2	3, 4	4, 3

Of the 11 pairs of outcomes, there are 4 pairs whose sum is prime. Therefore, the probability that the sum is prime of those that show a 3 on at least one roll is $\frac{4}{11}$.

b. This is an example of events that are NOT mutually exclusive. There are 36 possible outcomes when rolling a number cube twice.

List the combinations where at least one of the rolls is a 3.

1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
3, 1	3, 2	3, 4	3, 5	3, 6	

$P(\text{at least one roll is a 3}) = \frac{11}{36}$

List the combinations that have a prime sum.

1, 1	1, 2	1, 4	1, 6
2, 1	2, 3	2, 5	
3, 2	3, 4		
4, 1	4, 3		
5, 2	5, 6		
6, 1	6, 5		

$P(\text{a prime sum}) = \frac{15}{36}$

Identify the combinations that are in both lists.

2, 3	3, 2	3, 4	4, 3

The combinations in both lists represent the intersection. The probability of the intersection is the number of outcomes in the intersection divided by the total possible outcomes.

$P(\text{at least one roll is a 3 } \textbf{and} \text{ a prime sum}) = \frac{4}{36}$

If two events share outcomes, then outcomes in the intersection are counted twice when the probabilities of the events are added. So you must subtract the probability of the intersection from the sum of the probabilities.

$P(\text{at least one roll is a 3 } \textbf{or} \text{ a prime sum}) = \frac{11}{36} + \frac{15}{36} - \frac{4}{36} = \frac{22}{36} = \frac{11}{18}$

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SAMPLE ITEMS

1. Mrs. Klein surveyed 240 men and 285 women about their vehicles. Of those surveyed, 155 men and 70 women said they own a red vehicle. If a person is chosen at random from those surveyed, what is the probability of choosing a woman or a person who does NOT own a red vehicle?

A. $\frac{14}{57}$

B. $\frac{71}{105}$

C. $\frac{74}{105}$

D. $\frac{88}{105}$

2. Bianca spins two spinners that have four equal sections numbered 1 through 4. If she spins a 4 on at least one spin, what is the probability that the sum of her two spins is an odd number?

A. $\frac{1}{4}$

B. $\frac{7}{16}$

C. $\frac{4}{7}$

D. $\frac{11}{16}$

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3. Each letter of the alphabet is written on separate cards in red ink. The cards are placed in a container. Each letter of the alphabet is also written on separate cards in black ink. The cards are placed in the same container. What is the probability that a card randomly selected from the container has a letter written in black ink or the letter is A or Z?

A. $\frac{1}{2}$

B. $\frac{7}{13}$

C. $\frac{15}{26}$

D. $\frac{8}{13}$

Answers to Unit 6.2 Sample Items

1. C 2. C 3. B

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GEOMETRY ADDITIONAL PRACTICE ITEMS

This section has two parts. The first part is a set of 27 sample items for Geometry. The second part contains a table that shows for each item the standard assessed, the DOK level, the correct answer (key), and a rationale/explanation about the key and distractors. The sample items can be utilized as a mini-test to familiarize students with the item formats found on the assessment.

All example and sample items contained in this guide are the property of the Georgia Department of Education.

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Georgia Milestones Assessment System

Geometry Formula Sheet

Below are the formulas you may find useful as you take the test. However, you may find that you do not need to use all of the formulas. You may refer to this formula sheet as often as needed.

Geometry Formulas

Perimeter

The perimeter of a polygon is equal to the sum of the length of its sides.

Distance Formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Coordinates of point which partitions a directed line segment AB at the ratio of $a:b$ from $A(x_1, y_1)$ to $B(x_2, y_2)$

$(x, y) = \left(\frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a}\right)$

OR

$(x, y) = \left(x_1 + \frac{a}{a + b}(x_2 - x_1), y_1 + \frac{a}{a + b}(y_2 - y_1)\right)$

Circumference of a Circle

$C = \pi d$ or $C = 2\pi r$

$\pi \approx 3.14$

Arc Length of a Circle

Arc Length $= \frac{2\pi r\theta}{360}$

Area

Triangle	$A = \frac{1}{2}bh$
Rectangle	$A = bh$
Circle	$A = \pi r^2$

Area of a Sector of a Circle

Area of Sector $= \frac{\pi r^2\theta}{360}$

Pythagorean Theorem

$a^2 + b^2 = c^2$

Trigonometric Relationships

$\sin\theta = \frac{opp}{hyp}$; $\cos\theta = \frac{adj}{hyp}$; $\tan\theta = \frac{opp}{adj}$

Equation of a Circle

$(x - h)^2 + (y - k)^2 = r^2$

Volume

Cylinder	$V = \pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$
Cone	$V = \frac{1}{3}\pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$

Statistics Formulas

Conditional Probability

$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

Multiplication Rule for Independent Events

$P(A \text{ and } B) = P(A) \cdot P(B)$

Addition Rule

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

You can find mathematics formula sheets on the Georgia Milestones webpage at http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Milestones-EOC-Resources.aspx.

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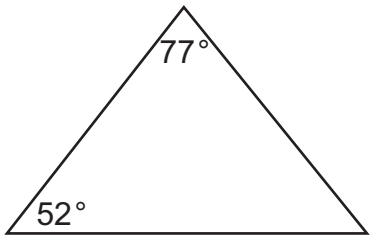
Item 1

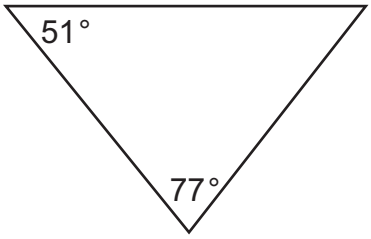
Selected-Response

Look at the triangle.

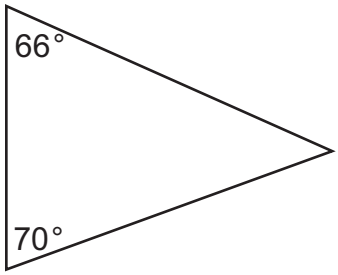


Which triangle is similar to the given triangle?

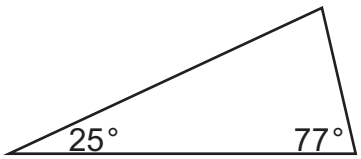
A.



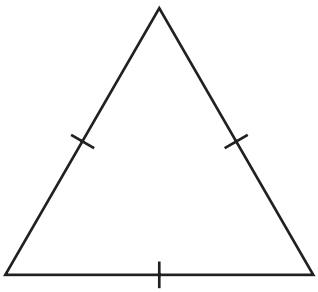
B.



C.



D.



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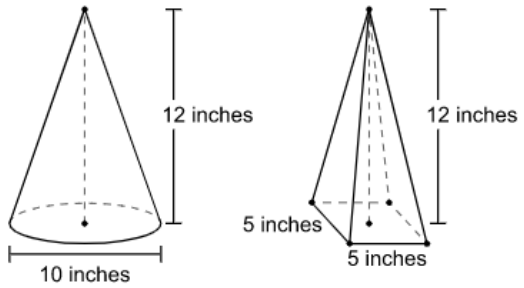
Item 2

Keypad-Input Technology-Enhanced

A cone and a pyramid are shown.



