GEORGIA'S K-12
MATHEMATICS STANDARDS
2021

Precalculus

MATHEMATICS
KEY COMPETENCIES &
COURSE STANDARDS
WITH
LEARNING OBJECTIVES
IN PROGRESSION ORDER

GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

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Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Precalculus Course, which is a fourth mathematics course option in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

Precalculus is a fourth-year math option for students who have completed Advanced Algebra (or the equivalent). The course provides students with the opportunity to develop a deeper understanding of concepts in Algebra that are critical to the study of Calculus as well as an understanding of trigonometry and its applications. Throughout the course there should be a focus on notational fluency and the use of multiple representations. The course includes the study and analysis of piecewise and rational functions; limits and continuity as related to piecewise and rational functions; sequences and series with the incorporation of convergence and divergence; conic sections as implicitly defined curves; the six trigonometric functions and their inverses; applications of trigonometry such as modeling periodic phenomena, modeling with vectors and parametric equations, solving oblique triangles in contextual situations, graphing in the Polar Plane; solutions of trigonometric equations in a variety of contexts; and the manipulation and application of trigonometric identities.

Topics should be analyzed in multiple ways, to include verbal and written, numerical, algebraic, and graphical presentations. Instruction and assessment should include the appropriate use of technology. Concepts should be introduced and investigated, where appropriate, in the context of realistic phenomena.

Prerequisite:

This course is designed for students who have successfully completed *Advanced Algebra / Algebra II.*

Georgia's K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL (QUANTITATIVE) REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

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The eight course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
PC.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.
PC.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
PC.FGR.2: Analyze the behaviors of rational and piecewise functions to model contextual mathematical problems.
PC.FGR.3: Utilize trigonometric expressions to solve problems and model periodic phenomena with trigonometric functions.
PC.AGR.4: Manipulate, prove, and apply trigonometric identities and equations to solve contextual mathematical problems.
PC.GSR.5: Analyze the behaviors of conic sections and polar equations to model contextual mathematical problems.
PC.AGR.6: Represent and model vector quantities to solve problems in contextual situations.
PC.PAR.7: Demonstrate how sequences and series apply to mathematical models in real-life situations.

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MATHEMATICAL MODELING		
PC.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.		
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)
PC.MM.1.1	Explain contextual, mathematical problems using a mathematical model.	***Fundamentals*** • Students should be provided with opportunities to learn mathematics in the context of real-life problems. • Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).
PC.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.	***Fundamentals*** • Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.
PC.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation.	
PC.MM.1.4	Use various mathematical representations and structures with this information to represent and solve real-life problems.	

FUNCTIONAL & GRAPHICAL REASONING – *Functions and their Characteristics*		
PC.FGR.2: Analyze the behaviors of rational and piecewise functions to model contextual mathematical problems.		
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)
Analyze piecewise-defined functions using different representations.		
PC.FGR.2.1	Graph piecewise-defined functions, including step functions and absolute value functions.	***Fundamentals*** • Students should be able to model real-life problems with piecewise-defined functions that incorporate linear, polynomial, logarithmic, exponential, and radical functions.
PC.FGR.2.2	Describe characteristics by interpreting the algebraic form and graph of a piecewise-defined function.	***Strategies and Methods*** • Students should be able to identify characteristics including domain, range, continuity, end behavior, intercepts, and intervals of increase and decrease.
PC.FGR.2.3	Represent the limit of a function using both the informal definition and the graphical interpretation in the context of piecewise-defined functions; interpret limits expressed in analytic notation.	***Fundamentals*** • Students should be able to determine if a limit exists or not and use appropriate limit notation. • Students should have opportunities to use one-sided limits to investigate continuity at a point.

