



Content Descriptions

Based on the Georgia Performance Standards

GPS Algebra



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"Making Education Work for All Georgians"

Introduction

The State Board of Education is required by Georgia law (A+ Educational Reform Act of 2000, O.C.G.A. §20-2-281) to adopt End-of-Course Tests (EOCT) designed to measure student achievement in core subjects in grades nine through twelve. With educator input and State Board of Education approval, eight content areas were designated in 2001 to be tested. The Georgia Performance Standards (GPS) in mathematics were adopted by the State Board of Education in July 2004, and the GPS Algebra EOCT was developed based on these standards.

Program Purpose

The EOCT are designed to improve student achievement by assessing student performance on the standards specific to each course tested. Student performance on each EOCT is provided to schools for diagnostic and remedial use. These results are used to help make instruction more effective and to ensure that all Georgia students have access to a rigorous curriculum that meets high academic standards. These results are also used for student accountability and to gauge the quality of education in the state. The EOCT are the final exams for each EOCT course. For students in grade 10 or above beginning the 2011-2012 school year, the final grade in each course is calculated by weighing the course grade 85% and the EOCT score 15%. For students in grade 9 beginning the 2011-2012 school year and later, the final grade in each course is calculated by weighing the course grade 80% and the EOCT score 20% (State Board Rule 160-4-2-.13). The student must have a final grade of at least 70 to pass the course and to earn credit toward graduation.

EOCT Content Descriptions

The EOCT Content Descriptions are provided to acquaint Georgia educators with the content coverage of the EOCT. Only the knowledge, concepts, and skills addressed in the GPS are assessed on the EOCT. Committees of Georgia educators reviewed the curriculum and provided guidance for the assessment program.

It is important to note that some curricular standards are better suited for classroom or individual assessment rather than large-scale, summative assessment. While those curricular standards designed for classroom/individual assessment are not included in the Content Descriptions, the knowledge, concepts, and skills outlined are often required for the mastery of the standards that are assessed. Therefore, the EOCT Content Descriptions are in *no way* intended to substitute for the GPS; they are provided to help educators better understand how the curriculum will be assessed. Further, the EOCT Content Descriptions *by no means* suggest *when* concepts and skills should be introduced in the instructional sequence; rather, their purpose is to communicate when concepts and skills will be assessed on the EOCT. Georgia law requires educators to teach the standards set forth in the state-adopted curriculum (i.e., the GPS). The GPS are located at <http://www.georgiastandards.org>.

GPS Algebra Domains

To provide reliable measures of student achievement and to give structure to the assessment program, the content standards contained in the GPS were grouped into content domains. Each domain was created by combining standards that share similar content characteristics. Two domains were identified for GPS Algebra.

- **Algebra (includes Number and Operations)**
Students will demonstrate the ability to use the complex number system; explore functions; solve radical, simple quadratic and rational equations; simplify and perform operations with radical, polynomial, and rational expressions; investigate piecewise and quadratic functions using numerical, analytical, and graphical approaches, focusing on the use of these functions in problem-solving situations; solve equations and inequalities related to these functions.
- **Data Analysis and Probability**
Students will demonstrate the ability to determine probability; use both permutations and combinations to find the number of outcomes; pose questions to be answered by collecting data; organize, represent, investigate, interpret, and make inferences from data; use linear and quadratic regressions to analyze data and to make inferences.

Process Standards

The GPS in mathematics require content to be taught in conjunction with process skills identified as the process standards. These process standards are necessary for students to master each of the mathematics content standards. Problem solving, reasoning, representation, connections, and communication are the critical dimensions of mathematical proficiency that all students need.

The concepts and skills inherent in the process standards are integrated in items across the two content domains.

Overview of the Process Standards

- Students will solve problems (using appropriate technology).
- Students will reason and evaluate mathematical arguments.
- Students will communicate mathematically.
- Students will make connections among mathematical ideas and with other disciplines.
- Students will represent mathematics in multiple ways.

