Scaffolding Instruction for English Learners:
A Georgia Mathematics Instructional Resource Guide
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Purpose

The purpose of this document is to provide mathematics teachers and leaders with evidence-based, pragmatic scaffolds and supports for English Learners (ELs). This guide is a useful tool to help teachers provide high-quality instruction aligned to Georgia’s K-12 Mathematics Standards, adopted in August 2021. The strategies presented in this guide provide support with the implementation of language scaffolds and mathematics frameworks that impact teaching and learning in Georgia. The goal is to help ELs master the newly adopted standards through accessible, useful, and easily implementable instructional strategies by mathematics content teachers and teachers of English to Speakers of Other Languages (ESOL).

Key Points Related to New Standards

➢ The philosophy of the Georgia Department of Education (GaDOE) is to reinforce a strong foundation of reasoning and fundamentals in the early grades that builds on relevant pathways for future success.
➢ Georgia’s K-12 Mathematics Standards are presented through a logical progression and provide detailed information as students, including ELs, work toward mastery of the key competencies and standards of the grade level or course.
➢ Standards-aligned instruction should allow all students to experience a sense of success and growth throughout the process of learning. Students should be provided with learning opportunities that focus on mastering fundamental topics and make connections that strengthen understanding.
➢ GaDOE does not mandate a specific strategy or approach to providing support for ELs; the content professional must determine the best approaches to teaching and learning that meet their individual students’ needs and best address the content objectives of the standards.
➢ The key competencies and standards presented for each grade level and course represent the ultimate expectation for mastery at each grade level for each big idea. Each standard is analyzed through learning expectations and evidence of student learning.
➢ The language strategies and evidence-based scaffolds and supports provided in this document can be used to help ELs master the newly adopted standards at high levels.
Overview

Teachers can help learners develop a strong foundation of the fundamentals by ensuring age- and developmentally appropriate concepts and content, thereby preparing students for life. Georgia’s K-12 standards will create more opportunities to engage ELs fully toward standards-based understanding and making sense of the mathematical ideas.

Georgia K-12 Mathematics Standards include key competencies that represent the standard expectation of learning for students in each grade level and course. The competencies (also known as standards) are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels and courses. Incorporating the eight Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards and apply in various ways throughout K-12 education. More details related to the eight Mathematical Practices and the Mathematical Modeling Framework can be found in the specific grade-level/course K-12 Mathematics Standards Curriculum Maps.

Mathematical Practices

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the “habits of mind” that help students become mathematical thinkers. There are eight Mathematical Practices, which apply to all grade levels and conceptual categories. These practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students’ capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Mathematical Practices present.

<table>
<thead>
<tr>
<th>Code</th>
<th>Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP.1</td>
<td>Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>MP.2</td>
<td>Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>MP.3</td>
<td>Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>MP.4</td>
<td>Model with mathematics.</td>
</tr>
<tr>
<td>MP.5</td>
<td>Use appropriate tools strategically.</td>
</tr>
<tr>
<td>MP.6</td>
<td>Attend to precision.</td>
</tr>
<tr>
<td>MP.7</td>
<td>Look for and make use of structure.</td>
</tr>
<tr>
<td>MP.8</td>
<td>Look for and express regularity in repeated reasoning.</td>
</tr>
</tbody>
</table>
Mathematical Modeling Framework

Teaching students to model with mathematics is engaging and builds confidence and competence. Also, experiences that foster this gives students the opportunity to collaborate while engaging in learning mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student’s education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make mathematical sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

Furthermore, mathematical modeling can provide ELs a critical bridge which associates mathematics with real world phenomena and offers them opportunities to develop and refine their decision-making process as they engage in the iterations of formulating a mathematical model.

Finally, teachers should set high academic expectations commensurate to the language proficiency level of each multilingual learner and evaluate not only the product outcomes (solutions/answers) of their respective mathematical models but also the logical and reasoning thought process evident in the various reiterations of the mathematical modeling cycle. The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a realistic problem or task.
Framework For Statistical Reasoning

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us. The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

To help ELs master Georgia’s K-12 Mathematics Standards, it will be imperative to have resources and scaffolds that more specifically support them in accessing and acquiring grade-level knowledge and skills.