How many times the volume of the pyramid is the volume of the cone? Use 3.14 for π and round your answer to the nearest tenth.

Use a mouse, touchpad, or touchscreen to enter a response.

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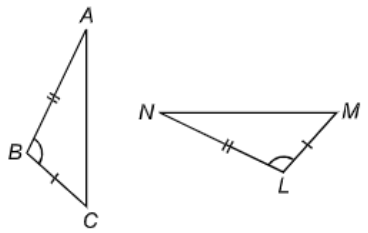
Item 3

Drop-Down Technology-Enhanced

Two triangles are shown.



Use the drop-down menus to complete the congruence statement.

△*ABC* ≅ △[ ] by [ ] congruence.

Use a mouse, touchpad, or touchscreen to click the arrow beside each of the two blank boxes. When you click the arrow, a drop-down menu will appear, showing you all the possible options for that blank box. Each drop-down menu with its options is shown below.

△*ABC* ≅ △[ ] by [ ] congruence.

LMN	AA
LNM	AAS
MLN	ASA
MNL	SAS
NLM	SSA
NML	SSS

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Item 4

Multi-Part Technology-Enhanced

Triangle *ABC* is similar but not congruent to triangle *DEF*.

Part A

Which series of transformations could map triangle *ABC* onto triangle *DEF*?

A. translation 4 units up, rotation 75° clockwise about the origin
B. reflection across the line $y = 2$, rotation 90° clockwise about the origin
C. translation 3 units left, dilation of scale factor 2 centered at the origin
D. reflection across the line $x = 1$, reflection across the line $y = 5$

Part B

Which equation must be true about triangle *ABC* and triangle *DEF*?

A. $AB = DE$
B. $AC = EF$
C. $m\angle A + m\angle B = m\angle D + m\angle F$
D. $m\angle A + m\angle C = m\angle D + m\angle F$

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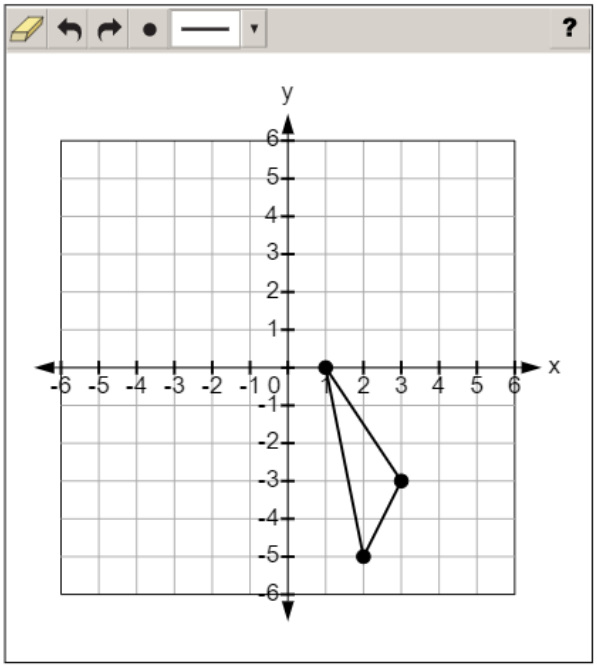
Item 5

Coordinate-Graph Technology-Enhanced

The triangle shown is reflected across the *y*-axis and rotated 90° clockwise around the origin.

Graph the image of the triangle after the transformations by plotting the vertices and line segments.



Use a mouse, touchpad, or touchscreen to graph the image of the triangle on the coordinate grid. At most 3 points and 3 line segments can be graphed.

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Item 6

Selected-Response

Which equation is true?

A. $\sin 40° = \tan 50°$
B. $\cos 40° = \cos 50°$
C. $\sin 40° = \sin 50°$
D. $\cos 40° = \sin 50°$

Item 7

Multi-Part Technology-Enhanced

Triangle *GHJ* is a right triangle. Angle *G* has a measure of *g*°, angle *H* has a measure of *h*°, and angle *J* is a right angle.

Part A

Which equation must be true?

A. $\sin(h°) = \sin(g°)$
B. $\cos(g°) = \sin(h°)$
C. $\cos(h°) = \cos(g°)$
D. $\sin(h°) + \cos(h°) = \tan(h°)$

Part B

Given that $\tan(g°) = \dfrac{\sin(g°)}{\cos(g°)}$, which ratio must have a value equivalent to the tangent of *g*°?

A. $\dfrac{\cos(h°)}{\sin(g°)}$

B. $\dfrac{\cos(h°)}{\sin(h°)}$

C. $\dfrac{\sin(h°)}{\cos(h°)}$

D. $\dfrac{\sin(h°)}{\cos(g°)}$

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Item 8

Selected-Response

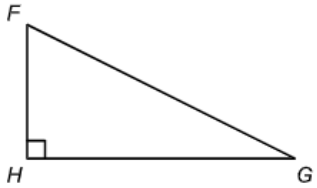
Which point is NOT on a circle with a center of (0, 0) and a radius of 10?

A. (0, 5)
B. (10, 0)
C. (0, –10)
D. (–8, 6)

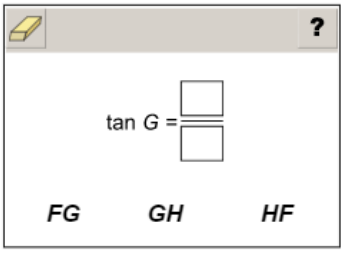
Item 9

Drag-and-Drop Technology-Enhanced

Right triangle *FGH* is shown.



Move the sides of triangle *FGH* into the boxes to complete the trigonometric ratio.



Use a mouse, touchpad, or touchscreen to move the side labels into the boxes. Each side label may be used 2 times.

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Item 10

Drag-and-Drop Technology-Enhanced

Sashima is drawing a regular hexagon inscribed in a circle.

Move each step into the correct order for Sashima to make her construction by using a compass and straightedge.

?

Step 1	
Step 2	
Step 3	
Step 4	
Step 5	
Step 6	

Draw an arc that intersects circle *C*.

Draw a circle with center *C* that passes through point *A*.

Place the compass on the intersection of the new arc and circle *C*.

Without changing the width of the compass, place the compass on point *A*.

Repeat the previous two steps until the point of the compass is placed on point *A* again.

Connect each point of intersection to the adjacent point of intersection with a line segment.

Use a mouse, touchpad, or touchscreen to move a step into each row. Each step may be used once.