		• Students are not expected to know nor derive the formal definition of a limit.
Analyze rational functions using different representations.		
PC.FGR.2.4	Divide polynomials using various methods.	***Strategies and Methods*** • Students should be provided opportunities with factoring and simplification, long division, and synthetic division. ***Fundamentals*** • Students should be able to connect the Remainder Theorem and Factor Theorem to the division process.
PC.FGR.2.5	Graph rational functions and identify key characteristics.	***Fundamentals*** • Students should have gained experience graphing and analyzing rational functions in the Advanced Algebra course. ***Strategies and Methods*** • Students should have opportunities to graph rational functions with and without the use of technology. • Students should be able to identify characteristics including zeros, asymptotes, domain, range, intercepts, intervals of increase and decrease, relative extrema, symmetries, discontinuities, and end behavior. • Students should be proficient using both interval and inequality notation to report key characteristics.
PC.FGR.2.6	Represent the behavior of a rational function using limit notation for vertical and horizontal asymptotes and end behavior.	***Examples*** • $\lim_{x \to 3^+} \frac{x-1}{x-3}$ for vertical asymptote. • $\lim_{x \to \infty} \frac{x-1}{x-3}$ for horizontal asymptote (end behavior). • $\lim_{x \to \infty} \frac{x^2-1}{x-3}$ for end behavior.
PC.FGR.2.7	Represent the limit of a function using both the informal definition and the graphical interpretation in the context of rational functions; interpret limits expressed in analytic notation.	***Fundamentals*** • Students are not expected to know nor derive the formal definition of a limit.
PC.FGR.2.8	Solve simple rational equations in one variable, and give examples showing how extraneous solutions may arise.	
PC.FGR.2.9	Perform partial fraction decomposition of rational functions using non-repeated linear factors.	***Relevance and Application*** • This is a relevant prerequisite skill for students intending to explore concepts of Calculus more deeply.

FUNCTIONAL & GRAPHICAL REASONING – *Trigonometric Relationships and Functions*		
PC.FGR.3: Utilize trigonometric expressions to solve problems and model periodic phenomena with trigonometric functions.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Define and analyze trigonometric relationships.		
PC.FGR.3.1	Use the concept of a radian as the ratio of the arc length to the radius of a circle to establish the existence of 2π radians in one revolution.	***Fundamentals*** • Students explored radian measures in Advanced Algebra and will build upon their understanding of radians with this learning objective. • Students should be able to fluently convert between degree and radian measures. • Students should be given the opportunity to develop the radian measure of the quadrantal angles. • Students should have opportunities to work with radian measures that are in terms of π and those not in terms of π.
PC.FGR.3.2	Utilize right triangles on the unit circle to determine the values of the six trigonometric ratios for $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$. Use reflections of the triangles as reference angles to establish known values in all four quadrants of the coordinate plane.	***Fundamentals*** • Students utilized radian measures corresponding to reference angles $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$ in one counterclockwise revolution in Advanced Algebra through conversions from degrees. • Students should connect the radian angle names on the 17-point unit circle to portions of 2π radians.
PC.FGR.3.3	Define the six trigonometric ratios in terms of *x*, *y*, and *r* using the unit circle centered at the origin of the coordinate plane. Interpret radian measures of angles as a rotation both counterclockwise and clockwise around the unit circle.	***Strategies and Methods*** • Students should utilize the parametric interpretation of the coordinates on the unit circle as (cos(t), sin(t)).
PC.FGR.3.4	Derive the fundamental trigonometric identities.	***Fundamentals*** • Students should be able to derive the fundamental trigonometric identities, including the quotient, reciprocal, and Pythagorean identities.
PC.FGR.3.5	Determine the value(s) of trigonometric functions for a set of given conditions.	***Examples*** • Given a point on the terminal side of an angle in standard position, students should be able to give the exact value of the six trigonometric functions. • Given one trigonometric ratio and the quadrant for the terminal side of an angle, students should be able to give the exact value of the other five trigonometric functions. • Given two trigonometric ratios, students should be able to give the other four trigonometric functions.

Analyze trigonometric functions and their inverses.		
PC.FGR.3.6	Graph and write equations of trigonometric functions using period, phase shift, and amplitude in modeling contexts.	***Strategies and Methods*** • Students should be given opportunities to graph with and without the use of technology. • Students should have the opportunity to construct equations for contexts such as a Ferris Wheel ride, pendulum motion, tides, predator-prey models, sound waves, etc.
PC.FGR.3.7	Classify the six trigonometric functions as even or odd and describe the symmetry.	**Strategies and Methods:** • Students should investigate the properties of the six trigonometric functions using the unit circle and the graphical representations of the trigonometric functions.
PC.FGR.3.8	Restrict the domain of a trigonometric function to create an invertible function and graph the inverse function. Evaluate inverse trigonometric expressions.	***Strategies and Methods*** • Students should relate the characteristics of inverse trigonometric functions to output values given with and without the use of technology.