The GaDOE Mathematics team has a wide variety of resources to support students, teachers, and leaders throughout the state. These supports offer the tools needed to meet the individualized learning needs of all students. These resources include Georgia’s K-12 Mathematics Standards, curriculum resources, comprehensive grade level/course curriculum overviews, instructional unit frameworks, parent and student resources, virtual mathematics professional learning, Guides for Effective Mathematics Instruction, instructional resources for mathematics teachers, mathematics resources for English Learners, Professional Learning Opportunities (PLO) courses, and Georgia Mathematics Strategies Toolkit to Address Learner Variability. Effective methods for enabling ELs to meet Georgia’s K-12 Mathematics Standards build on approaches that are supported by the GaDOE. In addition, the GaDOE provides additional support for multilingual learners, who are learning mathematics content and developing proficiency in the English language simultaneously.

The Mathematics Digital Learning Plans found in Georgia Home Classroom focusing on English Learner Supports can be used by teachers of multilingual learners as structured references to modify other tasks and lessons. Academic mathematics coaches and other mathematics content specialists, who provide professional development for building teacher capacity in engaging multilingual learners in deep mathematical discourse, may also use the English Learner Supports as exemplars.

Teachers may additionally compare the ELs scaffolds in multiple lessons and, in collaboration with their Professional Learning Communities (PLCs), make decisions about which scaffolds may be most beneficial to the student’s language proficiency level in specific classes or lessons.

The following table comprises a collection of three Mathematics Digital Learning Plans focusing on English Learner Supports, followed by an overview of malleable, vetted, high-yield scaffolds to engage and support ELs in meeting the rigors of Georgia’s K-12 Mathematics Standards.
These Mathematics Digital Learning Plans are as shown in the following table:

<table>
<thead>
<tr>
<th>Name of Digital Learning Plan</th>
<th>Grade</th>
<th>Content Addressed</th>
<th>Mathematical Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Language Scaffolds for English Learners – WIDA ELD Key Language Use - Argue</td>
<td>All grade levels</td>
<td>Students use academic English to produce a viable mathematical argument, to defend the validity of their mathematical reasoning, and to critique reasoning of others. (<em>WIDA English Language Development Standards, 2020, p. 233</em>)</td>
<td>MP.3 Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>Language Scaffolds for English Learners – WIDA ELD Key Language Use - Explain</td>
<td>All grade levels</td>
<td>Students use English language with precision to explain their mathematical reasoning and solutions. (<em>WIDA English Language Development Standards, 2020, p. 230</em>)</td>
<td>MP.6 Attend to precision.</td>
</tr>
<tr>
<td>Language Scaffolds for English Learners – WIDA ELD Key Language Use - Inform</td>
<td>All grade levels</td>
<td>Students use English language with precision to explain their mathematical reasoning and solutions. (<em>WIDA English Language Development Standards, 2020, p. 227</em>)</td>
<td>MP.6 Attend to precision.</td>
</tr>
</tbody>
</table>

**General Approach**

Scaffolds are displayed in two sets of Performance Definitions. One set of Performance Definitions scaffolds are for the interpretive language (viewing, listening, and reading) skills and represents how ELs process language to comprehend information, ideas, or concepts in either oral or written communication. The other set of Performance Definitions scaffolds are for the expressive language (representing or modeling, speaking, and writing) skills and shows how students use language to express information, ideas, or concepts about their mathematical thinking or seek clarification about mathematical content and/or academic mathematics language in either oral or written communication.

The suggested scaffolds may be used with ELs at either the emerging, developing, or expanding levels of English proficiency. It is expected that teachers identify, decide, and establish the level of variable academic support, accountability and precision expected for students based on the proficiency of the student measured by the WIDA ACCESS for English language learners (ELLs) Overall Composite Proficiency Levels (CPL) 1 - 5.

Students do not follow one common pathway toward language development. Many factors affect language development (e.g., student personality, language exposure, instructional design, service delivery, scaffolding, models for language). Therefore, the WIDA CPL Performance Definitions outline many pathways to students’ language development. We suggest collaboration between mathematics educators and language specialists who work together in supporting scaffolding mathematics teaching and learning for ELs.
Use Scaffolding Techniques and Protocols Consistent with Georgia’s K-12 Mathematics Standards

The scaffolding techniques and protocols are consistent with the Georgia’s K-12 Mathematics Standards by aligning them with criteria in the Georgia Mathematics Strategies Toolkit to Address Learner Variability and Guides for Effective Mathematics Instruction (K-12).