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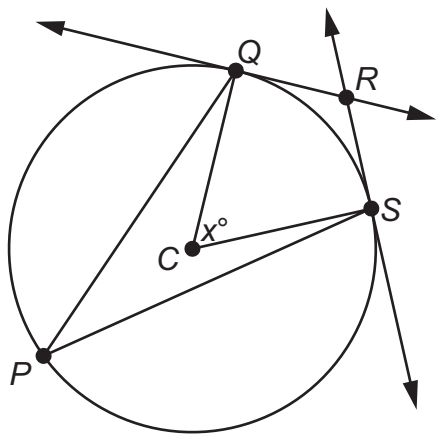
Item 11

Multi-Select Technology-Enhanced

The figure shows circle *C* with tangent lines $\overleftrightarrow{QR}$ and $\overleftrightarrow{SR}$.



The measure of $\angle QCS$ is $x°$.

Select THREE statements that are true about the figure.

A. The measure of $\angle QPS$ is $(90 - x)°$.

B. The measure of $\angle QPS$ is $\frac{1}{2}x°$.

C. The measure of $\angle PSR$ is 90°.

D. The measure of $\angle CQR$ is 90°.

E. The measure of $\angle QRS$ is $(180 - x)°$.

F. The measure of $\angle QRS$ is $2x°$.

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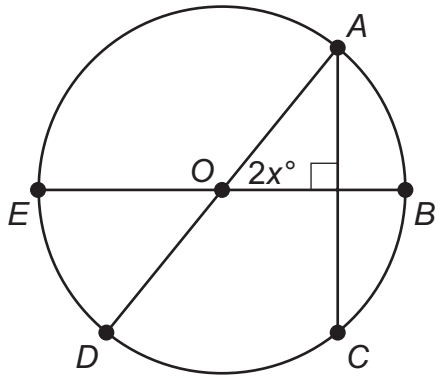
Item 12

Selected-Response

Points *A*, *B*, *C*, *D*, and *E* are located on circle *O*, as shown in this figure.



The measure of $\overset{\frown}{CD}$ is 80°. What is the value of *x*?

A. 50
B. 40
C. 35
D. 25

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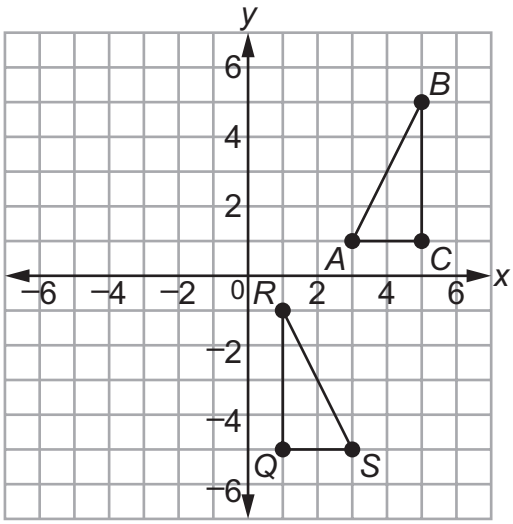
Item 13

Selected-Response

What is the sequence of transformations that carry triangle *ABC* to triangle *SRQ*?



A. Triangle *ABC* is reflected across the line $x = 3$. Then it is translated 2 units down.

B. Triangle *ABC* is reflected across the line $x = 3$. Then it is translated 6 units down.

C. Triangle *ABC* is translated 2 units to the left. Then it is rotated 90 degrees counterclockwise about the point (1, 1).

D. Triangle *ABC* is translated 2 units to the right. Then it is rotated 90 degrees counterclockwise about the point (1, 1).

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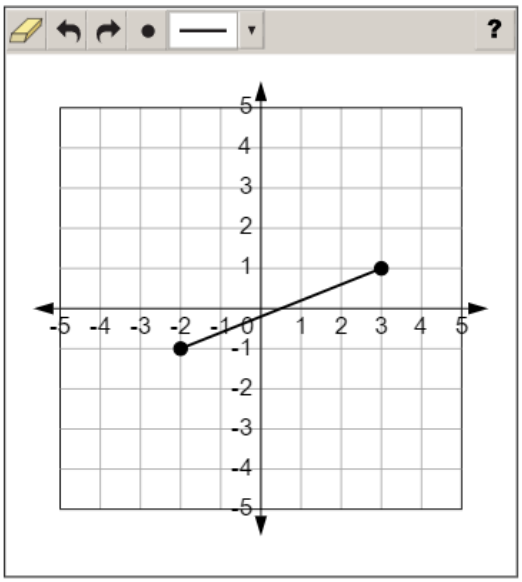
Item 14

Coordinate-Graph Technology-Enhanced

A line segment is shown on a coordinate plane.

Draw the image of the line segment after a 90° rotation counterclockwise about the point (–2, –1).



Use a mouse, touchpad, or touchscreen to graph a line on the coordinate grid. At most 1 line segment and 2 points can be graphed.

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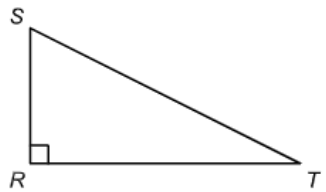
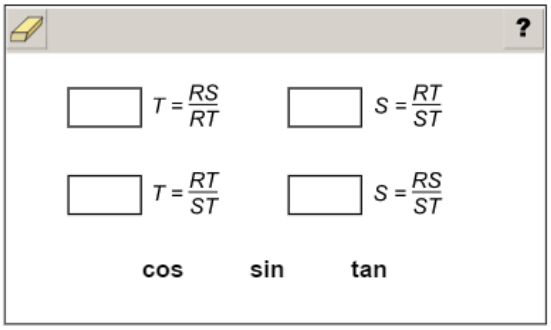
Item 15

Drag-and-Drop Technology-Enhanced

Right triangle *RST* is shown.



Move the correct trigonometric function into each box to complete the equations.



Use a mouse, touchpad, or touchscreen to move a trigonometric function into each box. Each trigonometric function may be used 4 times.

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Item 16

Selected-Response

Which transformation on quadrilateral *ABCD* produces an image that does not preserve the distances between points in quadrilateral *ABCD*?

A. reflection across $y = x$
B. translation 3 units down and 4 units to the right
C. dilation by a scale factor of 2
D. rotation of 270 degrees

