GEORGIA'S K-12
MATHEMATICS STANDARDS
2021

Advanced Finite Mathematics

MATHEMATICS
KEY COMPETENCIES &
COURSE STANDARDS

WITH
LEARNING OBJECTIVES
IN PROGRESSION ORDER

GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

Advanced Finite Mathematics

Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Advanced Finite Mathematics Course, which is a fourth mathematics course option in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

Advanced Finite Mathematics is designed to meet the needs of advanced students who have completed GSE Pre-Calculus or Accelerated GSE Pre-Calculus or the equivalent and will pursue careers which require the mastery of discrete mathematics topics often associated with modern computer science.

The course will examine mathematics in four areas through the lens of both pure mathematics and applied mathematics: set theory, number theory, probability/combinatorics, and graph theory. There will be a strong focus on the presentation of mathematical ideas through both written and oral communication, particularly through logic and proofs. Mathematical proofs will be presented through an abstract approach that characterizes upper-level mathematics courses. The goal is to give students the skills and techniques they will need as they study advanced mathematics or computer science at the college level. This is an alternative course for those students who do not wish to enroll in an Advanced Placement course, but who still wish to learn higher-level mathematics.

Prerequisite:

This course is designed for students who have successfully completed *Advanced Algebra / Algebra II*.

Georgia's K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL (QUANTITATIVE) REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)
LOGICAL REASONING (LR)
ABSTRACT & QUANTITATIVE REASONING (AQR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

Advanced Finite Mathematics

The eight course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
AFM.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.
AFM.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
AFM.LR.2: Apply methods of proof to prove or disprove mathematical statements; explain reasoning and justify thinking through mathematical induction when formulating mathematical arguments.
AFM.LR.3: Interpret, represent, and communicate logical arguments to explain reasoning and justify thinking when solving problems and to explain real-life phenomena.
AFM.NR.4: Apply number theory and number-theoretic operations to solve contextual, mathematical problems and to explain real-life phenomena.
AFM.AQR.5: Use set theory to describe relationships and equivalence when solving contextual, mathematical problems used to explain real-life phenomena.
AFM.AQR.6: Calculate and solve combinatorics problems to make sense of a real-life, contextual problem.
AFM.AQR.7: Apply graph theory to solve contextual, mathematical problems and to explain real-life phenomena.

Advanced Finite Mathematics

MATHEMATICAL MODELING		
AFM.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AFM.MM.1.1	Explain contextual, mathematical problems using a mathematical model.	***Fundamentals*** • Students should be provided with opportunities to learn mathematics in the context of real-life problems. • Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).
AFM.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.	***Fundamentals*** • Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.
AFM.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation.	
AFM.MM.1.4	Use various mathematical representations and structures with this information to represent and solve real-life problems.	

***LOGICAL REASONING** – Methods of Proof*		
AFM.LR.2: Apply methods of proof to prove or disprove mathematical statements; explain reasoning and justify thinking through mathematical induction when formulating mathematical arguments.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AFM.LR.2.1	Use a counterexample to disprove a statement.	***Example*** • The statement “there exists a positive integer n such that $n^2 + 3n + 2$ is prime” is disproved by the counterexample $n = 3$.
AFM.LR.2.2	Prove statements directly from definitions and previously proved statements.	***Examples*** • Using the definition of “rational number”, prove that every integer is rational; using the definition of “divides”, prove that if *a* divides *b* and *b* divides *a*, then *a* equals *b*.
AFM.LR.2.3	Prove statements indirectly by proving the contrapositive of the statement.	***Example*** • Prove that if the square of an integer is even, then the integer is even, by instead proving that if an integer is odd, then its square is odd.

AFM.LR.2.4	Apply the method of reductio ad absurdum (proof by contradiction) to prove statements.	***Example*** • Prove that there are infinitely many prime numbers.
AFM.LR.2.5	Use the method of mathematical induction to prove statements involving the positive integers.	***Examples*** • Prove that 3 divides $2^{2n} - 1$ for all positive integers *n.* • Prove that $1 + 2 + 3 + \ldots + n = n(n + 1)/2$ for all positive integers *n*.