The research-based scaffolding routines include the primacy of teaching academic vocabulary language intentionally and intensively using several best practices: incorporating oral and written English language instruction into content area teaching; creating frequent opportunities to develop written academic discourse; building student background content knowledge and capitalizing on multilingual learners’ home language skills and knowledge.

Teaching Academic Vocabulary Language for English Learners

Academic vocabulary language is primarily selected for instruction because it is the key for understanding and making meaning of the content. Academic vocabulary language also appears frequently across content at the target grade level and throughout a student’s K-12 mathematics and academic learning pathway.

More broadly, academic language is much more than academic vocabulary and it “refers to the language used in school to acquire new or deeper understanding of the content and to communicate that understanding to others” (Bailey & Heritage, 2008; Gottlieb, Katz, & Ernst-Slavit, 2009; Schleppegrell, 2004, p. 2).

Introducing new academic language is much more than introducing new vocabulary. It is expected that teachers do so in an explicit and intentional manner, attending to precision and using accessible and initially communicated (written & spoken) student-friendly language. It is important to provide students with opportunities to practice and make meaning of their new academic vocabulary language with classmates and teachers.

Nuggets from the Classroom:

Informal language is a great entry gateway for ELs; however, it is important to consistently move ELs quickly into the accurate, more complex, and technical meanings and definitions.
When presenting academic vocabulary language to ELs, it is suggested for teachers to explain it and purposefully teach it using one of many, crucial instructional routines:

- Pronounce academic terms clearly several times and have students repeat the term, chorally and then to an elbow partner. This could also be implemented using digital resources.

- Spotlight vocabulary words that are the same as words in other contexts or have homophones with different meanings (*for* and *four*, for example).

- Explain homonyms, vocabulary words, with the same sound and the same spelling but multiple meanings (*plane* – a machine that is built to fly & *plane* – a two-dimensional figure).

- Initially provide the academic vocabulary term definition in student-accessible language and have students record and develop their own academic language register in some way, such as in a foldable, Frayer model, or through digital resources.

- It is critically important for ELs to be provided with opportunities to practice the academic vocabulary language for mathematics – across all four modalities (speaking, reading, writing, and listening) each day – as they engage to solve real-world problems and navigate understanding and meaning making.

- ELs should be provided with an immediate opportunity to engage academic vocabulary language in a real-world context, using multiple modalities (writing and speaking) either by speaking to a partner or by writing a sentence or two using the new term. It could also be implemented using digital resources.

**Interactive Word Walls**

To develop academic discourse and help students communicate their mathematics reasoning and understanding, students need to learn and use appropriate academic mathematics language. Mathematics word banks and word/phrase walls are easily visible scaffolds that can provide an appropriate bridge to relevant general academic language, such as ‘hand it in’ and ‘answer table;’ specialized academic language, such as ‘times table’ and ‘graphing calculator;’ and with the technical academic language of mathematics, as in ‘hypotenuse’ and ‘commutative property.’

As teachers model indicating when they have used or will use terms from the word banks or word/phrase walls, words and phrases are added as they are introduced in the day’s lesson, and students can use appropriate words and terminology as they develop academic ownership of the content and knowledge of the mathematics register.
Concrete, Visual Models & Realia

For ELs to use their emerging, developing, or expanding English language to make meaning and engage with mathematics understanding, visual representations are important tools to make mathematics concepts more apparent and accessible.

Visual models can represent explicitly and concretely mathematical concepts that are abstract (Moyer, 2001: p. 176). These models may include manipulatives, illustrations, realia, or other opportunities to have hands-on experiences with the concepts (Fuson, Kalchman, & Bransford, 2005).

It is always important for students to reflect on the meaning of the mathematical concepts being represented through the manipulatives and/or illustrations and for teachers to help students make mathematical “connections” between the material and abstract symbols (Van de Walle et al., 2009). In fact, Van de Walle et al. (2009) stated “A mindless procedure with a good manipulative is still just a mindless procedure” (p. 34) (as cited in Rosli, R., Goldsby, D., & Capraro, M. M., 2015).

- Visual Math Improves Math Performance
- Seeing As Understanding: The Importance of Visual Mathematics for our Brain and Learning
- Teaching Mathematics to Young Children Through the Use of Concrete and Virtual Manipulatives

Visual Organizers, Foldables & the Frayer Model

The Frayer Model is a visual organizer that helps students understand technical academic language and concepts. It is divided into four sections which may hold a term or definition, characteristics/facts or engaging question, examples (illustrations or image), and non-examples (illustration or image) of the academic language or concept.

