# Making Mathematics Meaningful for Students with Learning Problems: Powerful Teaching Strategies that Work 

David Allsopp, Ph. D.
Department of Special Education University of South Florida dallsopp@tempest.coedu.usf.edu

## Agenda

- Introduction/Objectives
- Topic \#1: Importance of Meaning and Teacher SelfReflection
- Topic \#2: Why Do Students with Learning Disabilities Have Difficulty Learning Mathematics
- Topic \# 3: Ten Powerful Teaching Techniques
- Topic \#4: Long-Term Professional Development: Introduction to MatheVIDS
- Topic \# 5: Questions/Discussion


## Learning Objectives

8 Self-relect on beliefs and preparedness to teach mathematics to students with learning disabilities.

* Prioritize several important areas for long term professional development.
- Identify learning characteristics that make learning mathematics difficult for students with learning disabilities.
* Identify powerful/effective instructional strategies \& determine how you can use at least one to improve mathematics learning for your students.
* Learn about one resource, MathVIDS, and how to use it for my personal professional development goals.


## Topic \#1: Importance of Meaning \& Teacher Self-Reflection

- Meaning...
- Why is meaning important?
- Some examples



## Conceptual Meaning

$$
\begin{gathered}
2 \times 4=? \\
1 / 2 \times 1 / 4=?
\end{gathered}
$$

## Can you explain your answer for each problem?

## How can we enhance the meaning?

## Use Language

$2 \mathrm{x} \quad 4 \quad=\quad$ ?
two groups of four Nittany Lions is how many Lions

$$
\begin{array}{cccc}
1 / 2 & \mathrm{x} & 1 / 4 & = \\
\text { one-half } & \text { of } & \text { a one-fourth piece of pizza } & \text { is how much pizza }
\end{array}
$$

## Provide Concrete Experience

2 X 4 ?
two groups of four Nittany Lions is how many Lions


## Provide Concrete Experiences $1 / 2 \times 1 / 4 \quad=$ ?

 one-half of a one-fourth piece of pizza is how much pizza
## Contextual Meaning

- What is...
- $4+2+4+4+3+3+5+4+4$ ?


## Contextual Meaning

## Meaning and Disability

Write a title for a short story you might write for the following picture...


## Write Your Title

One thing I would like to remember from the learning activities for this topic is...

## Topic \#2: Why is it Difficult for Students with Learning Disabilities to Learn Mathematics?

-Brief reflection on what learning problems feel like

- 7 Learning Characteristics That Are Barriers for Learning Mathematics


# Let's Take A Quiz 

Informal Mathematics Teacher Competency Inventory

## Lets Take a Quiz!!

Directions: Solve the following basic facts. You have 1 minute to complete this quiz. Please remember that the + symbol means multiply, the - symbol means divide, the $\div$ symbol means add, and the $x$ symbol means subtract.

| $8+2=$ | $10-5=$ | $8 \times 7=$ |
| :--- | :--- | :--- |
| $14 \div 7=$ | $17 \times 2=$ | $2 \times 1=$ |
| $12 \times 2=$ | $8 \div 4=$ | $14-7=$ |
| $10-2=$ | $4 \times 3=$ | $6 \times 2=$ |
| $6 \times 5=$ | $15-3=$ | $8+5=$ |
| $9 \div 9=$ | $9 \div 2=$ | $6+1=$ |
| $9+6=$ | $9-3=$ | $8-4=$ |
| $12 \div 2=$ | $4+4=$ | $20-1-=$ |
| $5+6$ | $6 \div 6=$ | $8+2=$ |
| $16-4=$ | $8+3$ | $6 \div 2=$ |

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| $16-4=$ | $8+3$ | $6 \div 9-$ |

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| :---: | :---: | :---: |
| $14 \div 7=$ | $17 \times 2=$ | $2 \times 1=$ |
| $12 \times 2=$ | $8 \div 4=$ | Whork |
| 10-2 = | $4 \times 3=$ | $6 \times 2=$ |
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## 7 Learning Characteristics That Create Barriers for Learning Mathematics

Learned helplessness
Passive approach to learning
Metacognitive thinking deficits
Attention problems
Anxiety about learning mathematics
Cognitive Processing deficits (auditory, visual/spatial, fine motor)

Memory problems (nickname test)
Low level of academic achievement (gaps) (piglatin word problem)

## Let's Reflect On Our Experiences with the Math Quiz

Learned helplessness
Passive approach to learning
Metacognitive thinking deficits
Attention problems
Anxiety about learning mathematics

What is it?
How does it impact learning mathematics?