Associated GPS Standards

MM1P1 through MM1P5

Associated GPS Concepts and Skills

- Building new mathematical knowledge through problem solving.
- Solving problems that arise in mathematics and in other contexts.
- Applying and adapting a variety of appropriate strategies to solve problems.
- Reflecting on and monitoring the process of mathematical problem solving.
- Recognizing reasoning and proof as fundamental aspects of mathematics.
- Making and investigating mathematical conjectures.
- Developing and evaluating mathematical arguments and proofs.
- Selecting and using various types of reasoning and methods of proof.
- Organizing and consolidating mathematical thinking through communication.
- Communicating mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyzing and evaluating mathematical thinking and strategies of others.
- Using the language of mathematics to precisely express mathematical ideas.
- Recognizing and using connections among mathematical ideas.
- Understanding how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognizing and applying mathematics in contexts outside of mathematics.
- Creating and using representations to organize, record, and communicate mathematical ideas.
- Selecting, applying, and translating mathematical representations to solve problems.
- Using representations to model and interpret physical, social, and mathematical phenomena.

GPS Algebra

Domain: Algebra (includes Number and Operations)

Overview of the Domain

- Students will represent and operate with complex numbers.
- Students will use graphs, tables, and simple algebraic techniques to explore and interpret the characteristics of functions.
- Students will simplify and perform operations with radical expressions, polynomials, and rational expressions.
- Students will solve radical, simple quadratic, and rational equations.
- Students will investigate step and piecewise functions, including greatest integer and absolute value functions.
- Students will analyze quadratic functions in the forms $f(x) = ax^2 + bx + c$ and $f(x) = a(x - h)^2 + k$.
- Students will solve quadratic equations and inequalities in one variable.

Associated GPS Standards

MM2N1 MM1A1 MM1A2 MM1A3 MM2A1 MM2A3 MM2A4

Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- write square roots of negative numbers in imaginary form; write complex numbers in the form $a + bi$ in the context of solving quadratic equations
- add, subtract, multiply, and divide complex numbers
 - apply the associative, distributive, and commutative properties
 - identify and find conjugates of complex numbers
- simplify expressions involving complex numbers
 - understand that $i = \sqrt{-1}$ and $i^2 = -1$
- use function notation ($f(x)$ read as f of x) to represent functions and to convey functional relationships presented in tables, graphs, and algebraic form
- graph and identify graphs of basic functions (limited to $f(x) = x^n$, where $n = 1$ to 3 , $f(x) = |x|$, $f(x) = \sqrt{x}$ and $f(x) = \frac{1}{x}$)
 - select a graph that matches a particular function
 - select a function that matches a given graph
 - understand that graphs are geometric representations of functions
- graph transformations of basic functions
 - examine and identify vertical shifts, vertical stretches, and vertical shrinks of parent functions
 - explore and identify reflections across the x - and y -axes of parent functions
- investigate and explain the characteristics of quadratic, cubic, inverse, absolute value, and square root functions (using linear functions only as a building block)
 - identify a domain (the set of inputs) and a range (the set of outputs)