LOGICAL REASONING – *Logical Symbolism and Binary*		
AFM.LR.3: Interpret, represent, and communicate logical arguments to explain reasoning and justify thinking when solving problems and to explain real-life phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Represent and interpret statements using logical symbolism.		
AFM.LR.3.1	Construct truth tables that represent conditional, biconditional, and quantified statements; use truth tables to determine whether the statement is true or false and use Venn diagrams to illustrate the relationship represented by these truth tables.	
AFM.LR.3.2	Represent logic operations such as AND, OR, NOT, NOR, and XOR (exclusive OR) using logical symbolism, determine whether statements involving these operations are true or false, and interpret such symbols into English.	
AFM.LR.3.3	Apply modus ponens and modus tollens to analyze logical arguments to determine whether it is valid, invalid, a tautology, or a contradiction.	***Fundamentals*** • Students should be given opportunities to determine whether symbolic and verbal arguments are valid.
AFM.LR.3.4	Write the negation, converse, contrapositive, and inverse of a conditional statement and find the truth of each.	***Example*** • Write the negation, converse, and contrapositive of “If I want to keep this job, then I will be on time each day.”
Use binary to represent and interpret logical statements.		
AFM.LR.3.5	Represent the dichotomy between “true” and “false” with 1s and 0s. Use 1s and 0s to calculate whether a statement is true or false by constructing Boolean logic circuits.	***Relevance and Application*** • The Boolean logic circuit is a visual interpretation of the internal logical operation of a computer.
AFM.LR.3.6	Convert binary and hexadecimal numbers into decimal, and convert from binary to hexadecimal, and vice versa. Add binary integers and use 2’s complement to subtract binary integers.	***Examples*** • Add the binary numbers 1100 and 1010. Be able to give the result in binary. • Convert the hexadecimal number A7E9 to decimal and binary. • Use 2’s complement to find, in binary, the difference 11101110 – 10001010.

NUMERICAL REASONING – Number Theory		
AFM.NR.4: Apply number theory and number-theoretic operations to solve contextual, mathematical problems and to explain real-life phenomena.		
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)
Use number-theoretic operations.		
AFM.NR.4.1	Apply the divides relation to positive integers and calculate one integer modulo another integer.	***Examples*** • 3 | 12 implies 3n = 12 for some integer n; calculate a mod n for some integers a and n.
AFM.NR.4.2	Find the inverse of an integer for a certain modulus.	***Examples*** • Since 7·3 is congruent to 1 modulo 10, then 7 and 3 are inverses modulo 10. • Find the inverse of 43 modulo 60.
AFM.NR.4.3	Calculate the floor and the ceiling of a real number.	***Example*** • Find $\lfloor \pi \rfloor$ and $\lceil \pi \rceil$.
Prove statements in number theory.		
AFM.NR.4.4	Prove statements involving properties of numbers.	***Examples*** • Prove that the sum of two rational numbers is rational; prove that if a is even and b is odd, then $(a^2 + b^2 + 1)/2$ is an integer. • Prove that any odd number squared is of the form 8k + 1 for some integer k. • Prove that the square root of 2 is irrational.
AFM.NR.4.5	Prove statements involving the floor and ceiling functions.	***Example*** • For every integer n, prove that the floor of $n/2$ is equal to $n/2$ if n is even and equal to $(n – 1)/2$ if n is odd.
AFM.NR.4.6	Prove the Fundamental Theorem of Arithmetic, the Euclidean algorithm, and Fermat's Little Theorem.	***Relevance and Applications*** • Apply the RSA algorithm to encrypt a numerically coded message and to decode an encoded message.

ABSTRACT & QUANTITATIVE REASONING – Set Theory		
AFM.AQR.5: Use set theory to describe relationships and equivalence when solving contextual, mathematical problems used to explain real-life phenomena.		
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)
Use set theoretic operations.		
AFM.AQR.5.1	Find the union, intersection, difference, complement, and Cartesian product of sets, and classify sets as equal, subsets, and power sets.	

AFM.AQR.5.2	Justify whether the union of subsets of a set is a partition of that set.	
AFM.AQR.5.3	Given a relation on two sets, determine whether the relation is a function and find its inverse relation, if it exists.	
AFM.AQR.5.4	Determine the equivalence classes given an equivalence relation on a set; determine whether the union of equivalence classes of a set is a partition of that set.	***Examples*** • Find the equivalence classes over the integers of the "modulo 3" relation.
AFM.AQR.5.5	Prove set relations, including DeMorgan's Laws and equivalence relations.	***Examples*** • Given two sets *A* and *B*, prove that the intersection of *A* and *B* is a subset of *A* and a subset of *B.* • Prove that the empty set is unique and is a subset of every set.
Use and interpret Boolean algebra.		
AFM.AQR.5.6	Prove statements in Boolean algebra.	
AFM.AQR.5.7	Simplify Boolean algebra expressions using Karnaugh maps (K-maps).	