## Memory Problems

Storage
Retrieval

## Do you know the nicknames?

Directions: Write these university names on a sheet of paper aPenn State
-Pittsburgh
-Bucknell
aPenn
Ohio State
-Michigan
Mississippi State
Mississippi
F Forida
Florida State
Furman
aldaho
Northern Arizona
Ferrum
James Madison

## The National College Nickname Memory Test

Using your list of school names, write the nickname for each school name given orally.

## The National College Nickname Memory Test - Answers

${ }_{\square}$ a Penn State<br>a Pittsburgh<br>-Bucknell<br>$\square$<br>Nittany Lions<br>Panthers<br>-Ohio State<br>$\square$<br>Wolverines<br>Bulldogs<br>$\square$<br>©Florida<br>■<br>oFurman<br>uldaho<br>Northern Arizona<br>$\square$ Ferrum<br>«James Madison<br>Seminoles<br>Paladins<br>Vandals<br>Panthers<br>Dukes

## Reflection: Your Experiences

-How many nicknames were you able to recall?
-What either helped or inhibited your ability to recall the nicknames?
-What role does memory play in doing mathematics?
-Multi-step operations
-Multiple meanings for a math symbol or term
-Recalling formulas
-Employing problem solving strategies

## One thing I would like to

 remember from the learning activities in Topic \#2 is... \&
## Topic \#3 Ten Powerful Teaching Techniques

-What are they?
-Key instructional features

- Examples
-How do they help students with learning disabilities?


## 10 Powerful Teaching Techniques

1) Support Student Connections Between New Mathematics Concepts \& Prior Knowledge/Experiences
2) Provide
of Target Mathematics Concepts, Skills, \&
Processes
3) Imbed Instruction and Practice in
4) Teach Problem Solving
5) Cue Important Features of a Mathematics Concept/Skill
6) Use
to Enhance Meaning \& Understanding 7)Ground Abstract Concepts \& Processes in
7) Provide Students
8) Monitor Student Learning \& Provide Them Concrete Ways to Visualize Their Learning
10)Provide Continuous Maintenance Activities for Previously Mastered Concepts/Skills

Support Student Connections Between New Mathematics Concepts \& Prior Knowledge/Experiences

[^0]
## Meet Your Students Where They Are...


ink to prior knowledge/previous experiences.
dentify what students will learn.
rovide meaning/rationale.


## Provide Explicit Models of Target Mathematics Concepts, Skills, \& Processes



Student
Mathematics Concept

## Make the mathematics concept accessible to your students by...



# Imbed Instruction and Practice in Authentic Contexts 

Age/Grade<br>Interests/Hobbies<br>Family/Culture

Student Interest Inventory

## Student Interest Matrix (See Special Connections Website)

$\left.\begin{array}{l}\begin{array}{l}\text { Student Name: } \\ \text { Age/Grade Level: } \\ \text { Period/Class: }\end{array} \\ \hline \begin{array}{c}\text { Things I Like } \\ \text { To Do On My } \\ \text { Own }\end{array} \\ \hline\end{array} \begin{array}{c}\text { Special } \\ \text { Hobbies I } \\ \text { Have }\end{array} \quad \begin{array}{c}\text { Fun Things } \\ \text { My Family } \\ \text { Does }\end{array} \quad \begin{array}{c}\text { Things I Like } \\ \text { To Do With } \\ \text { My Friends }\end{array} \quad \begin{array}{c}\text { Things I Like } \\ \text { To Learn } \\ \text { About }\end{array}\right]$

## Mathematics Class Interest Matrix

 (see Special Connections Website)| Period/Class: <br> School Year: |  |  |
| :--- | :--- | :--- |
| Interests | Relevant Mathematics <br> Concepts/Skills I Teach <br> That Match Interest | Ideas for Creating <br> Authentic Contexts |
| Individual/Peer <br> Activities |  |  |
| 1. |  |  |
| 2. |  |  |
| Family Activities |  |  |
| 1. |  |  |
| 2. |  |  |

# PTT \# 4: <br> <br> Teach Problem Solving Strategies 

 <br> <br> Teach Problem Solving Strategies}

Students with learning disabilities do not naturally employ problem solving strategies like successful mathematics students do...