- understand set notation
- explore the zeros/solutions
- find x - and y -intercepts
- determine intervals of increase and decrease
- locate maximum and minimum values
- explain end behavior
- relate the characteristics of a function to a given context
 - utilize graphs, tables, and words to explain and predict the behavior of a function
 - understand the distinctions between discrete and continuous domains
- recognize sequences as functions with domains that are whole numbers greater than zero
 - examine sequences given in tables, algebraically, or by producing a context and identifying the corresponding function
 - understand the difference between finite and infinite sequences
 - explore how and when to use a recursive definition for a given pattern or sequence
- explore rates of change
 - compare graphs of functions that have a constant rate of change (i.e., slope) versus graphs that have variable rates of change
 - compare rates of change of linear, quadratic, square root, and other function families
 - explore average rates of change in regard to speed, cost, revenue, and other real-world applications
- determine graphically and algebraically whether a nonlinear function has symmetry
 - identify if a function is even, odd, or neither
 - interpret if a given function has symmetry
- understand that in any equation x can be interpreted as the equation $f(x) = g(x)$
 - interpret the solutions as the x -value(s) of the intersection points of the graphs of $y_1 = f(x)$ and $y_2 = g(x)$
 - use algebra to find the value of x that makes $f(x) = g(x)$ true
 - understand that functions are equal if they have the same domain and rule of correspondence
- simplify algebraic expressions involving square roots
- perform mathematical operations with square roots
 - understand when to rationalize a denominator
 - comprehend the equivalence of a simplified square root expression and the equivalence of a nonsimplified square root expression
- add, subtract, multiply, and divide polynomials
- use the Binomial Theorem to expand binomials, limited to the third power
- add, subtract, multiply, and divide rational algebraic expressions
- factor expressions involving the difference/sum of two squares, difference/sum of two cubes, and trinomials in the form $ax^2 + bx + c = 0$, where $a = 1$ and factor methods limited to the greatest common factor, grouping, trial and error, and special products

- utilize area formulas of polygons and volume models of prisms, cylinders, etc., for polynomial arithmetic
- use either factorization or square roots to solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$
- solve simple radical equations by isolating the variable and squaring both sides
- use technology, tables, and graphs to solve equations resulting from the investigation of $x^2 + bx + c = 0$
 - interpret the solution of a quadratic function from a graph of the data
 - identify and comprehend the meaning of the x -intercepts from a table of quadratic data
- solve simple rational equations that result in linear or quadratic equations with a leading coefficient of 1
- write and interpret absolute value functions as piecewise functions
 - recognize and write appropriate function rules for different parts of the domain
- investigate and explain characteristics of a variety of piecewise-defined functions, such as absolute value and greatest integer functions; relate these characteristics to a real-life situation modeled by such a function
 - translate fluently between graphical, algebraic, and numeric representations
 - identify the domain and range
 - find the vertex and axis of symmetry
 - identify the zeroes
 - find the x - and y -intercepts
 - identify points of discontinuity
 - identify intervals where the value of a function is constant, increasing, or decreasing
 - investigate rates of change for specified intervals
- solve absolute value equations and inequalities
 - use algebraic and analytical methods
 - determine solutions using graphs and/or number lines
- graph quadratic functions as transformations of the function $f(x) = x^2$
 - identify vertical and horizontal stretches and compressions, and vertical and horizontal translations
 - explore reflections across the x - and y -axes
- convert between standard $y = (ax^2 + bx + c)$ and vertex $y = (a(x - h)^2 + k)$ forms of a quadratic function using the roots of the quadratic and the symmetry properties of the parabola; use the vertex form to locate and graph a quadratic function, e.g., when using a quadratic function to model a data relationship; translate from vertex form back to standard form to identify the parameters a , b , and c
- investigate and explain characteristics of quadratic functions; use these characteristics to model and solve real-world problems
 - identify domain and range
 - identify the vertex and axis of symmetry

- find all zeroes
- find the x - and y -intercepts
- locate extrema using ordered pairs and be able to identify maximum and minimum values
- determine intervals of increase and decrease
- investigate rates of change for specific intervals
- explore arithmetic series and various ways of computing their sums
 - calculate the sum of a series by representing it as a function, using patterns, or using a formula, e.g., $S_n = \frac{n}{2}(a_1 + a_n)$
 - evaluate the sum of a finite series expressed in summation notation
- recognize sequences of partial sums of arithmetic series as examples of quadratic functions
 - calculate the partial sum of a finite series by representing it as a function, using patterns, or using a formula, e.g., $S_n = \frac{n}{2}[2a_1 + (n-1)d]$ or

