Engineering Calculus

MATHEMATICS KEY COMPETENCIES & COURSE STANDARDS

WITH LEARNING OBJECTIVES IN PROGRESSION ORDER

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

Engineering Calculus

Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Engineering Calculus Course, which is a fourth mathematics course option in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

Engineering Calculus is a fourth-year mathematics option for students who have completed AP Calculus BC. The course provides students with opportunities to develop an understanding of multivariable calculus as it applies to engineering systems, the history of engineering and its contributions to society. The course includes three-dimensional coordinate geometry; matrices and determinants; limits and continuity of functions with two independent variables; partial differentiation; multiple integration; the gradient; the divergence; the curl; Theorems of Green, Stokes, and Gauss; line integrals; integrals independent of path; and linear first-order differential equations.

Instruction and assessment should include the appropriate use of technology. Topics should be presented in multiple ways, such as verbal/written, numeric/data-based, algebraic, and graphical. Concepts should be introduced and used, where appropriate, in the context of realistic phenomena.

Prerequisite:

This course is designed for students who have successfully completed *AP Calculus BC*.

Georgia's K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL (QUANTITATIVE) REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)
ABSTRACT REASONING (AR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

Engineering Calculus

The six course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
EC.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.
EC.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
EC.AR.2: Using the engineering design process, apply mathematical concepts and procedures to solve problems in engineering contexts and research the impact of engineering and technological advancement on mathematics and society.
EC.AR.3: Using the engineering design process, express spatial and functional relationships with vectors, functions, and analytic geometry in three dimensions, and use these relationships to solve real-life, contextual, mathematical problems and to explain engineering-based phenomena.
EC.AR.4: Define, describe, and represent the differentiation of functions of two independent variables and differential vectors to solve contextual, mathematical problems and to explain engineering-based phenomena.
EC.AR.5: Interpret integrals of functions of two independent variables and of vector functions to solve contextual, mathematical problems and to explain engineering-based phenomena.

Engineering Calculus

MATHEMATICAL MODELING		
EC.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
EC.MM.1.1	Explain contextual, mathematical problems using a mathematical model.	***Fundamentals*** • Students should be provided with opportunities to learn mathematics in the context of real-life problems. • Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).
EC.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.	***Fundamentals*** • Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.
EC.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation.	
EC.MM.1.4	Use various mathematical representations and structures with this information to represent and solve real-life problems.	

ABSTRACT REASONING – *Impact of Engineering in Mathematics*		
EC.AR.2: Using the engineering design process, apply mathematical concepts and procedures to solve problems in engineering contexts and research the impact of engineering and technological advancement on mathematics and society.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Solve and explain engineering-based calculus problems; use mathematical and engineering models to explain real-life phenomena, using appropriate terminology and technology.		
EC.AR.2.1	Build new mathematical knowledge through problem solving that involves the engineering design process.	
EC.AR.2.2	Solve problems that arise in mathematics and in engineering contexts.	
EC.AR.2.3	Apply and adapt a variety of appropriate strategies to solve problems.	***Relevance and Application*** • Attend to constraints relevant to the design of a system, component, or process.
EC.AR.2.4	Use visual and written communication to organize, record, and articulate coherent, mathematical thinking and to express basic design elements.	

EC.AR.2.5	Monitor and reflect on the process of mathematical problem solving and interpret solutions that arise in engineering contexts.	
EC.AR.2.6	Produce multiple representations for mathematics presented in engineering contexts.	
EC.AR.2.7	Select, apply, and translate among mathematical representations to solve problems that arise in engineering contexts.	
EC.AR.2.8	Use mathematical representations to model and interpret physical and engineering phenomena.	
EC.AR.2.9	Demonstrate fundamentals of technical sketching using computer-generated visuals in appropriate mathematical scaling.	
EC.AR.2.10	Present a technical design, using computer-generated model, for an assigned design project utilizing the appropriate scientific units (US standards and SI units).	
EC.AR.2.11	Use connections among mathematics, technology, and engineering in contextual situations.	
Describe the impact of engineering and technological advancement on mathematics and society.		
EC.AR.2.12	Develop vocabulary and communication skills by reading materials associated with engineering and technology education.	***Examples*** • Journal articles, textbooks, codes and regulations, problem descriptions, project proposals. • Evaluate and present summaries of journal articles to incorporate the communication requirements needed of future engineers.
EC.AR.2.13	Describe the issues of necessity that have influenced innovation and technological development.	
EC.AR.2.14	Explain the impact of key persons and historical events and their impact on engineering and society.	
EC.AR.2.15	Investigate the educational requirements and professional expectations associated with engineering career paths.	

