2019 Fall Curriculum Leaders' Conference

NUMERACY STRATEGIES
for
ADVANCED STUDENTS

September 25, 2019

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Identify Needs
Select Interventions
Plan Implementation
Implement Plan
Examine Progress
Coherent Instruction
Professional Capacity
Supportive Learning Environment
Family & Community Engagement
Effective Leadership
READY TO LEARN
READY TO LIVE
READY TO LEAD
WHOLE CHILD

Building a Foundation

- A student's level of numeracy will determine success in middle and high school mathematics
- Students retained as early as K-4 are 5 times more likely to drop out of school
- 6th graders who fail English or Math have only a 10% chance of graduating on time
- Interventions – get at root causes for misunderstanding
- To get to college-level courses such as Calculus, a student must accelerate their curriculum either during middle school or high school
- Important to have a K-16 perspective
- GaDOE Numeracy Project – to help diagnose gaps and how to support students
- 8 Standards of Mathematical Practice (means by which we teach numeracy)

(www.gadoe.org/Curriculum-instruction-and-Assessment/Curriculum-and-instruction/Pages/Mathematics.aspx)

Standards for Mathematical Practice

- Habits of Mind for a mathematical thinker

1) To make sense of problems and persevere in solving them
2) Reason abstractly and quantitatively
3) Construct viable arguments and critique the reasoning of others
4) Model with mathematics
5) Use appropriate tools strategically
6) Attend to precision
7) Look for and make use of structure
8) Look for and express regularity in repeated reasoning

Mathematical Practices for AP Calculus

MPAC 1: Reasoning with definitions and theorems

MPAC 2: Connecting concepts

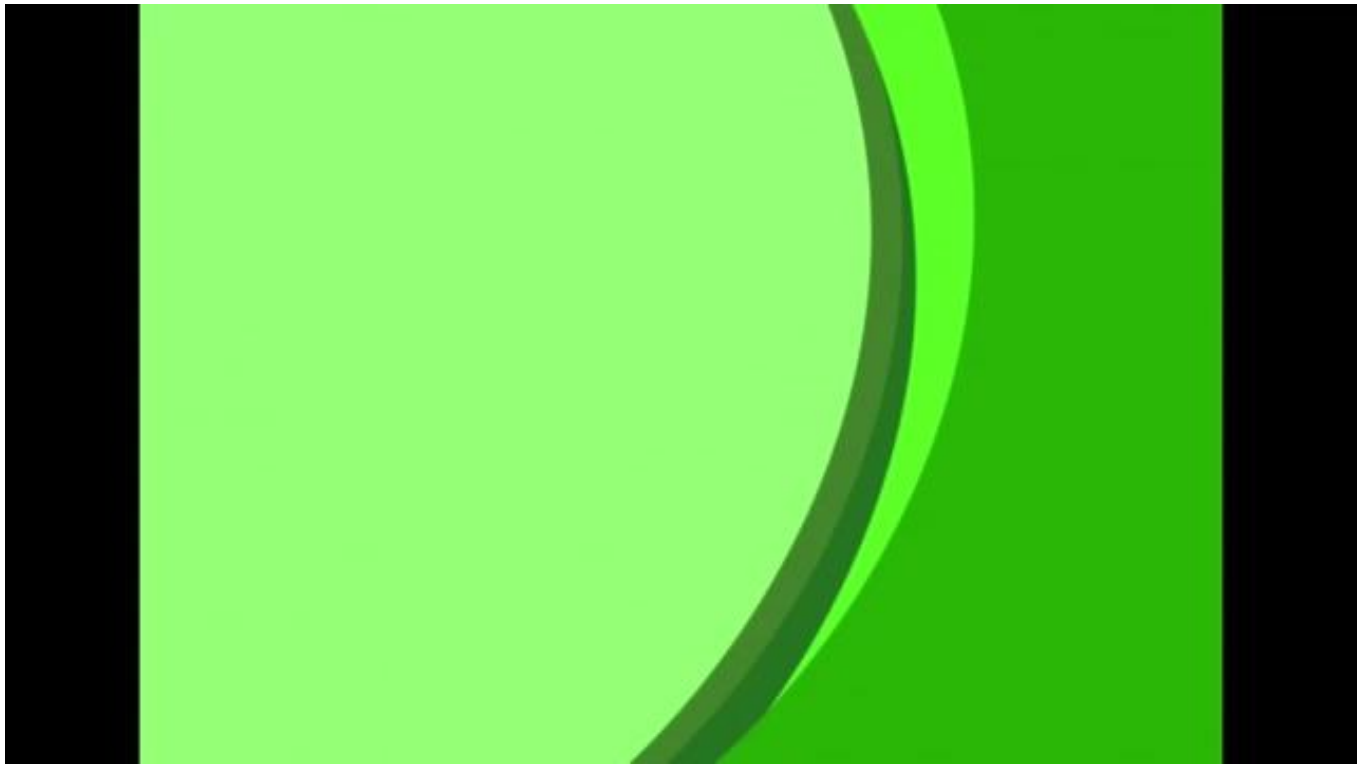
MPAC 3: Implementing algebraic/computational processes