***ALGEBRAIC & GEOMETRIC REASONING** – Trigonometric Identities and Equations*		
PC.AGR.4: Manipulate, prove, and apply trigonometric identities and equations to solve contextual mathematical problems.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Verify trigonometric identities and solve trigonometric equations.		
PC.AGR.4.1	Apply the fundamental trigonometric identities to simplify expressions and verify other identities.	***Fundamentals*** • Students should be able to use the quotient, reciprocal, and Pythagorean identities.
PC.AGR.4.2	Use sum, difference, double-angle, and half-angle formulas for sine, cosine, and tangent to establish other identities and apply them to solve problems.	***Relevance and Application*** • Students should investigate the connections between the identities as they are derived.
PC.AGR.4.3	Solve trigonometric equations arising in modeling contexts.	***Strategies and Methods*** • Students should extend their understanding of solving trigonometric equations on one counterclockwise revolution of the unit circle from Advanced Algebra. • Students should be given opportunities to represent solutions using the general solution, on a given interval, exact values from the unit circle, and ones obtained with technology. • Students should be given opportunities to investigate the visual idea that solving sin(x) = ½ finds the graphical intersection of y = sin(x) and y = ½. • Students should have the opportunity to investigate contexts such as a Ferris Wheel ride, pendulum motion, tides, predator-prey models, sound waves, etc.

		• Students should have the opportunity to solve trigonometric equations using algebraic techniques such as factoring, root methods, etc.
Apply trigonometry to general triangles.		
PC.AGR.4.4	Prove and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.	***Relevance and Application*** • Students should have the opportunity to investigate surveying problems, resultant forces, etc.
PC.AGR.4.5	Determine the area of an oblique triangle.	***Strategies and Methods*** • Students should use trigonometric area formulas or Heron's Formula.

GEOMETRIC & SPATIAL REASONING *– Conic Sections and Polar Equations*			
PC.GSR.5: Analyze the behaviors of conic sections and polar equations to model contextual mathematical problems.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
Analyze conic sections using different representations.			
PC.GSR.5.1	Identify and graph different conic sections given the equations in standard form.	***Relevance and Application*** • Students should explore circles, parabolas, ellipses, and hyperbolas.	
PC.GSR.5.2	Identify different conic sections in general form and complete the square to convert the equation of a conic section into standard form.	***Relevance and Application*** • Students should explore contexts like orbital paths, whispering galleries, satellite dish, nuclear cooling tower, parabolic hot dog cooker, etc.	
Extend trigonometry to the polar plane.			
PC.GSR.5.3	Define polar coordinates and relate polar coordinates to Cartesian coordinates.	***Strategies and Methods*** • Students should be able to connect the trigonometric function in the Cartesian Plane to the corresponding polar function in the Polar Plane.	
PC.GSR.5.4	Classify special polar equations and apply to contextual situations.	***Relevance and Application*** • Students should investigate circles, cardioids, limaçons, and rose curves.	***Example*** • Students are able to explore and explain the connection to cardioid microphone sound patterns.
PC.GSR.5.5	Graph equations in the polar coordinate plane with and without the use of technology.	***Relevance and Application*** • Students should be able to graph the trigonometric function in the Cartesian Plane using the corresponding polar function in the Polar Plane.	

ALGEBRAIC & GRAPHICAL REASONING – *Vectors and Parametric Equations*			
PC.AGR.6: Represent and model vector quantities to solve problems in contextual situations.			
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)	
Perform operations with vectors in context.			
PC.AGR.6.1	Represent vector quantities as directed line segments; represent magnitude and direction of vectors in component form using appropriate mathematical notation.	***Fundamentals*** • Students should be able to use appropriate and varied symbols for vectors and their magnitudes.	
PC.AGR.6.2	Add and subtract vectors and multiply vectors by a scalar to find the resultant vector.	***Fundamentals*** • Students should be able to perform operations with vectors to find the resultant vector that solves a real-life problem.	
PC.AGR.6.3	Add and subtract vectors on a coordinate plane using different methods.	***Strategies and Methods*** • Students should explore, understand, and explain tail-to-head, component-wise, and the parallelogram law of vector addition.	
PC.AGR.6.4	Solve contextual vector problems, such as those involving velocity, force, and other quantities.		
Model situations with parametric equations.			
PC.AGR.6.5	Sketch the graph of a curve represented parametrically, indicating the direction of motion.	***Relevance and Application*** • Students should be able to graph a pathway like $x(t)=3t$ and $y(t) = t^2 + 5$ that is traced from left to right as t increases and a pathway like $x(t)=-3t$ and $y(t) = t^2 + 5$ that is traced from right to left as t increases.	
PC.AGR.6.6	Apply parametric equations to contextual problems.	***Strategies and Methods*** • Students should identify, represent, and graph circles, ellipses, and hyperbolas in parametric form.	***Examples*** • Students should explore situations like the position on a Ferris Wheel, movement along a curve in the Cartesian Plane, projectile motion, etc.