ABSTRACT REASONING *– Algebra in Engineering*		
EC.AR.3: Using the engineering design process, express spatial and functional relationships with vectors, functions, and analytic geometry in three dimensions, and use these relationships to solve real-life, mathematical problems and to explain engineering-based phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Express the relationships between points, lines, and planes in three dimensions.		
EC.AR.3.1	Determine the equations of lines and surfaces using vectors and 3D graphing.	
EC.AR.3.2	Apply dot and cross products of vectors to express equations of planes, parallelism, perpendicularity, angles.	
EC.AR.3.3	Describe the role of vectors in engineering applications, such as modeling the velocity of moving objects or static forces on structures and objects.	
EC.AR.3.4	Evaluate matrices and apply their properties to solve problems expressed as matrix equations.	***Fundamentals*** • Calculate determinants, express systems of linear equations using matrices, solve systems of linear equations using Gaussian elimination.
Investigate functions of two and three independent variables to model engineering systems.		
EC.AR.3.5	Compute limits of scalar and vector-valued functions.	***Example*** • Determine whether two vectors are orthogonal using the dot product; find the projection of a vector along another; and find the angle between two vectors.
EC.AR.3.6	Identify and graph level curves of multivariate functions.	
EC.AR.3.7	Find the regions of continuity of multivariate functions.	

ABSTRACT REASONING *– Partial Differentiation in Engineering*		
EC.AR.4: Define, describe, and represent the differentiation of functions of two independent variables and differential vectors to solve contextual, mathematical problems and to explain engineering-based phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Apply partial differentiation of functions of two or more independent variables.		
EC.AR.4.1	Compute the first and second partial derivatives of a function.	
EC.AR.4.2	Use the general chain rule to determine the partial derivatives of composite functions.	
EC.AR.4.3	Compute and apply the gradient of multivariable functions.	
EC.AR.4.4	Solve engineering optimization problems by applying partial differentiation or Lagrange multipliers.	

EC.AR.4.5	Utilize partial derivatives in developing the appropriate system balances in engineering problems.	***Terminology*** • System balances refers to stability and equilibrium.

ABSTRACT REASONING *– Integration in Engineering*		
EC.AR.5: Interpret integrals of functions of two independent variables and of vector functions to solve contextual, mathematical problems and to explain engineering-based phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
EC.AR.5.1	Manipulate integrals by changing the order of integration, introducing variable substitutions, or changing to curvilinear coordinates.	
EC.AR.5.2	Evaluate and apply line integrals that are independent of path.	
EC.AR.5.3	Apply properties of integrals to calculate and represent area, volume, or mass.	
EC.AR.5.4	Use integrals of vectors to define and apply the gradient, divergence, and the curl.	
EC.AR.5.5	Interpret the theorems of Green, Stokes, and Gauss and apply them to the study of real-world phenomena.	

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

Mathematical Practices	
EC.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
EC.MP.1	Make sense of problems and persevere in solving them.
EC.MP.2	Reason abstractly and quantitatively.
EC.MP.3	Construct viable arguments and critique the reasoning of others.
EC.MP.4	Model with mathematics.
EC.MP.5	Use appropriate tools strategically.
EC.MP.6	Attend to precision.
EC.MP.7	Look for and make use of structure.
EC.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