Complete a Frayer Model using the word _____________________.

![Frayer Model Diagram](image)
Example Lesson for Teachers

Teacher Moves

**TM:** We are going to use Frayer models (see suggested image) to focus and record some of the important vocabulary information we will need for the sample lesson on functions. You will record these in your graphic organizer for today’s vocabulary.

Terms: domain, inputs, independent variable, range, outputs, dependent variable, and image.

The suggested layout includes –

- Front page – key academic words or terms written on the center of the page, and the four corners of the page list the definition or engaging question, characteristics of term (graph/visual), example (illustration) of the definition, and a nonexample (illustration) of the definition.
- Back page – could be modified to include a side-by-side definition in two languages, if applicable.

Additional modifications could include the back page to include side-by-side definition in two languages stressing the *cognate* relationships that may exist in both definitions. Whether they are provided to the students partially completed or students construct and populate them completely themselves depends on the level of rigor that best approaches – Vygotsky’s notion of the ELs’ zone of proximal development.
Foldables are visual organizers that fold in some type of manner. A foldable poses questions and students can selectively reveal the answers by lifting the flaps, as in a lift-the-flap book. Foldables are an organized way for students to help study and retain new mathematics academic content. Visual organizers are a high-yield support for multilingual learners because they provide an engaging hands-on learning experience and a means of organizing and representing complex information succinctly and graphically appealing. Printed Visual Organizers.

Multimedia to Enhance Comprehension
Multimedia can enhance instruction. The use of media and visuals can help engage and support ELs’ understanding of a mathematics academic topic that may be unknown to them.


Teachers could use short snippets from selectively vetted videos to support ELs’ understanding of procedural fluency, help students to build background content knowledge as well as to understand and practice a unit’s concepts, so long as they understand that the multimedia snippets or still shots do not, alone, convey the meaning of mathematical concepts.

Incorporating Oral and Written English Language Instruction into Content Area Teaching

Productive Discourse Opportunities to Speak with a Partner or Small Group
When using collaborative grouping strategies such as: Think-Pair-Share, Small groups, or Do-Talk-Record, partnering ELs with more proficient English speakers provides them with English language models.

However, ELs work better when partnered with peers that are within one or two English proficiency levels different. There are several excellent resources that provide guidance related to supporting academic meaning-making in mathematics with ELs and productive conversation moves to help ELs of all proficiency levels acquire the skills they need to engage in mathematics academic discourse. Students who reflect on what
they do and communicate with others about it are in the best position to build useful connections in mathematics. (Hiebert et al., 1997, p. 6).

The following links provide examples of suggested student discourse moves supported with sentence starters and sentence frames. As students participate in discourse, these suggested moves will help them frame and develop academic communicative skills, such as: elaborate and justify, support ideas with examples, build on or challenge a partner’s ideas, paraphrase, and synthesize conversation points.

With their classmates, ELs will explore mathematics concepts, explain their understanding, critique & justify claims, and make connections to the mathematics involved while they develop in mathematics and academic discourse simultaneously.

**DOING AND TALKING MATH AND SCIENCE: Strengthening Reasoning, Strengthening Language**


Through collaborative planning, teachers can anticipate the range of student approaches and then monitor them while circulating among student groups. They can then select and sequence specific approaches for the whole group to consider. In this way the teacher can use evidence of student thinking to build a bridge to the mathematical concepts at the heart of the lesson (Smith & Stein, 2011). To assist ELs, provide students with sentence frames and sentence starters to prompt and promote discourse.
The following are suggested Student Discourse Moves:

<table>
<thead>
<tr>
<th>WORKING WITH IDEAS: Student Discourse Moves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emerging proficiency (PL2)</td>
</tr>
<tr>
<td><strong>Tell and explain a new idea</strong></td>
</tr>
<tr>
<td>I think…My reason is…</td>
</tr>
<tr>
<td>My idea is…</td>
</tr>
<tr>
<td>I think Layla is right. My reason is I got the same answer.</td>
</tr>
<tr>
<td><strong>Clarify</strong></td>
</tr>
<tr>
<td>Say again, please.</td>
</tr>
<tr>
<td>(Seeking repetition)</td>
</tr>
<tr>
<td>What is …? (Seeking more information)</td>
</tr>
<tr>
<td>Why did you draw that?</td>
</tr>
<tr>
<td>(Seeking rationale)</td>
</tr>
<tr>
<td><strong>Restate or summarize an idea</strong></td>
</tr>
<tr>
<td>He said “…” (may attempt direct quotation; not likely to produce full details)</td>
</tr>
<tr>
<td>It means …</td>
</tr>
<tr>
<td>It is the weight. (Statement of main point, not embedded into another framing sentence.)</td>
</tr>
<tr>
<td>Ismail said he thinks Layla is wrong.</td>
</tr>
<tr>
<td><strong>Compare ideas</strong></td>
</tr>
<tr>
<td>It always works</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
Creating Opportunities for Student Discourse –

Figure 2 WIDA (2017). Teacher Discourse Moves [Images]. Retrieved from wida.wisc.edu

Engaging in Opportunities for Discourse –

Figure 3 WIDA (2017). Student Discourse Moves [Images]. Retrieved from wida.wisc.edu

<table>
<thead>
<tr>
<th>Support someone’s idea</th>
<th>Good idea because…</th>
<th>Remember, it said in our book that…</th>
<th>That explanation is a good one, since it accounts for…</th>
</tr>
</thead>
<tbody>
<tr>
<td>I agree with her because….</td>
<td>I am sure that would work since…</td>
<td>The advantage of the proposed change is…</td>
<td></td>
</tr>
<tr>
<td>I like what Daniela said. I drew the same picture</td>
<td>That idea makes sense because last week, we learned…</td>
<td>Jonathan’s explanation is the most powerful because he used both equations and an array to demonstrate why Lupita’s strategy did not work.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Safiyyah’s idea makes sense because her array shows that there are way more than 124 squares.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Build on someone’s idea

| Let us try that. We can draw a picture too. | What if we try it this way, instead of…? Let us change the model to show that. Jasmine’s idea made me wonder about… After we draw the array, we could write some equations that show how we figured out the answer. | The obvious next move would be to… Based on what Juan said, I think it would probably be helpful to build the array with base 10 blocks so that everyone else can see clearly which partial products Layla was missing. |

## Question or challenge someone’s idea

| I think that is wrong because… It will not work because… What is your evidence? | But your explanation will not account for… Did you think about…? How does the evidence support your idea that______ | Unfortunately, that explanation does not fit every situation. The situation you have neglected to account for is… Had you considered…? Do you think that there might be a more efficient way that you could multiply 13 and 18? |

To further facilitate mathematics discourse in the classroom teachers may post the following [Student Discourse Moves](#) and [Teacher Discourse Moves](#) graphs in the classroom, as a whole class reference, and students could also use laminated cutouts.
**Productive Discourse Opportunities to Write with a Partner or Small Group**

Techniques include providing writing assignments that are anchored in mathematics academic content and focused on developing mathematics academic language as well as writing skills; providing language-based supports to facilitate students’ access into and continued development of writing; using pairs, triads, and small collaborative groups to create discourse opportunities for students to work and converse together on varied aspects of academic writing; and assessing students’ writing periodically to identify instructional needs and provide positive constructive feedback in response (Baker et al., 2014, p. 6) (as cited in August, D. A., Fenner, D.S.F., Bright, A.B., 2014).

As students are given opportunities to build academic discourse using multiple modalities (writing, reading, listening, and speaking), teachers create opportunities for students, through collaborative small groupings, and through the Socratic method of questioning that require students to talk in pairs and then write, to work and talk together on varied aspects of writing.

Teachers are encouraged to provide writing assignments that include language-based scaffolded supports, such as visual organizers – Frayer Models, sentence frames and sentence starters to facilitate students’ continual development of academic discourse.

**Sentence Frames**

Sentence frames provide students with language chunks and a small range of options to complete sentences (Kinsella, 2012). Teachers may also make available a word bank of relevant academic vocabulary.

<table>
<thead>
<tr>
<th>Example Lesson for Teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Suggested Teacher Moves</strong></td>
</tr>
<tr>
<td><strong>TM:</strong> In this lesson, you will practice independently or with a partner, to complete the following example sentence frames for this sample lesson on functions. Teachers may also make available a word bank of relevant academic vocabulary.</td>
</tr>
</tbody>
</table>

Example Sentence Frames:
The _________ of a **function** is the ______ of all _________ for which the function rule can give an ____________.