MPAC 4: Connecting multiple representations

MPAC 5: Building notational fluency

MPAC 6: Communicating

Fostering Academic Conversations and Ownership of Learning



DOK Levels

- DOK 1: Recall and Reproduction
- DOK 2: Skills & Concepts
- DOK 3: Strategic Thinking and Reasoning
- DOK 4: Extended Thinking

DOK is about the intended outcome, not the difficulty…

DOK is a reference to the complexity of mental processing that must occur to answer a question, perform a task, or generate a product:

- Addition is a mental process.
- Knowing the rule for addition is the intended outcome that influences the DOK level.
- Once a student learns the "rule" of addition, 2 + 2 is DOK 1 and is also easy.
- Adding 2,351,789 + 9,874,320 is still a DOK 1 but may be more "difficult."

DOK 1 (Recall and Reproduction)

- Requires recall of information, such as a fact, definition, term, or performance of a simple process or procedure

Find the area of a rectangle whose length is 4 cm and width is 7 cm.

DOK 2 (Basic Application of Concepts & Skills)

- Includes the engagement of some mental processing beyond recall or reproducing a response requiring students to make some decisions as to how to approach the question or problem

Find the area of a right triangle whose hypotenuse has length 13 cm and one leg has length 5 cm.

DOK 3: Strategic Thinking and Reasoning

- Requires deep understanding exhibited through planning, using evidence, and more demanding cognitive reasoning that is complex and abstract

- An item that has more than one possible answer and/or requires students to justify the response would most likely be Level 3.

Using each of the four mathematical operations one time, create three different expressions that simplify to an answer of 24. Parentheses are allowed.

DOK 4: Extended Thinking

- Requires high cognitive demand and is very complex and should require in-depth analysis using multiple sources or describing real-world phenomena.

- Students are expected to make connections, relate ideas within the content or among content area, and select or devise one approach among many alternatives on how the situation can be solved.

Is Georgia experiencing global warming? Use historical temperature data to investigate the best-fit sine curves for annual temperature trends. Has there been a significant shift in temperature readings in recent years?