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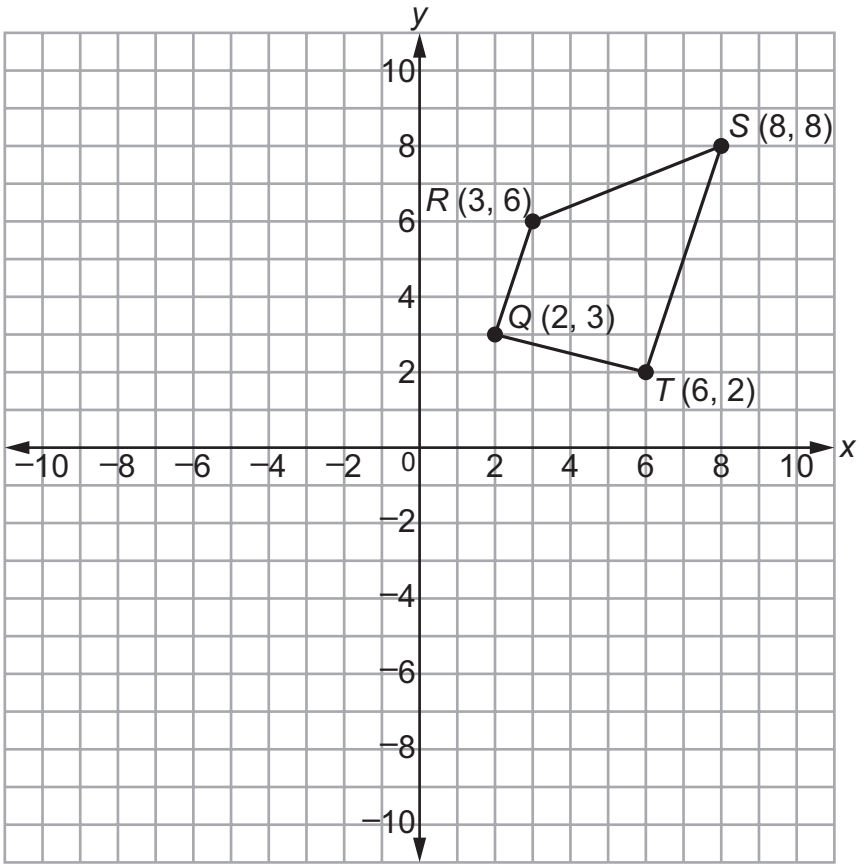
Item 17

Selected-Response

Look at quadrilateral *QRST*.



What is the image of point *R* after a counterclockwise rotation of 270 degrees about the origin?

A. (6, –3)
B. (–3, 6)
C. (–6, 3)
D. (3, –6)

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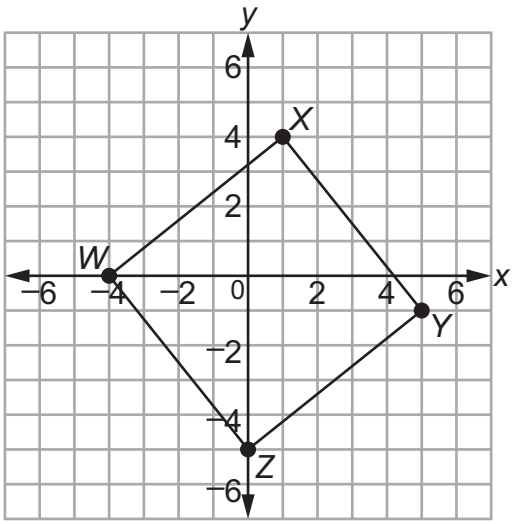
Item 18

Selected-Response

Look at the square *WXYZ* on this coordinate plane.



Which measure is closest to the perimeter of square *WXYZ*?

A. 20 units
B. 25.6 units
C. 32 units
D. 40.9 units

Item 19

Selected-Response

What are the coordinates of a point that lies along the directed line segment from *Q*(2, 5) to *R*(7, 12) and partitions the segment in the ratio of 3 to 2?

A. (3, 4.2)
B. (4.5, 8.5)
C. (5, 9.2)
D. (5, 7)

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Item 20

Drop-Down Technology-Enhanced

Some probabilities are listed below.

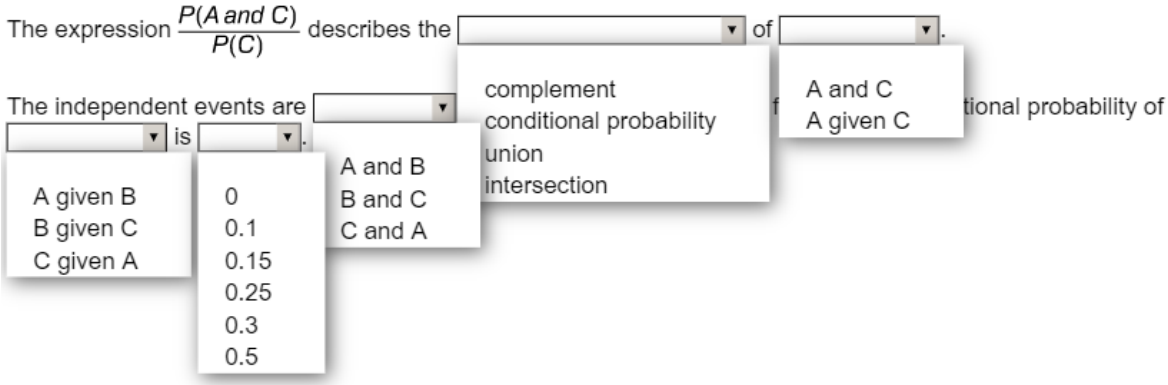
- $P(A) = 0.3$
- $P(B) = 0.5$
- $P(C) = 0.25$
- $P(A \text{ and } B) = 0.15$
- $P(B \text{ and } C) = 0$
- $P(C \text{ and } A) = 0.1$

Use the drop-down menus to complete the statements.

The expression $\frac{P(A \text{ and } C)}{P(C)}$ describes the [ ] of [ ].

The independent events are [ ]. This is demonstrated by the fact that the conditional probability of [ ] is [ ].

Use a mouse, touchpad, or touchscreen to click the arrow beside each of the five blank boxes. When you click the arrow, a drop-down menu will appear, showing you all the possible options for that blank box. Each drop-down menu with its options is shown below.



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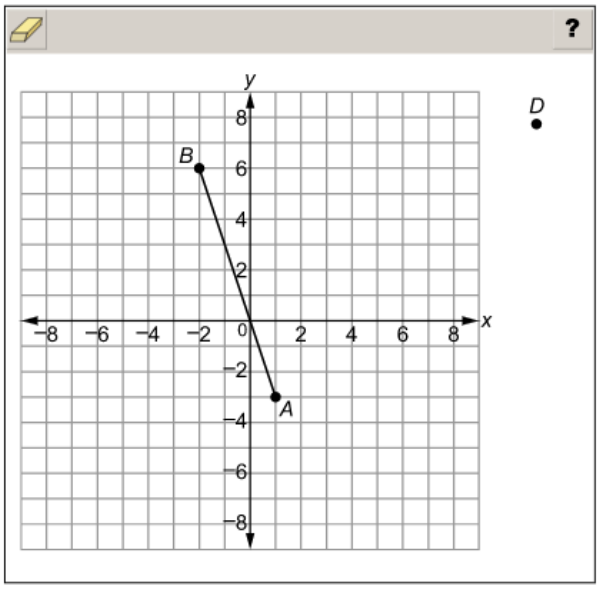
Item 21

Drag-and-Drop Technology-Enhanced

Point *A* is located at (1, –3). Point *B* is located at (–2, 6). Line segment *AB* is shown on the coordinate grid.

Point *D* partitions line segment *AB* such that the ratio *AD*:*DB* is 7:2.

Move point *D* to plot it on the coordinate grid.



Use a mouse, touchpad, or touchscreen to move the labeled point onto the grid. The labeled point may be used once.

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Item 22

Selected-Response

What is the equation of the line that is perpendicular to $y = \frac{1}{2}x - 6$ and passes through the point (6, 4)?