**Word bank:** domain, set, inputs, output

Example Sentence Frames:
The _________ of a **function** is the ______ of all _________ for which there is an __________ that results in them.

**Word bank:** range, set, outputs, input.
### Sentence Starters

To further facilitate mathematics discourse in the classroom teachers may use the following **Student Discourse Moves** to help students communicate their ideas in a more academic register.

### Example Lesson for Teachers

#### Suggested Teacher Moves

**TM:** In this lesson, teachers will use a Desmos activity on Exponential Growth to leverage various multilingual supports under one interactive platform.

#### Notice and Wonder

[Image of a Desmos activity for Exponential Growth]

**Teacher Moves**

Encourage students to play the animation as many times as they need to understand the context. Consider using pacing to restrict students to this screen.

**Sample Responses**

- I notice that the cell is splitting in half.
- I wonder how many cells there will be at Stage 100.

#### Suggested Teacher Moves

**TM:** In this lesson, teachers will use multilingual supports such as a sentence starter to help students develop Mathematics Meaning Making.

### Notice and Wonder

[Image of a Desmos activity for Exponential Growth]

**Teacher Moves:** Help a Student Clarify

Wait time (20-30 seconds) after questions and after responses.

**Student Moves:** Tell and explain

I notice that the cell is ________________.

I wonder how many cells ________________.

**T M:** Make an Idea Public

“Tell us more about what you’re thinking”

**S M:** Restate or Summarize

“In other words,...”
Background Knowledge for English Learners and their Teachers

Teachers are encouraged to not solely rely on prerequisite content knowledge nor pose leading questions about “real life” contexts used in mathematics problems such as “You know what a Hot Air Balloon is, right?” and rely on students’ affirmative responses as sufficient to ensure that all learners are aware of the contextual situation being presented.

Because students may be unsure and self-conscious about the multitude of English contextual references that are unfamiliar to them, they may not be entirely forthright in publicly admitting the information that is not known to them. It is important to present or reinforce contextual references using multimedia, pictures, and visuals to help build background knowledge.

In addition to building background knowledge for ELs, teachers, themselves, are encouraged to simultaneously develop their own background knowledge capacity, particularly about the populations and families they serve and the mathematics contributions that have, historically, been provided by various peoples and cultures throughout the world.

In addition, teachers building their knowledge capacity may also afford themselves opportunities to discover or uncover novel or more intuitive methods of engaging and solving mathematics problems and an awareness to “the use of home language instruction for helping ELs develop literacy and content knowledge in English” (e.g., Francis, Lesaux, & August, 2006, p. 6, as cited in August, D. A., Fenner, D.S.F., Bright, A.B., 2014).

Ensuring that all students understand the “real life” contextual evidence being presented in class is an essential prerequisite for development of understanding and meaning making. Teachers are encouraged to expand the references presented to make broader connections to universal lived experiences.

A large body of research indicates that ELs draw on conceptual knowledge and skills acquired in their home language in learning their new language (Dressler, 2006) and that instructional methods that help ELs draw on home language knowledge and skills promote literacy development in a new language (August et al., 2009; Carlo et al., 2004; Liang, Peterson, & Graves, 2005, as cited in August, D. A., Fenner, D.S.F., Bright, A.B., 2014).

Nuggets from the Classroom:

BE YOURSELF! When you show that you want the best for your students, they will see that you care and that they are important. Students can sense when you care about their success, and they will rise or fall to your expectations!
Clarify Content Delivered in a Second Language

It is imperative that ELs understand the necessary contextual mathematics references in English. This can be accomplished by clarifying key concepts through the reduction of the complexity and language demand of academic text without reducing the rigor of the mathematics standards.

Clarifying Key Concepts

At times, the linguistic complexity of the language commonly used in mathematics may impede student access to the content being taught. By attending to the language load in the texts shared with students and in the explanations provided orally, key content can be highlighted and clarified (Abedi & Lord, 2001).

Here is an example of a diagnostic assessment from an eighth grade Sample Mathematics Learning Plan, with a scaffolded, contextually modified scenario for comparison:

**Original Text**

Dr. West, a sports medicine doctor, provides diagnostic treatments and therapeutic treatments. Each diagnostic treatment consists of 4 injections, and each therapeutic treatment consists of 2 injections. Last year, Dr. West gave a total of 60 treatments that consisted of a total of 184 injections.