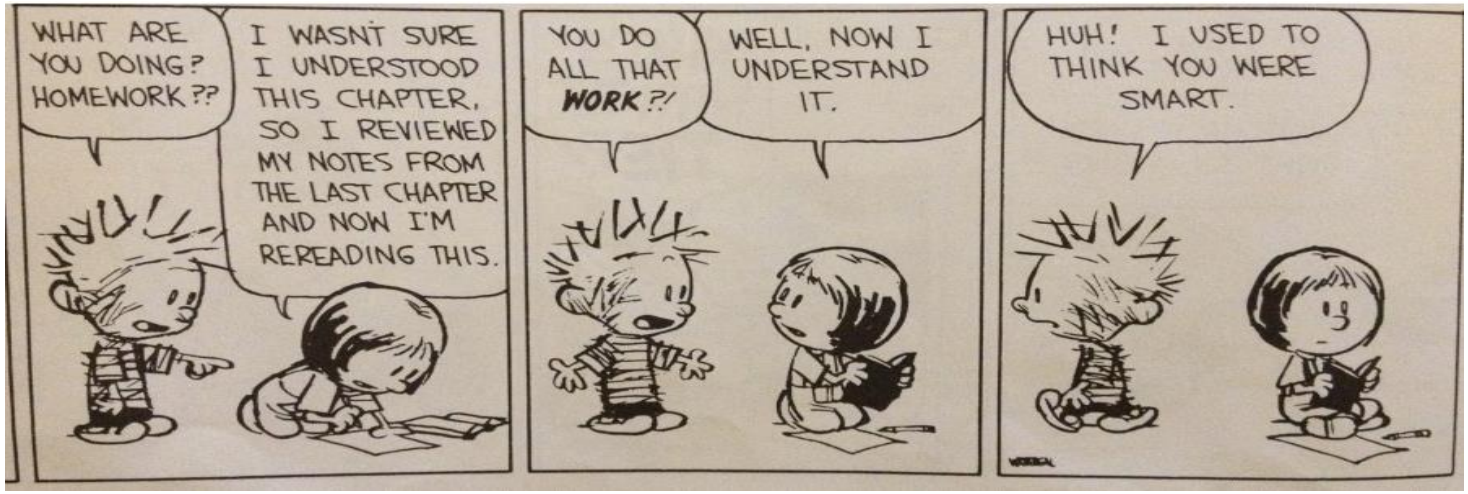
Hess Cognitive Rigor Matrix

Depth + Thinking	Level 1 Recall & Reproduction	Level 2 Skills & Concepts	Level 3 Strategic Thinking	Level 4 Extended Thinking
Remember				
Understand				
Apply				
Analyze				
Evaluate				
Create				