## What Are They?

## An efficient \& learnable process for:

1) solving a particular type of problem
2) developing conceptual understanding of important mathematics concepts

## What do they include?

- limited number of steps ( 3 to 7 steps) saccurately reflect the problem/concept sprovide cueing
-actions \& thinking
॰they are taught

Strategy Examples

## Mnemonic Strategies

## Examples:

DRAW-operations
FASTDRAW- story problems
SPIES - greater than/less than
ADD - adding + \& - integers
DRAW for Algebra - one-variable equations
FASTDRAW for Algebra - algebra story problems
(See MathVIDS for these and others)

## DRAW for Algebra

## D iscover the variable

Read the problem
A nswer the problem or draw \& check
$\underline{W}$ rite the answer

## SPIES

S ay the integer out loud.
P oint to each integer and circle negative signs.
Identify whether integer is pos. or neg.
E valuate the magnitude of each integer.
S_elect integer of greatest value.

Rules of Value

1) $+\&-=$ positive integer is of greater value
2) $+\&+=$ integer farthest from zero on \#line is of greatest value
3)     - \& - = integer closest to zero on \# line is of greatest value

## Example: Chart to Help Students Generate ProblemSolving Strategies

| Ways We Know to Add |  |  |  |
| :---: | :---: | :---: | :---: |
| Counting Up | Make a Ten | Near Doubles | Other |
| Start at 8 . <br> Count 9, 10, 11, 12, <br> 13. | $\begin{array}{r} 8 \rightarrow 10 \\ +5 \rightarrow 3 \\ \cline { 1 - 3 } 13 \end{array}$ | $\begin{array}{r} 55 \\ +6 \\ +\frac{5}{10}+1=11 \end{array}$ | $\begin{array}{cc} \hline 8 & 8 \\ +9 & +10 \\ \hline & 18-1=17 \end{array}$ |
|  |  |  |  |

## Example: Helping Students Think About/Monitor Their Use of Different Strategies

## STRATEGY SHEET

## Paste problem here.

I solved the problem by:


## PPT \# 5:

Cue Important Features of a Mathematics Concept/Skill Using Multisensory Methods

Visual<br>Auditory<br>Tactile (Touch)

Kinesthetic (Movement)
Cognitive/Thinking

## Examples of Visual Cuing



## Example: Cue Sheet to Enhance A Student's Independent Practice

Goal: To solve a math problem.
Checklist
You are to:

1. Write the problem at the top of the page.
2.     * Use a strategy to solve the problem.

* Use pictures or words to explain your strategy.

3. Write your answer in the blank.
4. Write the items or ideas you used to solve the problem.

## PPT \# 6: <br> Use Language Experiences to Enhance Meaning \& Understanding

Incorporate the following language expression modalities:
Speech
Writing
Drawing
Acting Out/Drama
Song
Videotape, phoography/technology

How?

# Examples - Describe what numbers \& symbols mean: 

Describe what numbers \& symbols mean:
$1 / 2 \times 1 / 4=1 / 8$
one-half of one-fourth is one-eighth


# Examples: Create Stories/Examples That Communicate Understanding 

 Example: Intersect of x \& y coordinates on a plane.Speech/Words - The place on the football field where the Bucs kickoff.