A. $y = -\frac{1}{2}x + 1$

B. $y = -\frac{1}{2}x + 7$

C. $y = -2x - 8$

D. $y = -2x + 16$

Item 23

Selected-Response

Study this equation of a circle.

$$x^2 - 6x + y^2 + 2y + 6 = 0$$

Which of these represents the center and radius of the circle?

A. center: (3, –1), radius: 4
B. center: (–3, 1), radius: 4
C. center: (3, –1), radius: 2
D. center: (–3, 1), radius: 2

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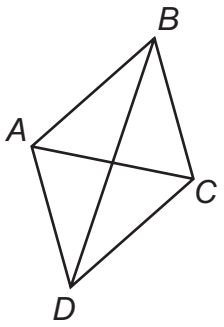
Item 24

Selected-Response

What proves that figure *ABCD* is a parallelogram?



A. Diagonal *BD* bisects angle *ABC*.
B. Side *AB* is equal to diagonal *AC*.
C. Diagonals *BD* and *AC* bisect one another.
D. Diagonal *BD* is greater than diagonal *AC*.

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Item 25

Selected-Response

When rolling a number cube with sides labeled 1 through 6, what is the probability of rolling an even number or a number less than 3?

A. $\frac{5}{6}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

Item 26

Selected-Response

What is the probability of having rolled a 5 on a number cube with sides labeled 1 through 6 if you know that you rolled an odd number?

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

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Item 27

Drag-and-Drop Technology-Enhanced

A class of 23 students is surveyed about their favorite colors and what grades they are in. A partial two-way table has been created.

Move a number into each blank cell to complete the two-way table.

Students	Favorite Color: Red	Green	Blue	Total
10th Grade		2	8	14
11th Grade	3		1	
Total		7	9	23

1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23

Use a mouse, touchpad, or touchscreen to move a number into each blank cell. Each number may be used 4 times.

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ADDITIONAL PRACTICE ITEMS ANSWER KEY

Item	Standard/Element	DOK Level	Correct Answer	Explanation
1	MGSE9-12.G.SRT.3	1	A	The correct answer is choice (A). The missing angle of the triangle in choice (A) is 52°, making it similar to the triangle given. Choices (B), (C), and (D) are incorrect because they have angle measures that are different than the original triangle.
2	MGSE9-12.G.GMD.3	2	N/A	See scoring rubric and exemplar response on page 176.
3	MGSE9-12.G.CO.8	2	N/A	See scoring rubric and exemplar response on page 177.
4	MGSE9-12.G.SRT.2	2	Part A: C Part B: D	Part A: The correct answer is choice (C) translation 3 units left, dilation of scale factor 2 centered at the origin. It is the only transformation that has a scale factor, resulting in triangles that are similar but not congruent. Choices (A), (B), and (D) are incorrect because they all result in triangles that would be congruent. Part B: The correct answer is choice (D) $m\angle A + m\angle C = m\angle D + m\angle F$. Corresponding angles in similar triangles are equal. $\angle A$ and $\angle D$ are equal and $\angle C$ and $\angle F$ are equal, so adding the respective angle measures together will result in an equal amount. Choices (A) and (B) are incorrect because similar triangles do not have equal side lengths. Choice (C) is incorrect because the angles given do not correspond to each other, so they might not add up to equal amounts.
5	MGSE9-12.G.CO.5	2	N/A	See scoring rubric and exemplar response on page 178.
6	MGSE9-12.G.SRT.7	1	D	The correct answer is choice (D) cos 40° = sin 50°. The sine of an angle is equal to the cosine of the angle's complement. Choices (A), (B), and (C) are incorrect because they do not correspond to any trigonometric identities.

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Item	Standard/Element	DOK Level	Correct Answer	Explanation
7	MGSE9-12.G.SRT.7	2	Part A: B Part B: B	Part A: The correct answer is choice (B) $\cos(g°) = \sin(h°)$. The acute angles in a right triangle are always complementary, which means the cosine of one is equal to the sine of the other. Choices (A) and (C) are incorrect because the sines and cosines of the acute angles in a right triangle are not necessarily equal to each other. Choice (D) is incorrect because the tangent of an angle is not equal to the sum of the sine and cosine of the same angle. Part B: The correct answer is choice (B) $\frac{\cos(h°)}{\sin(h°)}$. The sine of $g°$ is equal to the cosine of $h°$, and the cosine of $g°$ is equal to the sine of $h°$. Choices (A), (C), and (D) are incorrect because they substitute values in for the sine of $g°$ and the cosine of $g°$ that are not equal to the original values.
8	MGSE9-12.G.GPE.4	2	A	The correct answer is choice (A) (0, 5). The point is only 5 units away from the center of the circle. Choices (B), (C), and (D) are incorrect because they are 10 units away from the center of the circle.
9	MGSE9-12.G.SRT.6	1	N/A	See scoring rubric and exemplar response on page 179.
10	MGSE9-12.G.CO.13	2	N/A	See scoring rubric and exemplar response on page 180.
11	MGSE9-12.G.C.2	3	B/D/E	The correct choices are (B), (D) and (E). Choice (B) is correct because an inscribed angle measure is half the measure of the intercepted arc. Choice (D) is correct because a line that is tangent to a circle is perpendicular to the radius drawn to the point of tangency. Choice (E) is correct because a circumscribed angle measure is equal to 180° minus the measure of the central angle that forms the intercepted arc. Choice (A) is incorrect because the measure of an inscribed angle is half of the measure of the intercepted arc, rather than the difference between 90° and the central angle. Choice (C) is incorrect because the measure of the angle made by a tangent and a secant line segment cannot be 90°. Choice (F) is incorrect because the measure of a circumscribed angle is the difference between 180° and the central angle, rather than twice the central angle.

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Item	Standard/Element	DOK Level	Correct Answer	Explanation
12	MGSE9-12.G.C.2	2	D	The correct answer is choice (D) 25. The measure of arc *CD* is 80° and the measure of angle *DAC* is 40°. Since the angles in a triangle add to 180°, the measure of angle *AOB* is 50°; if $2x = 50$, then $x = 25$. Choice (A) is incorrect because it is the measure of angle *AOB*. Choice (B) is incorrect because the answer is true only if the triangle is isosceles. Choice (C) is incorrect because a computation error was made when determining the value of x.
13	MGSE9-12.G.CO.5	1	B	The correct answer is choice (B) Triangle *ABC* is reflected across the line $x = 3$. Then it is translated 6 units down. Choice (A) is incorrect because the triangle is translated more than 2 units down. Choices (C) and (D) are incorrect because rotating the triangle after the translation would not yield the correct orientation of triangle *SRQ*.
14	MGSE9-12.G.CO.5	2	N/A	See scoring rubric and exemplar response on page 181.
15	MGSE9-12.G.SRT.6	1	N/A	See scoring rubric and exemplar response on page 182.
16	MGSE9-12.G.CO.2	1	C	The correct answer is choice (C) dilation by a scale factor of 2. When a figure is dilated, its line segments are either increased or decreased by a scale factor to form a similar figure. Choices (A), (B), and (D) are incorrect because these are rigid transformations that move the figure on the plane without affecting side lengths.
17	MGSE9-12.G.CO.2	2	A	The correct answer is choice (A) (6, –3). Rotating the figure 270 degrees counterclockwise about the origin is the same as rotating it clockwise 90 degrees. Therefore, the figure would be in the 4th quadrant, and point *R* would be at (6, –3). Choices (B) and (C) are incorrect because they are in the 2nd quadrant. Choice (D) is incorrect because even though it is in the 4th quadrant, the coordinate points are wrong.