Revised Bloom's Taxonomy	Webb's DOK Level 1 Recall & Reproduction	Webb's DOK Level 2 Skills & Concepts	Webb's DOK Level 3 Strategic Thinking/Reasoning	Webb's DOK Level 4 Extended Thinking
Remember Retrieve knowledge from long-term memory, recognize, recall, locate, identify	o Recall, observe, & recognize facts, principles, properties o Recall/ identify conversions among representations or numbers (e.g., customary and metric measures)	Use these Hess CRM curricular examples with most mathematics or science assignments or assessments.		
Understand Construct meaning, clarify, paraphrase, represent, translate, illustrate, give examples, classify, categorize, summarize, generalize, infer a logical conclusion), predict, compare/contrast, match like ideas, explain, construct models	o Evaluate an expression o Locate points on a grid or number on number line o Solve a one-step problem o Represent math relationships in words, pictures, or symbols o Read, write, compare decimals in scientific notation	o Specify and explain relationships (e.g., non-examples/examples; cause-effect) o Make and record observations o Explain steps followed o Summarize results or concepts o Make basic inferences or logical predictions from data/observations o Use models /diagrams to represent or explain mathematical concepts o Make and explain estimates	o Use concepts to solve non-routine problems o Explain, generalize, or connect ideas using supporting evidence o Make and justify conjectures o Explain thinking/reasoning when more than one solution or approach is possible o Explain phenomena in terms of concepts	o Relate mathematical or scientific concepts to other content areas, other domains, or other concepts o Develop generalizations of the results obtained and the strategies used (from investigation or readings) and apply them to new problem situations
Apply Carry out or use a procedure in a given situation; carry out (apply to a familiar task), or use (apply) to an unfamiliar task	o Follow simple procedures (recipe-type directions) o Calculate, measure, apply a rule (e.g., rounding) o Apply algorithm or formula (e.g., area, perimeter) o Solve linear equations o Make conversions among representations or numbers, or within and between customary and metric measures	o Select a procedure according to criteria and perform it o Solve routine problem applying multiple concepts or decision points o Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps o Translate between tables, graphs, words, and symbolic notations (e.g., graph data from a table) o Construct models given criteria	o Design investigation for a specific purpose or research question o Conduct a designed investigation o Use concepts to solve non-routine problems o Use & show reasoning, planning, and evidence o Translate between problem & symbolic notation when not a direct translation	o Select or devise approach among many alternatives to solve a problem o Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze Break into constituent parts, determine how parts relate, differentiate between relevant-irrelevant, distinguish, focus, select, organize, outline, find coherence, deconstruct	o Retrieve information from a table or graph to answer a question o Identify whether specific information is contained in graphic representations (e.g., table, graph, T-chart, diagram) o Identify a pattern/trend	o Categorize, classify materials, data, figures based on characteristics o Organize or order data o Compare/ contrast figures or data o Select appropriate graph and organize & display data o Interpret data from a simple graph o Extend a pattern	o Compare information within or across data sets or texts o Analyze and draw conclusions from data, citing evidence o Generalize a pattern o Interpret data from complex graph o Analyze similarities/differences between procedures or solutions	o Analyze multiple sources of evidence o Analyze complex/abstract themes o Gather, analyze, and evaluate information
Evaluate Make judgments based on criteria, check, detect inconsistencies or fallacies, judge, critique	"UG" – unsubstantiated generalizations = stating an opinion without providing any support for it!		o Cite evidence and develop a logical argument for concepts or solutions o Describe, compare, and contrast solution methods o Verify reasonableness of results	o Gather, analyze, & evaluate information to draw conclusions o Apply understanding in a novel way, provide argument or justification for the application
Create Reorganize elements into new patterns/structures, generate, hypothesize, design, plan, produce	o Brainstorm ideas, concepts, or perspectives related to a topic	o Generate conjectures or hypotheses based on observations or prior knowledge and experience	o Synthesize information within one data set, source, or text o Formulate an original problem given a situation o Develop a scientific/mathematical model for a complex situation	o Synthesize information across multiple sources or texts o Design a mathematical model to inform and solve a practical or abstract situation

What do we know about advanced math students?

- Need opportunities to make mistakes and less pressure to be perfect
- Need opportunities to see that hard work and effort, rather than innate mental ability, are factors that can lead to growth and success (growth mindset vs fixed mindset)
- Need to develop flexibility in mathematical thinking
- Need to develop persistence and self-regulation when faced with challenging tasks



For deep learning, with understanding, students need recurring and sustained opportunities for:

Productive struggle – with important mathematics

Explicit connections – among concepts, procedures, problems, situations

Deliberate practice – increasing variation and complexity over time

Jo Boaler, Stanford University, talks about “Number Sense”

www.youcubed.org

- *Struggle is good. But what do we need our students to struggle with?*

It's the core concepts that underlie the domain they are learning, and that will make their knowledge flexible and usable.

Building Algebraic Thinking…

Using the same number for each square, what number could replace each square to make a true statement? You must use the same number for each square.
How many solutions can you find?

$$\square \cdot \square = 7 \cdot \square$$

$$x^2 = 7x$$

Common error:

$$x^2 = 7x$$

$$x \cdot x = 7 \cdot x$$

$$\frac{x \cdot x}{x} = \frac{7 \cdot x}{x}$$

$$x = 7$$

MGSE9-12.A.SSE.3a: Factor any quadratic expression to reveal the zeros of the function defined by the expression.