$$S_n = \frac{n}{2}(a_1 + a_n)$$
- solve quadratic equations graphically; use graphs to estimate solutions to quadratic functions and identify the number of real solutions and intervals on which the solutions occur
 - interpret the x -intercepts of the function in the context of a real-life situation that is modeled by the function
- find real and complex solutions of quadratic equations analytically; be familiar with multiple methods and recognize when a certain method is most appropriate
 - use factoring methods and the zero-product property
 - apply the quadratic formula
 - explore both exact and approximate solutions; recognize when each type of solution is appropriate and why
 - recognize how the solutions of quadratic equations apply to a real-world situation modeled by the quadratic function, e.g., when one or both roots are meaningless in context
- analyze the nature of roots of quadratic equations
 - use a graph, a table of values, and/or the discriminant, $b^2 - 4ac$, to determine the number and nature of real and imaginary solutions; relate the value of the discriminant to the behavior of the graph and/or the context
- solve quadratic inequalities and describe the solutions using linear inequalities
 - determine the solutions using one-variable inequalities obtained from factoring and apply the zero-product property; relate solutions to graphs on a number line; use the number line to investigate and identify solution intervals
 - understand and apply conventions related to graphical representations of inequalities, e.g., shading and dotted lines vs. solid lines; use various strategies to test which part of a graph should be shaded and identify correct shading

GPS Algebra

Domain: Data Analysis and Probability

Overview of the Domain

- Students will determine the number of outcomes related to a given event.
- Students will use the basic laws of probability.
- Students will relate samples to a population.
- Students will explore variability of data by determining the mean absolute deviation (the average of the absolute values of the deviations).
- Students will determine an algebraic (limited to linear or quadratic) model to quantify the association between two quantitative variables.

Associated GPS Standards

MM1D1 MM1D2 MM1D3 MM1D4 MM2D2

Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- apply the addition and multiplication principles of counting
- calculate and use simple permutations and combinations
 - integrate the multiplication principle to clarify the difference between permutations and combinations and when each is appropriate to use for a situation
 - use diagrams to justify the classification
 - utilize permutation and combination formulas to determine the number of possible arrangements of real-world events
- understand when an event is mutually exclusive and use diagrams, tables, and the formula $P(A \text{ or } B) = P(A) + P(B)$ to calculate the probability of mutually exclusive events
- use diagrams, tables, and the formula $P(A \text{ and } B) = P(A) \bullet P(B|A)$ to find the probabilities of dependent events and understand when an event is dependent
- use diagrams, tables, and the formula $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ to calculate conditional probabilities of real-world events
- use expected value to predict outcomes and make inferences
- compare summary statistics from one sample data distribution to another sample data distribution
 - interpret the mean, median, quartiles, and interquartile range of multiple data sets
 - understand normal and binomial data distributions
 - describe center and variability of data distributions
- compare the averages of summary statistics from a large number of samples to the corresponding population parameters
 - extract information from two data sets (such as the set from an overall population to the set for a sample)

- interpret histograms to compare data sets
- understand that a random sample is used to improve the chance of selecting a representative sample
 - determine the type of sampling to be used, given a scenario, so that a survey yields results from a random population sample
 - understand that a random sample will yield unbiased results
- explore the variability of data by determining the mean absolute deviation (the average of the absolute values of the deviations) and find the greatest or least mean absolute deviation of a data set
- use linear and quadratic functions to model relationships between two quantitative variables presented numerically and/or graphically
 - decide whether a linear or quadratic model or neither is appropriate for data presented in a table or graph
 - recognize an appropriate algebraic model given a table or graph (note: in the absence of appropriate technology, students will be expected to estimate the correct parameters for a linear or quadratic function presented in the form $y = ax + b$ or $y = ax^2 + bx + c$)
 - decide whether or not a particular set of data is appropriately modeled by a given function
- understand issues that arise when using data to explore the relationship between two variables, including correlation, e.g., recognizing whether the fit of an algebraic model is strong, weak, or nonexistent; confusing correlation and causation