***ABSTRACT & QUANTITATIVE REASONING** – Probabilities and Combinatorics*		
AFM.AQR.6: Calculate and solve combinatorics problems to make sense of a real-life, contextual problem.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Calculate the probability of events.		
AFM.AQR.6.1	Use the addition rule to count the number of outcomes in a disjoint set of sample spaces. Use the principle of inclusion-exclusion to count the number of outcomes in the union of sample spaces.	
AFM.AQR.6.2	Apply the axioms of probability to determine the probability of dependent and independent events, including use of the multiplication rule for independent events.	
AFM.AQR.6.3	Find expected value.	
AFM.AQR.6.4	Apply Bayes' Theorem to determine conditional probability.	
Use methods of counting to solve problems involving permutations and combinations.		
AFM.AQR.6.5	Calculate the number of permutations of a set with *n* elements. Calculate the number of permutations of *r* elements taken from a set of *n* elements.	
AFM.AQR.6.6	Calculate the number of subsets of size *r* that can be chosen from a set of *n* elements.	
AFM.AQR.6.7	Calculate the number of combinations with repetitions of *r* elements from a set of *n* elements	

Prove statements involving combinatorics.		
AFM.AQR.6.8	Prove combinatorial identities.	***Examples*** • Prove that *n* + 1 choose r is equal to *n* choose *r* – 1 plus *n* choose *r.* • Prove that the sum of the entries in the *k*th row of Pascal's triangle is 2^k (where the first row is row 0).
AFM.AQR.6.9	Apply a combinatorial argument to prove the binomial theorem.	
AFM.AQR.6.10	Use the pigeonhole principle to prove statements about counting.	

***ABSTRACT & QUANTITATIVE REASONING** – Graph Theory*		
AFM.AQR.7: Apply graph theory to solve contextual, mathematical problems and to explain real-life phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Use, apply, and prove graph properties.		
AFM.AQR.7.1	Identify simple graphs, complete graphs, complete bipartite graphs, and trees. Identify graphs that have Euler and Hamiltonian cycles.	
AFM.AQR.7.2	Construct the complement and the line graph of a graph.	
AFM.AQR.7.3	Use the adjacency matrix of a graph to determine the number of walks of length *n* in a graph.	
AFM.AQR.7.4	Prove statements about graph properties.	***Examples*** • Prove that a graph has an even number of vertices of odd degree. • Prove that a graph has an Euler cycle if and only if the graph is connected and every vertex has even degree. • Prove that any tree with *n* vertices has *n* – 1 edges.
Apply graph theory in context.		
AFM.AQR.7.5	Prove that every connected graph has a minimal spanning tree.	
AFM.AQR.7.6	Use Kruskal's algorithm and Prim's algorithm to determine the minimal spanning tree of a weighted graph.	***Relevance and Application*** • Students will find the shortest path between to vertices of a graph that represents a delivery of goods between cities • Students will find the minimal spanning tree of a graph that represents the cost of travel between cities.

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

Mathematical Practices	
AFM.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
AFM.MP.1	Make sense of problems and persevere in solving them.
AFM.MP.2	Reason abstractly and quantitatively.
AFM.MP.3	Construct viable arguments and critique the reasoning of others.
AFM.MP.4	Model with mathematics.
AFM.MP.5	Use appropriate tools strategically.
AFM.MP.6	Attend to precision.
AFM.MP.7	Look for and make use of structure.
AFM.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