Multiple Choice:

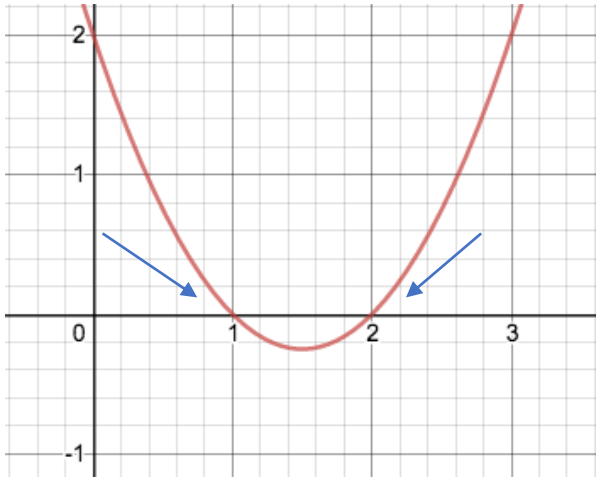
Solve $x^2 - 3x + 2 = 0$ $\square \cdot \square = 0$

(A) $x = 0, x = 1$

(B) $x = 1, x = 2$ 😊

(C) $x = -1, x = -2$

(D) $x = 2, x = 3$



Should these two solutions earn the same score?

Solve $x^2 + 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

$x - 5 = 0$ or $x + 2 = 0$

$x = 5$ or $x = -2$

Solve $x^2 + 3x - 10 = 0$

$x^2 - 3x = 10$

$x(x - 3) = 10$

$x = 10$ or $x - 3 = 10$

$x = 10$ or $x = 13$

Conceptual Understanding

Connect to prior knowledge ←→ Set the stage for future learning

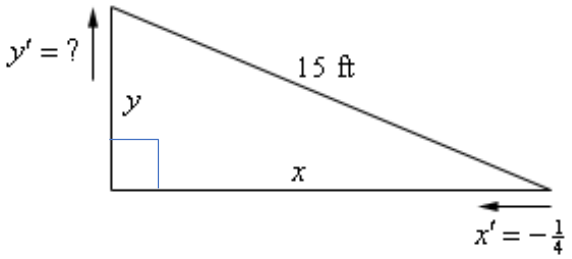
Right Triangles

- Elementary
  - Classify shapes, identify attributes, draw perpendicular lines/segments, recognize right triangles
- Middle School
  - Area of right triangles, Pythagorean Theorem, Congruence/Similarity
- Algebra/Geometry
  - Similarity, Congruence, Area, Right Triangle Trigonometry, Special Right Triangles, Distance Formula, Equation of a Circle
- Precalculus
  - Unit Circle, Graphs of trig functions, Pythagorean Identities, Vectors, Conics
- Calculus
  - Related Rates, Area of Equilateral Triangle, Arc length (based on distance formula), Rectangular to Polar, Vector-valued functions

Vertical Team – Concept Development

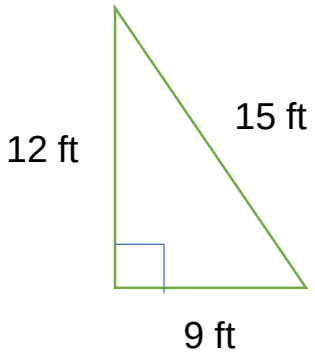
AP Calculus Problem – backwards alignment

A 15 foot ladder is resting against the wall. The bottom is initially 12 feet away from the wall and is being pushed towards the wall at a rate of 1/4 ft/sec. How fast is the top of the ladder moving up the wall 12 seconds after we start pushing? At what rate is the angle between the bottom of the ladder and the ground changing? At what rate is the area of the triangle changing?

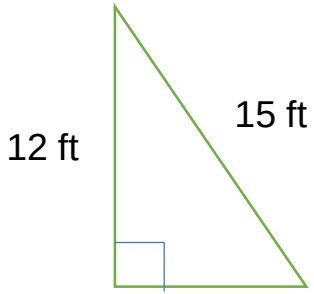


Elementary:

Identify this figure.
What attribute(s) help you identify the figure?
Using correct units, find the perimeter.
Using correct units, find the area.