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Item	Standard/Element	DOK Level	Correct Answer	Explanation
18	MGSE9-12.G.GPE.7	2	B	The correct answer is choice (B) 25.6 units. Apply the distance formula to find the length of one side, which is 6.4. Since this is a square, multiply 6.4 by 4 to obtain the perimeter. Choice (A) is incorrect because the number of unit squares on a line segment was counted to estimate the length and then multiplied by 4. Choice (C) is incorrect because the number of unit squares across the square was counted to estimate the length and then multiplied by 2. Choice (D) is incorrect because it is the approximate area of the square.
19	MGSE9-12.G.GPE.6	2	C	The correct answer is choice (C) (5, 9.2). The rise from *Q* to *R* is 5 and the run from *Q* to *R* is 7. Multiply each value by the ratio $\frac{3}{5}$ and then add that amount to the original coordinates (starting point) to find the *x*- and *y*-values for point *P*. Choice (A) is incorrect because these are the *x*- and *y*-values that need to be added to point *Q*. Choice (B) is incorrect because this is the midpoint of the line segment *QR*. Choice (D) is incorrect because this is the difference of the *x*-coordinates and *y*-coordinates.
20	MGSE9-12.S.CP.3	3	N/A	See scoring rubric and exemplar response on page 183.
21	MGSE9-12.G.GPE.6	1	N/A	See scoring rubric and exemplar response on page 184.
22	MSGE9-12.G.GPE.5	2	D	The correct answer is choice (D) $y = -2x + 16$. This response is correct because the given equation has a slope of $\frac{1}{2}$, and the slope of a perpendicular line will be the negative reciprocal, in this case –2. This equation will also pass through the point (6, 4). Choices (A) and (B) are incorrect because they do not have slopes that are perpendicular to the given line. Choice (C) is incorrect because the line does not pass through the point (6, 4).

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Item	Standard/Element	DOK Level	Correct Answer	Explanation
23	MGSE9-12.G.GPE.1	2	C	The correct answer is choice (C) center: (3, –1), radius: 2. When the equation is changed to standard form using completing the square, the *h*- and *k*-values are 3 and –1 and $r^2 = 4$, so $r = 2$. Choices (A) and (B) are incorrect because the radius comes from taking the square root of the constant in standard form. Choice (D) is incorrect because the signs of the center are opposite.
24	MGSE9-12.G.CO.11	2	C	The correct answer is choice (C) Diagonals *BD* and *AC* bisect one another. If diagonals bisect one another, then the quadrilateral is a parallelogram. Choices (A), (B), and (D) are incorrect because there is not enough information to prove the quadrilateral is a parallelogram.
25	MGSE9-12.S.CP.7	1	B	The correct answer is choice (B) $\frac{2}{3}$. An even number or a number less than 3 includes the outcomes of 1, 2, 4, and 6, and there are 6 outcomes in the sample space; $\frac{4}{6}$ simplifies to $\frac{2}{3}$. Choice (A) is incorrect because the probability of rolling a 1 and the probability of rolling a number less than 3 were added together without subtracting the overlap. Choice (C) is incorrect because it is the probability of an even number only. Choice (D) is incorrect because it is the probability of a number less than 3 only.
26	MGSE9-12.S.CP.3	2	B	The correct answer is choice (B) $\frac{1}{3}$. With the conditional probability, we assume that an odd number was rolled, which reduces our sample space to 1, 3, and 5. Out of those possibilities, the probability of rolling a 5 is $\frac{1}{3}$, 1 successful outcome out of 3 total outcomes. Choice (A) is incorrect because it is the probability of rolling a 5 without knowing an odd number was rolled. Choice (C) is incorrect because it is the probability of rolling an odd number. Choice (D) is incorrect because it is the complement of the correct answer.

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Item	Standard/Element	DOK Level	Correct Answer	Explanation
27	MGSE9-12.S.CP.4	1	N/A	See scoring rubric and exemplar response on page 185.

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ADDITIONAL PRACTICE ITEMS SCORING RUBRICS AND EXEMPLAR RESPONSES

Item 2

Scoring Rubric

Points	Description
1	The student correctly answers the question.
0	The student does not correctly answer the question.

Exemplar Response

The correct response is shown below.

The volume of a square pyramid is found by multiplying one-third times the height of the pyramid times the square of the side length of the base. For this pyramid, the volume is $\frac{1}{3} \times 12 \times 5 \times 5$. The volume of a cone is found by multiplying one-third times the height of the cone times pi times the square of the radius of the base. For this cone, the volume is $\frac{1}{3} \times 12 \times \text{pi} \times 5 \times 5$. Both volumes have factors of $\frac{1}{3}$, 12, 5, and 5, so the volume of the cone is pi times the volume of the pyramid.

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Item 3

Scoring Rubric

Points	Description
1	The student selects the correct options in both drop-down menus.
0	The student does not select the correct options in both drop-down menus.

Exemplar Response

The correct response is shown below.

$\triangle ABC \cong \triangle$ [NLM ▾] by [SAS ▾] congruence.

"*NLM*" is correct because of the correspondence between the markings of the two triangles: the double tick mark is on side *AB* and on side *NL*, an angle mark is on angle *B* and on angle *L*, and the single tick mark is on side *BC* and on side *LM*. "SAS" is correct because the markings show corresponding congruent angles between pairs of corresponding congruent sides.