Drawing: $(x, y)$ where $=15 \& y=0 ;=-15 \& y=0$

|  | 1 | 0 | 2 | 0 | 3 | 0 | 4 | 0 | 5 | 0 | 4 | 0 | 3 | 0 | 2 | 0 | 1 | 0 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

# PTT \# 7 <br> Ground Abstract Concepts \& <br> Processes in Concrete Experiences 

## Manipulatives

Teach Drawing Strategies

## Manipulative Examples (Discrete):

- Attributes more "accessible"
- Can be "manipulated" more easily



## Teach Drawing Strategies

- Start with concrete experiences
- Move to drawing representations of concrete experiences


Drawing Example：

$$
3 \mathrm{x}+4=16
$$



IIII
｜｜｜｜｜｜｜｜｜｜｜｜｜｜｜｜｜


それ才
｜｜｜｜｜｜｜｜｜｜｜｜㲦
（1111）（1111）$\rightarrow x=4$

## Other Drawing Examples (see MathVIDS website)



## Enhancing Abstract Understanding

- Reasons for Difficulty

Lack of conceptual understanding Memory problems
Organization/writing/visual processing problems
-Provide Many Practice Opportunities/Link to concrete \& drawing experiences! $\leftrightarrows$ MASTERY

## Provide Students Many Opportunities to Respond

The more opportunities students with learning disabilities have to respond to a learning task, the more likely it is they will master that learning task...
initial acquisition $\longrightarrow$ advanced acquisition (Teacher Directed Instruction)
proficiency maintenance (Student Practice)
generalization $\longmapsto$ adaption (Extension)

## How?

## Ensure that...

-students have a motivational context ๑the practice activity focuses on the target math concept/skill -students have multiple opportunities to respond -teachers have a way to evaluate student responses

## Examples

Instructional Games
Self-correcting Materials
Structured Peer-Mediated Learning Groups
Structured Language Experiences Planned Discovery Experiences

## Math Instructional Games/SelfCorrecting Materials - A Few Ideas

| Instructional Games |  |
| :---: | :--- |
| Board Games |  |
| Checker Board/Checkers |  |
| Spinners |  |
| Dice |  |
| Cards |  |

(See MathVIDS for more ideas)

PPT \# 9:
Monitor Student Learning \& Provide Them Concrete Ways to Visualize
Their Learning
-At least 2-3 times weekly

- Incorporate at concrete, drawing \& abstract levels
-Use short, easy to evaluate "probes"
-Pinpoint key concepts for monitoring
- Teach students to chart their learning
-Use as a way to engage students in setting learning goals
-CELEBRATE SUCCESS!!


## How?

| Level of <br> Understanding | Method | Criterion |
| :---: | :---: | :---: |
| Abstract | $1-2$ minute timings | Fluency <br> (Rate \& Accuracy |
| Drawing | $8-10$ tasks | Accuracy <br> $90-100 \% ~ 3 ~ t i m e s ~$ |
| Concrete | 3 tasks | Accuracy <br> $100 \% ~ 3 ~ t i m e s ~$ |

## Examples

## Abstract Level

Probe/Curriculum Slice


## Visual Display

"corrects"
"incorrects"


Name: Carlos
Concept/Skill: Two-Digit Addition Strategies

| \# of Different Strategies Used | 5 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4 |  |  |  |  | $\bigcirc$ |
|  | 3 |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| - Successful <br> Unsuccessful | 2 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
|  | 1 | $\bigcirc$ | - | - | - | - |
|  |  | Week 1 | Week 2 | Week 3 | Week 4 | Week 5 |
| Strategy/ies used: | - | Add Tens | Add Tens | Add Tens <br> Make a Ten | Add Tens <br> Make a <br> Ten | Add Tens <br> Make a Ten <br> Use easier number |
|  | - | Make a Ten | Make a <br> Ten | Use easier number | Use easier number | Compensation |

## PTT \# 10:

## Provide Continuous Maintenance Activities for Previously Mastered Concepts/Skills

-Purposefully plan maintenance opportunities
-Emphasize foundational concepts for the mathematics curriculum you teach
-Make sure target maintenance concepts are ones students have previously mastered

- Vary the type of activities
-Avoid "drill \& practice"
slnclude students in developing ideas for maintenance activities
-Emphasize connections between abstract \& concrete -Engage students in "talking/writing/drawing" about target maintenance concepts


## One thing I would like to

 remember from the learning activities in Topic \#3 is ... 如
## Topic \#4 Long-term Professional Development Resources <br>  <br> http://coe.jmu.edu/mathvidsr



http://coe.jmu.edu/learnngtoolbox
http://www.specialconnections.ku.edu/cgi-bin/cgiwrap/specconn/index.php

Topic\#5: Questions/Discussion


[^0]:    Where our students are