PATTERNING & ALGEBRAIC REASONING – *Sequences and Series*		
PC.PAR.7: Demonstrate how sequences and series apply to mathematical models in real-life situations.		
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)
Analyze sequences using multiple representations.		
PC.PAR.7.1	Demonstrate that sequences are functions whose domain is the set of natural numbers.	
PC.PAR.7.2	Represent sequences graphically, numerically, and symbolically.	***Strategies and Methods*** • Students should be able to fluently work with representations that are presented numerically, analytically or algebraically, symbolically, and graphically.

PC.PAR.7.3	Determine the limit of a sequence if it exists.	***Strategies and Methods*** • Students should check for convergence or divergence with and without the use of technology.
Analyze series using multiple representations.		
PC.PAR.7.4	Demonstrate that a series is the sum of the sequence and represent series graphically, numerically, and symbolically.	***Fundamentals*** • Students should be able to use sigma notation.
PC.PAR.7.5	Describe the behavior of a series in terms of the limit of its partial sums.	**Relevance and Application** • Students should explore the partial sums using technology. • Students should correctly use limit notation to represent the convergence of the partial sums.
PC.PAR.7.6	Derive and use the sum formula of a finite geometric series to solve contextual problems to model real-life situations.	
PC.PAR.7.7	Derive and use the sum formula of an infinite geometric series to solve contextual problems to model real-life situations.	***Strategies and Methods*** • Students should have opportunities to check for convergence or divergence with and without the use of technology.

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

Mathematical Practices	
PC.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
PC.MP.1	Make sense of problems and persevere in solving them.
PC.MP.2	Reason abstractly and quantitatively.
PC.MP.3	Construct viable arguments and critique the reasoning of others.
PC.MP.4	Model with mathematics.
PC.MP.5	Use appropriate tools strategically.
PC.MP.6	Attend to precision.
PC.MP.7	Look for and make use of structure.
PC.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