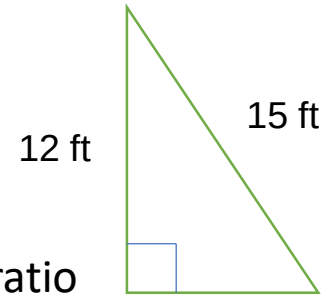


Middle



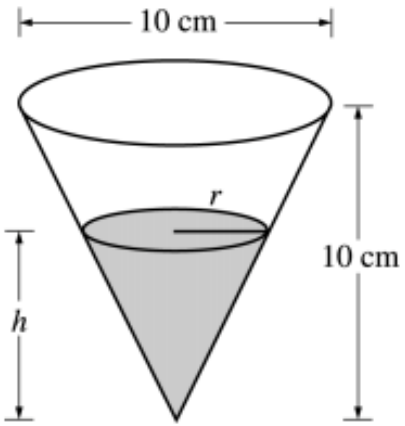
Using correct units, find the perimeter.
Using correct units, find the area.
Give the measures of a similar triangle.

Algebra/Geometry



Find the measure of the missing side.
Find the measure of the two acute angles.
Give the measures of two similar triangles – find the ratio of the three sides of each triangle. What do you notice?
If the length of the hypotenuse remains 15 feet, but the vertical leg is now 10 feet, what is the length of the horizontal leg?
If you rotate the triangle about the vertical leg, what 3-dimensional shape would be generated?

2002 AP® CALCULUS AB FREE-RESPONSE QUESTIONS



5. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

   (Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

   (a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

   (b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

   (c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

(https://secure-media.collegeboard.org/apc/calculus_ab_frq_02_10315.pdf)

Building Conceptual Understanding…

- Verbally
- Numerically
- Analytically
- Graphically

Common Errors in Precision

- Parentheses
- Negative signs
- Exponents
- Calculations with fractions
- Decimal answers - don’t round until the final calculation
- Connecting unequal quantities with equal signs
- Simplifying square roots and powers of sums/differences incorrectly
- Dividing by a variable
- Not using the Zero Product Property when solving a quadratic equation
- Written interpretation, with correct units, of a calculation in the context of the problem

Evidence of Academic Rigor

- Do assignments/tasks require multiple steps?
- Do questions/assignments require students to question, probe, apply problem solving skills, analyze, or evaluate meaning?
- Is there evidence of relevancy in activities and assignments?
- Is it clear the teacher is not satisfied with superficial answers and allowing answers without demonstration of understanding?
- Are students relying on the teacher's prompts when responding?
- Does the teacher expect students to use academic and domain-specific vocabulary?

Evidence of Academic Rigor

- Are assignments and/or questions designed to require higher level thinking skills (especially evaluation, creativity, or DOK 4)? Are the students creating rather than replying?

- Does the teacher allow time for a thoughtful response? Area questions higher level or lower level? (Research shows more than 50% of most teachers' questions are lower level and teachers typically wait less than 1 second for responses after asking a question.)

- Are students required to support assertions or claims with evidence such as data, text references to examples, or reasons/sources?

(Adapted from GaDOE document "*Administrator's Reference Tool for Evidence of Academic Rigor/Critical Thinking*")

"La mathematica el'alfabeto nel quale Dio ha scritto l'universo." ~ Galileo

Resources

- College Board AP Calculus MPACs: https://apcentral.collegeboard.org/courses/resources/mathematical-practices-ap-calculus-mpacs
- GaDOE Mathematics: https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/Mathematics.aspx
- HP Reveal Augmented Reality: https://studio.hpreveal.com/landing
- Jo Boaler, Stanford University: www.youcubed.org
- Karin Hess' Cognitive Rigor Matrices: https://www.karin-hess.com/free-resources
- Project M^3: Mentoring Mathematical Minds: https://k12.kendallhunt.com/content/20604/prog-feature

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