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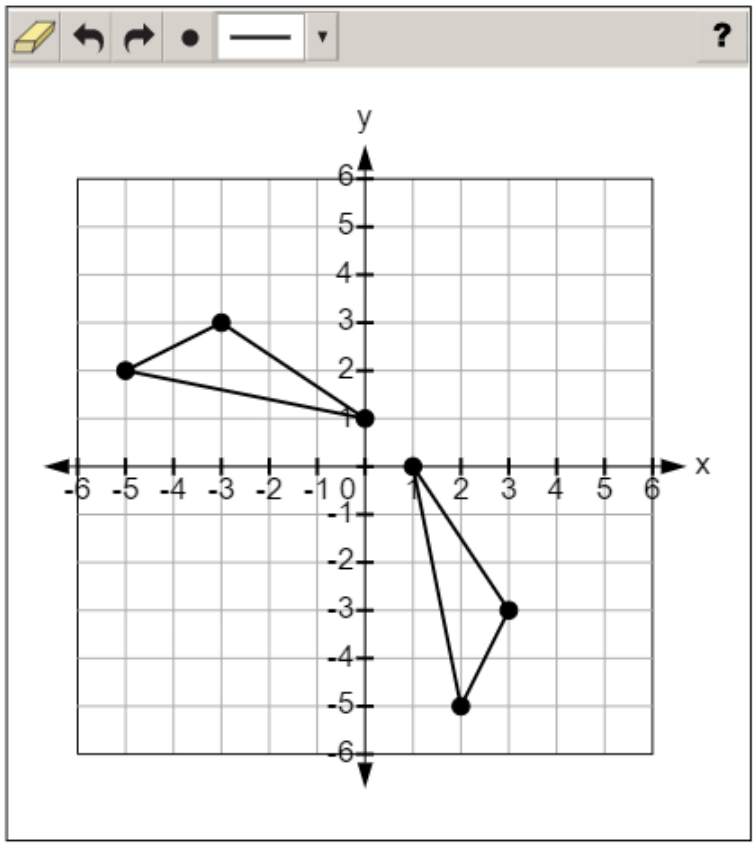
Item 5

Scoring Rubric

Points	Description
2	The student correctly graphs the image of the triangle.
1	The student correctly graphs one or two of the three vertices of the triangle.
0	The student does not correctly graph at least one of the three vertices of the triangle.

Exemplar Response

The correct response is shown below.



Reflecting the triangle across the y-axis can be modeled by the rule $(x, y) \rightarrow (-x, y)$. Rotating the triangle 90° clockwise about the origin can be modeled by the rule $(x, y) \rightarrow (y, -x)$. Combining these rules gives the rule $(x, y) \rightarrow (y, x)$. After applying this rule to the original coordinates, the image of the triangle has vertices located at (0, 1), (–3, 3), and (–5, 2).

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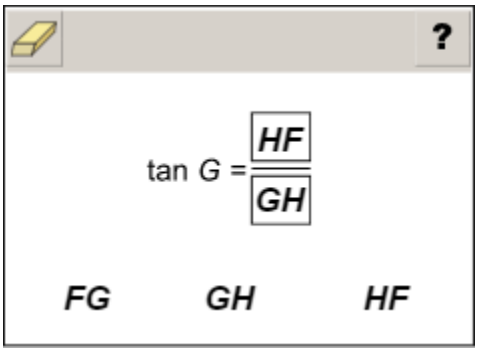
Item 9

Scoring Rubric

Points	Description
1	The student correctly completes the ratio.
0	The student does not correctly complete the ratio.

Exemplar Response

The correct response is shown below.



This is the correct response because the tangent of an angle is the ratio of the opposite side length to the adjacent side length. In this triangle, the side opposite angle *G* is side *HF* and the side adjacent to angle *G* is side *GH*.

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Item 10

Scoring Rubric

Points	Description
2	The student correctly places the six steps.
1	The student correctly places four or five of the six steps.
0	The student does not correctly place at least four of the six steps.

Exemplar Response

The correct response is shown below.

Sashima is drawing a regular hexagon inscribed in a circle.

Move each step into the correct order for Sashima to make her construction by using a compass and straightedge.

Step 1	**Draw a circle with center *C* that passes through point *A*.**
Step 2	**Without changing the width of the compass, place the compass on point *A*.**
Step 3	**Draw an arc that intersects circle *C*.**
Step 4	**Place the compass on the intersection of the new arc and circle *C*.**
Step 5	**Repeat the previous two steps until the point of the compass is placed on point *A* again.**
Step 6	**Connect each point of intersection to the adjacent point of intersection with a line segment.**

The first step of drawing a regular hexagon inscribed in a circle is to draw the circle. Since a regular hexagon can be composed of six equilateral triangles, the radius of the circle can be used as the sides of the triangles. The width of the compass stays the same while the arcs that intersect the circle are drawn. Finally, connecting these intersections creates the regular hexagon.

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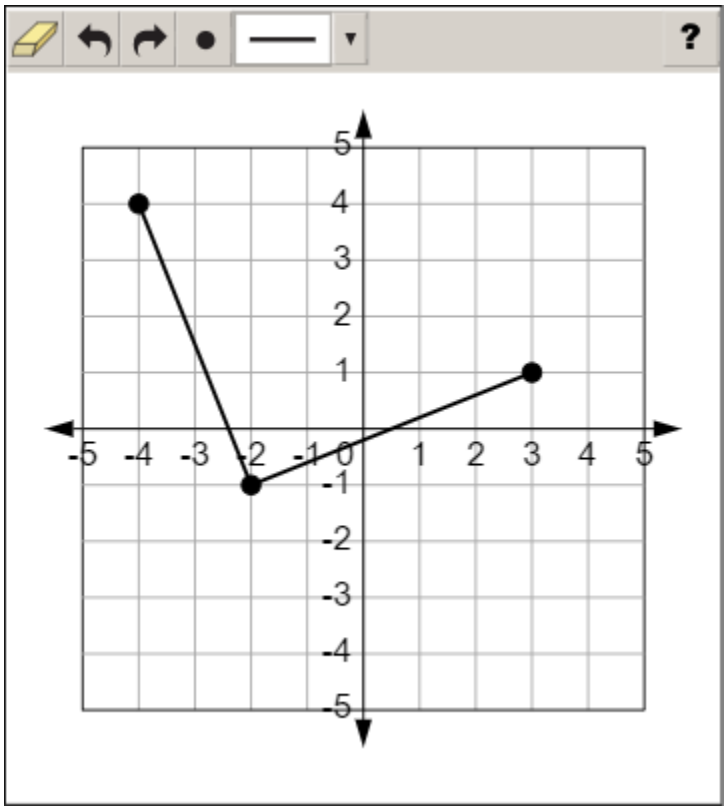
Item 14

Scoring Rubric

Points	Description
1	The student correctly graphs the line segment.
0	The student does not correctly graph the line segment.

Exemplar Response

The correct response is shown below.



This is the correct response because rotating a line segment 90 degrees about one of its endpoints results in a right angle. Therefore, the slopes of the line segments must be opposite reciprocals. The slope of the original line segment is $\frac{2}{5}$, and the slope of the rotated line segment is $-\frac{5}{2}$. The line segments are also congruent because rotations preserve length.

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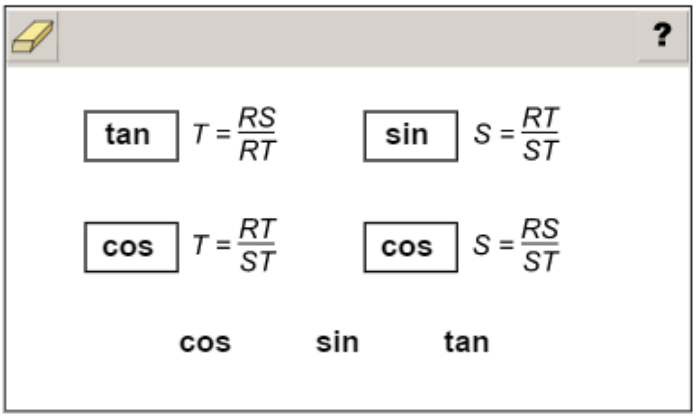
Item 15

Scoring Rubric

Points	Description
2	The student correctly places trigonometric functions in all four boxes.
1	The student correctly places trigonometric functions in two or three of the four boxes.
0	The student does not correctly place trigonometric functions in at least two of the four boxes.