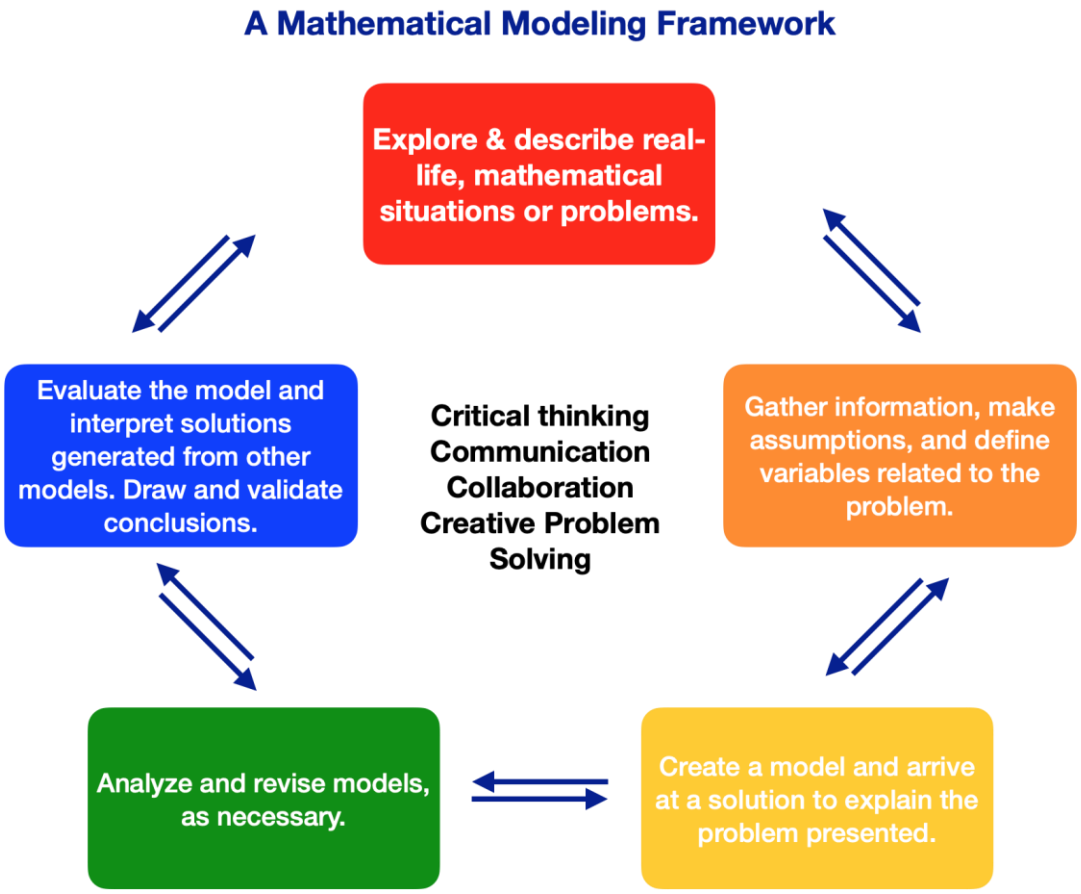

Image adapted from: Suh, Matson, Seshaiyer, 2017

FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

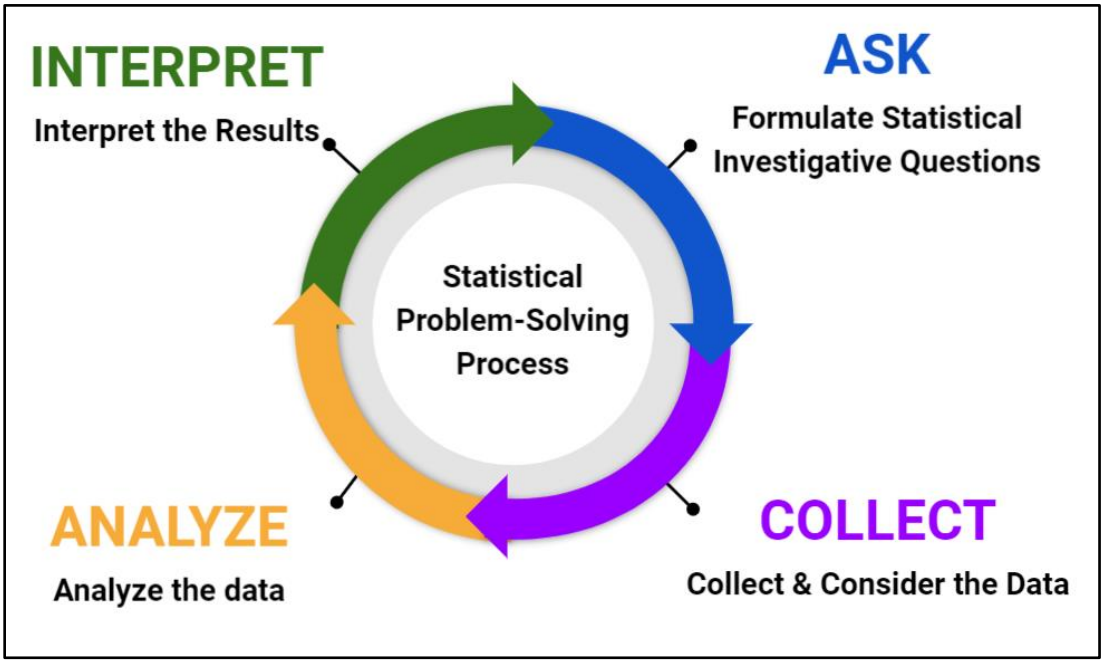

Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. Formulate Statistical Investigative Questions
Ask questions that anticipate variability.

II. Collect & Consider the Data
Ensure that data collection designs acknowledge variability.

III. Analyze the Data
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. Interpret the Results
Answer statistical investigative questions based on the collected data.