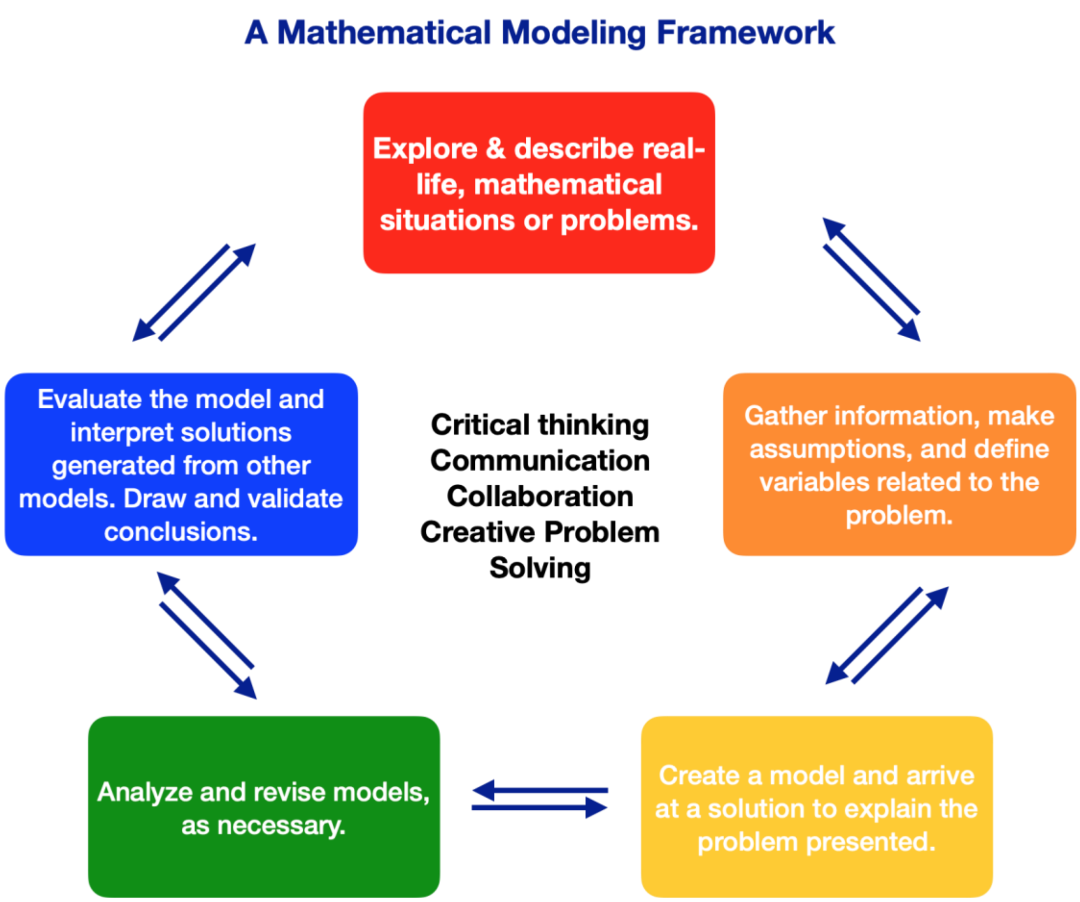

Image adapted from: Suh, Matson, Seshaiyer, 2017

FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

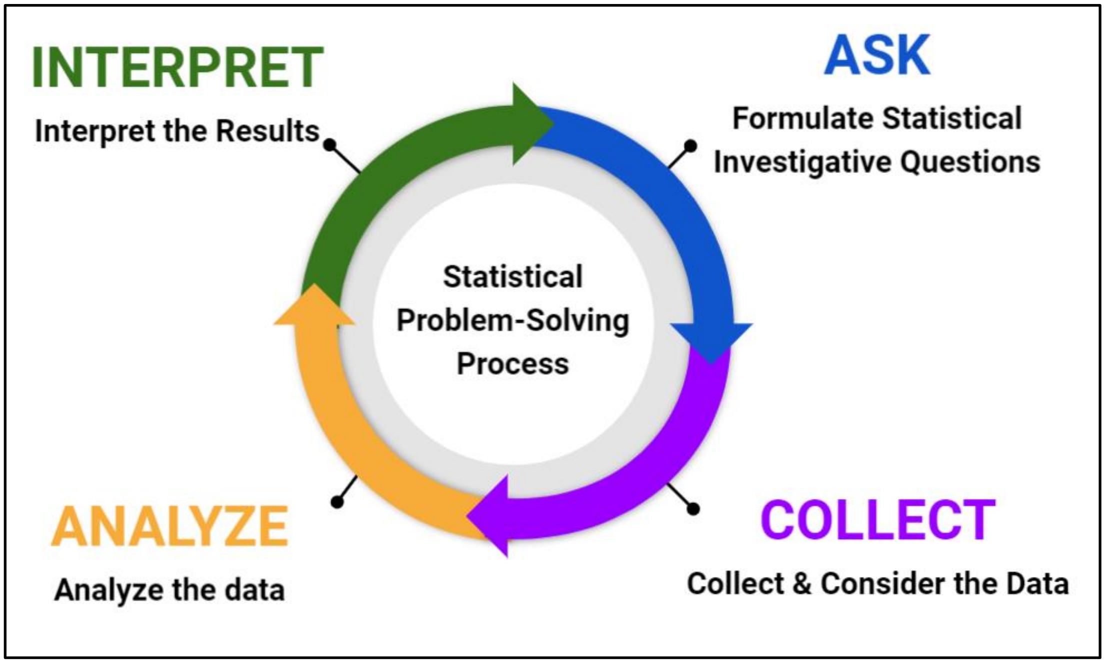

Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. Formulate Statistical Investigative Questions
Ask questions that anticipate variability.

II. Collect & Consider the Data
Ensure that data collection designs acknowledge variability.

III. Analyze the Data
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. Interpret the Results
Answer statistical investigative questions based on the collected data.