Exemplar Response

The correct response is shown below.



The sine of an angle is equal to the ratio of the opposite leg to the hypotenuse. The cosine of an angle is equal to the ratio of the adjacent leg to the hypotenuse. The tangent of an angle is equal to the ratio of the opposite leg to the adjacent leg. For angle T, RS is the opposite leg, RT is the adjacent leg, and ST is the hypotenuse, which means $\frac{RS}{RT}$ is the "tan" of angle T and $\frac{RT}{ST}$ is the "cos" of angle T. For angle S, RT is the opposite leg and RS is the adjacent leg, which means $\frac{RT}{ST}$ is the "sin" of angle S and $\frac{RS}{ST}$ is the "cos" of angle S.

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Item 20

Scoring Rubric

Points	Description
2	The student correctly completes both paragraphs.
1	The student correctly completes either the first paragraph or the second paragraph.
0	The student does not correctly complete either paragraph.

Exemplar Response

The correct response is shown below.

The expression $\frac{P(A \text{ and } C)}{P(C)}$ describes the [conditional probability] of [A given C].

The independent events are [A and B]. This is demonstrated by the fact that the conditional probability of [A given B] is [0.3].

"Conditional probability" and "A given C" correctly complete the first paragraph because the expression is the definition of conditional probability. The conditional probability of A given B can be found by using the formula in the first paragraph and replacing C with B. The events are independent because the conditional probability is equal to the probability of A.

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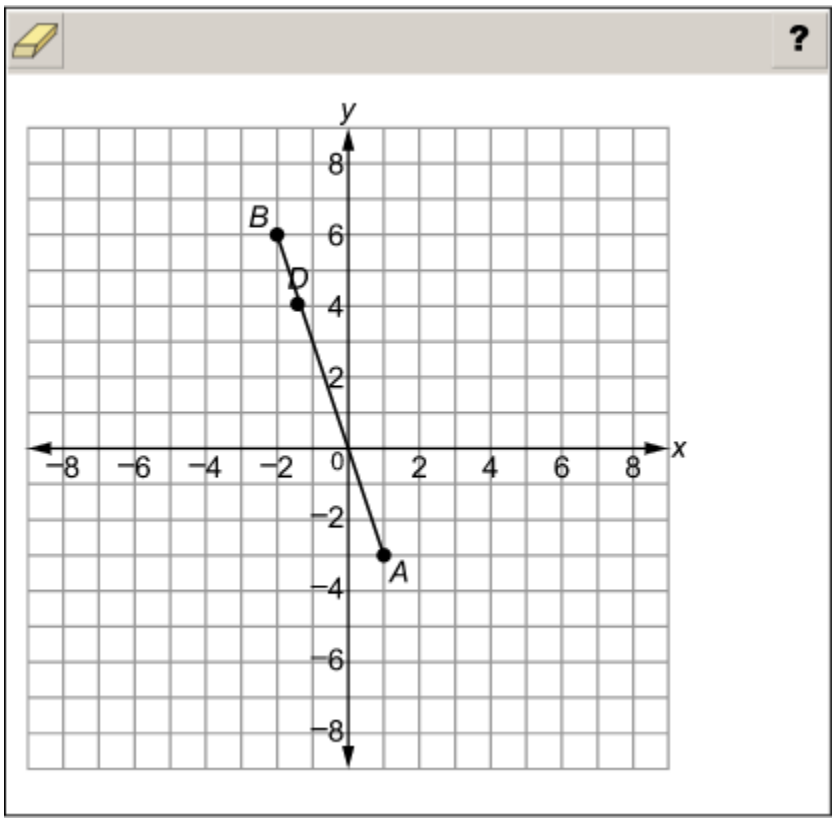
Item 21

Scoring Rubric

Points	Description
1	The student correctly places the point.
0	The student does not correctly place the point.

Exemplar Response

The correct response is shown below.



In the formula for partitioning a line segment substitute the following values, $a = 7$, $b = 2$, $x_1 = 1$, $y_1 = -3$, $x_2 = -2$, and $y_2 = 6$, to determine the location of point D. Solving the formula gives the location $(-\frac{4}{3}, 4)$ for point D.

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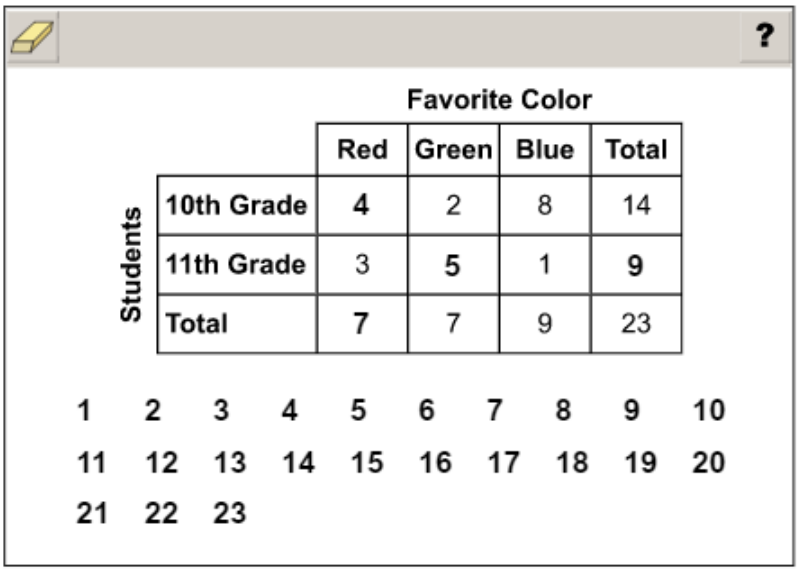
Item 27

Scoring Rubric

Points	Description
2	The student correctly places numbers in all four blank cells.
1	The student correctly places numbers in two or three of the four blank cells.
0	The student does not correctly place numbers in at least two of the blank cells.

Exemplar Response

The correct response is shown below.



Favorite Color

Students	Red	Green	Blue	Total
10th Grade	4	2	8	14
11th Grade	3	5	1	9
Total	7	7	9	23

1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23

There are 23 students in the group, and 14 of those students are in 10th grade, which means there are 9 students in 11th grade. Of the students in 11th grade, 3 students like red and 1 student likes blue, so there must be 5 students remaining who like green. Of the 14 students in 10th grade, 8 students like blue and 2 students like green, so there must be 4 students who like red. Finally, since there are now 4 students who like red in 10th grade and 3 students who like red in 11th grade, there must be a total of 7 students who like red.

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Study/Resource Guide
for Students and Parents
Geometry
End-of-Course