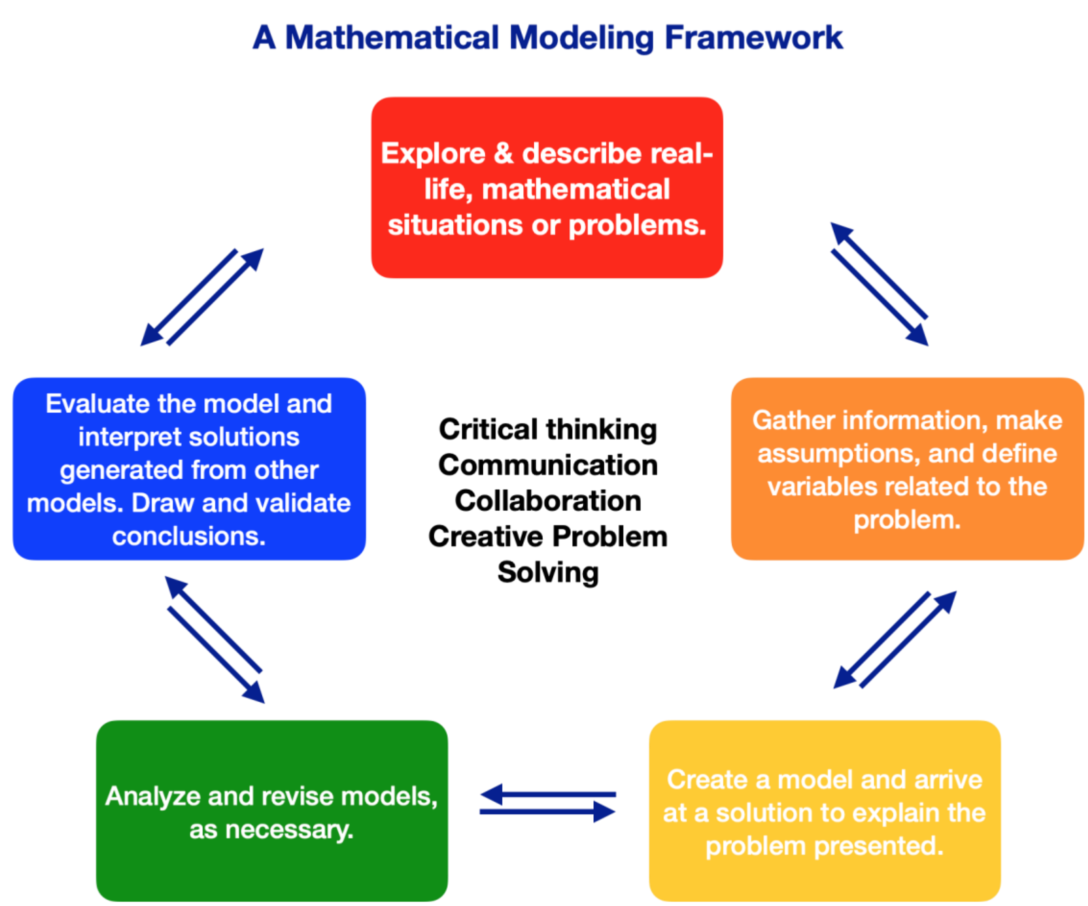

Image adapted from: Suh, Matson, Seshaiyer, 2017

FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

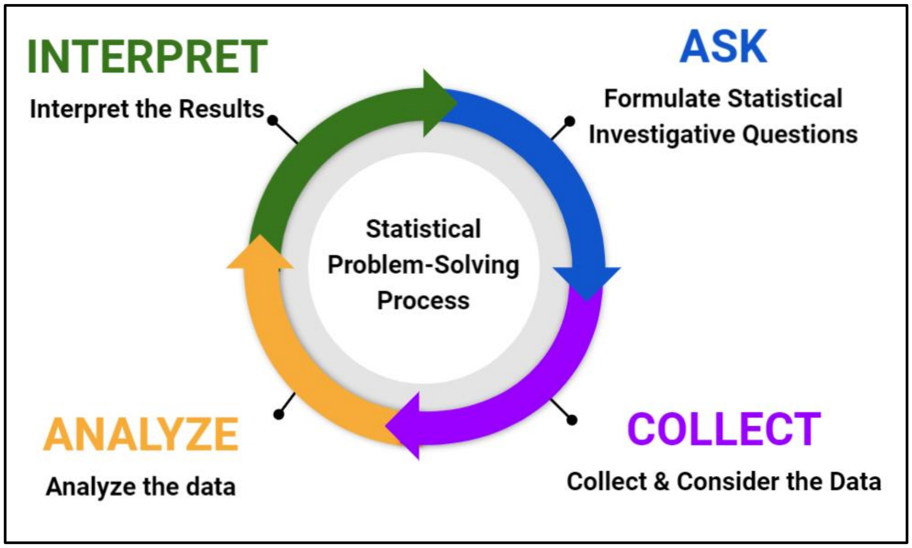

Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. Formulate Statistical Investigative Questions
Ask questions that anticipate variability.

II. Collect & Consider the Data
Ensure that data collection designs acknowledge variability.

III. Analyze the Data
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. Interpret the Results
Answer statistical investigative questions based on the collected data.