- How many diagnostic treatments did Dr. West give last year?
- How many therapeutic treatments did Dr. West give last year?
- Dr. West is planning ahead for next year. How many of each injection do you recommend she order? Explain how you arrived at your answers.

**Scaffolded Text**

A doctor gives injections for diagnostic and therapeutic treatments.

One diagnostic treatment has 4 injections.

One therapeutic treatment has 2 injections.

The doctor gave 60 treatments AND 184 injections.

- How many diagnostic treatments did the doctor give?
- How many therapeutic treatments did the doctor give?
- How many of each injection do you recommend the doctor order?
- Explain how you arrived at your answers.

### Example Lesson for Teachers

<table>
<thead>
<tr>
<th>Suggested Teacher Moves</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TM:</strong> In this lesson, teachers will provide multilingual learners with the graphic organizer that reduces the number of treatments to fifteen (15) for students to...</td>
</tr>
</tbody>
</table>
develop patterns and make sense of the content of this problem. Use the suggested modified language to help focus attention on key conceptual ideas. Because we reduced the number of treatments to fifteen, teachers should also direct students to reduce the number of injections to forty-six (46).

a) Complete the following table.

<table>
<thead>
<tr>
<th># of Treatments</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Diagnostic Injections</td>
<td>4</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of Therapeutic Injections</td>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total # of Injections</td>
<td>6</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) When will there be more than forty-six (46) total injections? (On how many # of treatments?)

c) Write a formula that models the total number of Diagnostic and Therapeutic treatments. (Let the # of Diagnostic treatments to be \(x\) and the # of Therapeutic treatments to be \(y\))

d) Write an expression that models the number of Diagnostic injections. (Let the # of Diagnostic treatments to be \(x\))

e) Write an expression that models the number of Therapeutic injections. (Let the # of Therapeutic treatments to be \(y\))

f) Use your results from d & e to write a formula that models the total number of Diagnostic and Therapeutic injections.

g) Use the results from a through f to answer the following questions:
   - The doctor gave 60 treatments AND 184 injections.
   - How many Diagnostic treatments did the doctor give?
   - How many Therapeutic treatments did the doctor give?
   - How many of each injection do you recommend the doctor order?

Explain how you arrived at your answers.
Teacher Modeling and Explanation

Evidence-based research has shown that when teaching mathematics learners, it is important for teachers to establish learning environments and classroom norms that support the active engagement of all students, including ELs. In these types of learning environments, the teacher honors the diverse ways in which students approach mathematics, communicate their mathematical thinking (Moschkovich, 2009), and record their strategies and solutions to problem-solving tasks (Tankersley, 1993).

- Teaching Math Their Way
- Use and Connect Mathematical Representations

Nuggets from the Classroom:

Discourse used in the process of meaning making may be simplified and ELs should be provided with the necessary scaffolds to make sense of challenging content, while maintaining the rigor and intentional objectives of the K-12 mathematics standards.
Capitalize on Student’s Home Language Skills and Knowledge

A large body of research indicates that ELs draw on conceptual knowledge and skills acquired in their home language in learning their new language (Dressler, 2006) and that instructional methods that help ELs draw on home language knowledge and skills promote literacy development in a new language (August et al., 2009; Carlo et al., 2004; Liang, Peterson, & Graves, 2005, as cited in August, D. A., Fenner, D.S.F., Bright, A.B., 2014).

Differentiate Instruction for Students at Variable Levels of English Language Proficiency for English (as a 2nd language) Learners

Students use their emerging, developing, or expanding English language to make meaning and engage with mathematics understandings. When classroom teachers encourage mathematics classroom discussions in ways that support acquisition of mathematics concepts and language development, student learning increases (Smith & Stein, 2011).

The student scaffolds presented in this Resource Guide and illustrated in the Digital Learning Plans can be used on students at the emerging, developing, and expanding levels of English language proficiency (ELP) as measured by the WIDA ACCESS for ELLs (Overall Composite Proficiency Levels 1 - 5). In addition, examples for presenting a lesson and explaining how students of a variety of proficiency levels engage with Mathematics content and produce language can be referenced by reviewing the suggestion provided in WORKING WITH IDEAS: Student Discourse Moves.

It is expected that teachers will identify their students’ level of English proficiency to select the appropriate teacher actions.
References


Jenny, (2017). Math Foldables [Image Figure 1]. Retrieved from Math Foldables - Math in Demand


